A newtonian-vortex cosmology model from solar system to galaxy to large scale structures: Navier-stokes-inspired cosmography

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INTRODUCTION

Some years ago, Matt Visser asked the following interesting questions: How much of modern cosmology is really cosmography? How much of modern cosmology is independent of the Einstein equations? (Independent of the Friedmann equations?) These questions are becoming increasingly germane as the model’s cosmologists use for the stress-energy content of the universe become increasingly baroque. Therefore, in this paper we will discuss a novel Newtonian cosmology model with vortex, which offers wide implications from solar system, galaxy modeling up to large scale structures of the Universe. The basic starting point is very simple: It has been known for long time that most of the existing cosmology models have singularity problem. Cosmological singularity has been a consequence of excessive symmetry of flow, such as “Hubble’s law.” More realistic one is suggested, based on Newtonian cosmology model but here we include the vortical-rotational effect of the whole Universe. We review an Ermakov-type equation obtained by Nurgaliev, and solve the equation numerically with Mathematica. A potential application is also considered, namely for understanding tornado dynamics using 3D Navier-Stokes equations. It is our hope that the new proposed method can be verified with observations, in order to open new possibilities of more realistic nonlinear cosmology models.

Key Words: Ermakov-type equation; Nonlinear cosmology; Newtonian cosmology; Vortex dynamics, turbulence; Navier-Stokes equations; Spiral galaxy

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results which may be comparable with the observed orbits of planetoids beyond Pluto, including what is dubbed as Sedna (7). And it seems that the proposed model is slightly better compared to Nottale-Schumacher’s gravitational Schrödinger model and also Titius-Bode’s empirical law.

SPIRAL GALAXY MODEL

In this section, we discuss a simple model of galaxies based on a postulate of turbulence vortices which govern the galaxy dynamics. Abstract of Vatistas’ paper told clearly (8).

Expanding our previous work on turbulent whirls (1) we have uncovered a similarity within the similarity shared by intense vortices. Using the new information we compress the tangential velocity profiles of a diverse set of vortices into one and thus identify those that belong to the same genus. Examining the Laser Doppler Anemometer (LDA) results of mechanically produced vortices and radar data of several tropical cyclones, we find that the uplift and flattening effect of tangential velocity is a consequence of turbulence. Reasoning by analogy we conclude that turbulence in the interstellar medium could indeed introduce a flattening effect in the galactic rotation curves.

The result of his model equation can yield prediction which is close to observation (without invoking dark matter hypothesis), as shown in the following diagram (Figure 1):

\[
\frac{\dot{R}}{R^3} = \frac{GM}{R^2}, \quad (2)
\]

or

\[
\frac{\ddot{R}}{R^2} = -\frac{\dot{H}}{R}, \quad (3)
\]

Equation (3) may be written as Ermakov-type nonlinear equation as follows;

\[
\frac{\ddot{R}}{R^2} = -\frac{\dot{H}}{R}, \quad (4)
\]

Nurgaliev tried to integrate equation (3), but now we will solve the above equation with Mathematica 11. First, we will rewrite this equation by replacing GM=A, K^2=B, so we get:

\[
\frac{\ddot{R}}{R^2} = -\frac{\dot{H}}{R}, \quad (5)
\]

As with what Nurgaliev did in (1,2), we also tried different sets of A and B values, as follows:

(A) A < 0, B > 0; A=-10; B=10; ODE=x''[t]+A/x[t]^2-B/x[t]^3==0; sol=ND

Solve[{ODE,x[0]==1,x'[0]==1},x[t],{t,-10,10}]; Plot[x[t]/.sol,{t,-10,10}].

(B) A < 0, B > 0; A=-10; B=10; ODE=x''[t]+A/x[t]^2-B/x[t]^3==0; sol=ND

Solve[{ODE,x[0]==1,x'[0]==1},x[t],{t,-10,10}]; Plot[x[t]/.sol,{t,-10,10}].

Therefore, it appears possible to model galaxies without invoking numerous ad hoc assumptions, once we accept the existence of turbulent interstellar medium. The model is also governed by Navier-Stokes equations (8).

DERIVING ERMAKOV-TYPE EQUATION FOR NEWTONIAN UNIVERSE WITH VORTEX

It has been known for long time that most of the existing cosmology models have singularity problem. Cosmological singularity has been a consequence of excessive symmetry of flow, such as “Hubble’s law”. A more realistic one is suggested, based on Newtonian cosmology model but here we include the vortical-rotational effect of the whole Universe.

In this section, we will derive an Ermakov-type equation following Nurgaliev (2). Then we will solve it numerically using Mathematica 11.

After he proceeds with some initial assumptions, Nurgaliev obtained a new simple local cosmological equation (1)

\[
\dot{H} + H^2 = \frac{\omega^2}{3} + \frac{4\pi G}{3} \rho, \quad (1)
\]

Where Here, \( \dot{H} = dH / dt \), stand for Hubble constant, Newtonian gravitational constant, angular speed, and density, respectively.

The angular momentum conservation law \( \omega R = const = K \) and the mass conservation law \( (4\pi/3) pR^3 = const = M \) makes equation (1) solvable: (2)

\[
\frac{\dot{H}+H^2}{R^2} = \frac{\frac{K^2}{R^2} - \frac{GM}{R^2}}{3}, \quad (2)
\]

or

\[
\frac{\ddot{R}}{R^2} = -\frac{\dot{H}}{R}, \quad (3)
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Solve[{ODE,x[0]==1,x'[0]==1},x[t],{t,-10,10}]; Plot[x[t]/.sol,{t,-10,10}].

(C) A > 0, B < 0; A=1; B=-10; ODE=x''[t]+A/x[t]^2-B/x[t]^3==0; sol=ND

Solve[{ODE,x[0]==1,x'[0]==1},x[t],{t,-10,10}]; Plot[x[t]/.sol,{t,-10,10}].
The coupled Riccati ODEs read as follows: (11)

\[ \frac{\partial u}{\partial t} + \left( u \cdot \nabla \right) u = -\frac{\nabla p}{\rho} + v \cdot \nabla^2 u + \mathbf{F}, \]

where \( u \) is the flow velocity, a vector field; \( \rho \) is the fluid density, \( p \) is the pressure, \( v \) is the kinematic viscosity, and \( \mathbf{F} \) represents external force (per unit mass of volume) acting on the fluid (11).

In general case, such a solution should be obtained for the Navier–Stokes equations in the case of incompressible flow, which was suggested earlier. In fluid mechanics, there is an essential deficiency of the analytical solutions of Navier–Stokes equations for 3D case of non-stationary flow. The Navier–Stokes system of equations for incompressible flow of Newtonian fluids should be presented in the Cartesian coordinates as below (under the proper initial conditions): (11)

\[ \nabla \cdot u = 0, \]

\[ \frac{\partial u}{\partial t} + \left( u \cdot \nabla \right) u = -\frac{\nabla p}{\rho} + v \cdot \nabla^2 u + \mathbf{F}, \]

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The solutions obtained here open up new ways to interpret existing solutions of known 3D Navier-Stokes equations numerically. It is our hope that the above numerical solution of 3D Navier-Stokes equations can be found useful for engineering purposes, such as controlling large hurricanes which happen quite often in various regions each year.

**ENGINEERING APPLICATION: HURRICANE DYNAMICS AND SOLUTION OF 3D NAVIER-STOKES**

Various methods to describe hurricane dynamics have been proposed in the literature, but most of them are based on 3D Navier-Stokes. Some existing models of tornado dynamics can be found in (9,10).

Now, we will discuss a simplified numerical solution of 3D Navier-Stokes equations based on Sergey Ershkov’s papers (11,12).

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In (11), Ershkov explores the ansatz of derivation of non-stationary solution for the Navier-Stokes equations in the case of incompressible flow, which was suggested earlier. In general case, such a solution should be obtained from the mixed system of \( 2 \) coupled Riccati ordinary differential equations (in regard to the time-parameter \( t \)). But instead of solving the problem analytically, we will try to find a numerical solution (12-15).

The coupled Riccati ODEs read as follows: (11)

\[ a' = \frac{w_r}{2} a^2 - (w_z \cdot b) \cdot a - \frac{w_r}{2} (b^2 - 1) + w_z \cdot b \]

\[ b' = -\frac{w_z}{2} b^2 - (w_z \cdot a) \cdot b - \frac{w_z}{2} (a^2 - 1) + w_z \cdot a. \]

First, equations (8) and (9) can be rewritten in the form as follows:

\[ x(t) = \frac{v}{2} t \cdot y(t) - \frac{v}{2} \cdot y(t) \cdot x(t) - \frac{v}{2} (y(t)^2 - 1) + w \cdot y(t), \]

\[ y(t) = -\frac{u}{2} t \cdot y(t) - \frac{v}{2} \cdot x(t) \cdot y(t) - \frac{u}{2} (x(t)^2 - 1) + w \cdot x(t). \]

Then we can put the above equations into Mathematica expression:

\[ v=1; u=1; w=1; \quad \text{Flatten[NDsolve[\{x'[t] = (v/2)*x[t] + u*y[t] + w, \quad y'[t] = -u/2*t + w\}, \quad \{x[0], y[0] \} = \{1, 0 \}, \quad x[t], y[t], \{t, 0, 10 \}] \].

The result is as shown below Figure 2.

![Figure 2](image-url)


