RESEARCH

A probabilistic proof of the multinomial theorem following the number A_n^p

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ABSTRACT

In this note, we give an alternate proof of the multinomial theorem following the number A_n^p using probabilistic approach. Although the multinomial theorem following the number A_n^p is a combinatorial result, our proof may be simple for a student familiar with only basic probability concepts.

Key words: *Probabilistic Proof; Multinomial Theorem; Probability; Number*

INTRODUCTION

The following multinomial theorem based on number n^p (development based on a power of a number) is an important result with many applications in mathematics statistics and computations. The theorem states as follows:

Theorem 1

Let n and m be non-zero natural numbers, $x_1, x_2, ..., x_m$ real numbers:

$$(x_1 + x_2 + ... + x_m)^n = \sum_{\sum_{i=1}^m k_i = n} \frac{n!}{k_1!k_2!...k_m!} x_1^{k1} x_2^{k2} ... x_m^{km} \quad (1.1)$$

where k_i are natural integers.

For readers interested, they can see reference for further interpretation [1]. Recently, we have published another type of multinomial theorem based on numbers A_n^p and given some applications in the case of binomials [2]. (see [1] for more details). In each case, the first demonstrations are based on a proof by induction using the binomial formula. A. Rosalsky proposed a probabilistic approach to this proof in the case of binomials which will be generalized to the multinomial theorem by Kuldeep Kumar Kataria [3-4]. An urn contains x_1 balls numbered 1, x_2 balls numbered 2, x_m balls numbered m, such that the total number of balls is $N = \sum_{i=1}^m x_i$. Consider an experiment where we draw a ball from the urn without replacement, and note the number on it each time. By repeating this experiment n times [5-6]. The probability mass function of the variables $X_1, X_2, ..., X_m$ is:

$$P[X_1 = k_1, X_2 = k_2, ..., X_m = k_m] = n! \prod_{i=1}^m \frac{P_i^{k_j}}{k_j!} \quad (1.2)$$

Where, $N = \sum_{i=1}^m k_i = n$ and $P_i^{k_j}$ the probability of having the balls numbered $i\ k_j$ times.

From (1.2) we have:

$$1 = \sum_{\sum_{i=1}^m k_i = n} n! \prod_{i=1}^m \frac{P_i^{k_j}}{k_j!} \quad (1.3)$$

Next, we will establish and prove the multinomial theorem following the number A_n^p.

A PROBABILISTIC PROOF OF THE MULTINOMIAL THEOREM FOLLOWING THE NUMBER A_n^p

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Theorem 2

Let m and n be two non-zero natural numbers and $x_1, x_2, ..., x_m$ natural numbers. Then,

$$A^n_{(x_1, x_2, ..., x_m)} = \sum_{\Sigma_{i=1}^m k_i = n} \frac{n!}{k_1!k_2!...k_m!} A^{k_1}_{x_1} A^{k_2}_{x_2} ... A^{km}_{x_m}$$

Where k_i are non-negative integers, $k_i \le x_i$ and

$$A^{ki}_n = k_i ! C^{ki}_n = k_i ! \binom{n}{k_i}$$

Proof: Let us consider:

$$A^n_{(x_1 + x_2 + ... + x_m)} = (x_1 + x_2 + ... + x_m)(x_1 + x_2 + ... + x_m - 1)...(x_1 + x_2 + ... + x_m - n + 1)$$

Using the distributives property without resuming the number on the right side of the equation, it follows that for any natural numbers x_i we have :

$$A^n_{(x_1 + x_2 + ... + x_m)} = \sum_{\Sigma_{i=1}^m k_i = n} C_n^{k_1, k_2, ..., k_m} A^{k_1}_{x_1} A^{k_2}_{x_2} ....A^{k_m}_{x_m} \quad (2.1)$$

Where $C_n^{k_1, k_2, ..., k_m}$ are positive integers and x_i are non-negative integer's satisfying $\sum_{i=1}^{m} k_i = n$. We just need to show that.

$$C_n^{k_1, k_2, ..., k_m} = \frac{n!}{k_1!k_2!...k_m!} \quad (2.2)$$

We have $n \ge x_i$ for $i = 1, 2, ..., m$ Let's put:

$$P_i(j) = \frac{x_i^j}{x_1 + x_2 + ... + x_m - j + 1} \quad (2.3)$$

Where x_i^j i is the remaining number of the balls numbered i before the jth draw. $0 \le p_i(j) \le 1$ substituting (2.3) in (1.3) we obtain

$$A^n_{(x_1 + x_2 + ... + x_m)} = \sum_{\Sigma_{i=1}^m k_i = n} \frac{n!}{k_1!k_2!...k_m!} A^{k_1}_{x_1} A^{k_2}_{x_2} ...A^{km}_{x_m} \quad (2.4)$$

finally the subtraction of (2.4) from (2.1) gives:

$$\sum_{\Sigma_{i=1}^m k_i = n} \left(C_n^{k_1, k_2, ..., k_m} - \frac{n!}{k_1!k_2!...k_m!}\right) A^{k_1}_{x_1} A^{k_2}_{x_2} ...A^{km}_{x_m} = 0 \quad (2.5)$$

$A^{k_i}_{xi} \ge 0$ since (2.5) is a zero polynomial in m variables, (2.2) follows and the proof is complete.

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