SHORT COMMUNICATION

A Proof of Collatz's Conjecture

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INTRODUCTION

The proof of Collatz conjecture is divided in to three parts. In the first part it is proved for the set of numbers {1, 5, 21, 85, ---}. In the second part, using the above it is proved for odd numbers of the form (4k+1). In the third part it is shown how numbers of the form (4k+3) iterates in to numbers of the form (4k+1). Thus the conjecture is proved for all odd 'n'. As even numbers iterate into odd numbers, the conjecture is proved for all positive integers [1-3].

DESCRIPTION

Consider the following equation for a positive integer 'n'.

n (n, 1)=n/2, if 'n' is even and=(3n+1) if 'n' is odd. let f (n,r)=f [f(n,r-1), 1] f or n ≥ 2.

Now consider the set A(n)={f(n, 1), − − − − − − − −, f(n, r), − − − − − −−} Consider an imaginary set B (n)={f (n, 1), − − − − ,1, 4, 2, 1, 4, 2, 1, −−} According to Collatz's conjecture, both the sets are identical. That is A(n)=B(n).

Therefore, A(n)=B(n), means Collatz's conjecture is true for 'n'. It is obvious that Collatz's conjecture has to be proved for odd value of 'n', as for even value, it will iterate in to an odd number and ultimately ending in the odd number '1'.

Part 1: An analysis of numbers 1, 5, 21, 85, etc.

Consider the number n=[4r−1 + 4r−2 + 4r−3 + − − − − − − − +1] f (n, 1)=(4 −1) [4r−1 + − − − − − − − − + 1] + 1=4r Therefore, f (n, 2r+1)=1. Hence A(n)=B(n). Thus, a set s1={1, 5, 21, 85, 341, − − − − − − − −−} is generated. s1={sr: sr=(4r−1)/3, r ≥ 1}.

This set is the 'primary set', because every other number iterates into a member of this set greater than '1' before iterating into '1' [4-7].

Part 2: An analysis of n=4k+1

As only odd value of 'n' is analyzed it can be divided into two forms, that is, n=4k+1 and n=4k+3. An analysis of n=4k+3 is dealt in part 3. But, part 2 and part 3 are interconnected. Consider, n=4k+1 [8-10].

f (n, 1)=f [(4k+1), 1]

=12k+6

f (n, 2)=6k+3

f (n, 3)=3k+1

If 'k' is odd, then f (k)=3k+1.

Or if 'n' is an odd number and A(n)=B(n),

Then, A(4n+1)=B(4n+1).

This is an important result and it can even derive the primary set.

S1={1, 4 + 1, 5 (4) + 1, 21 (4) + 1, − − − − −−}

S1={1, 5, 21, − − − − − − − − − − − −−}, similarly

S3={3, 13, 53, − − − − −−}

S7={7, 29, 117, − − − − −}

S9={9,37,149, − − − − −} or in general

S2n+1={2n + 1, 8n + 5, 32n + 21, − − −}.

These sets are important in the understanding of the problem. In these sets, except the first member, all the other members are, obviously, of the (4k+1) form. Consider the numbers below 21.

The proof for (3, 11, 19,---); (7, 23,--); (15,47-) etc are given in part-3. What is left out is '9 and 17' which are of the (8k+1) form, which iterate into (6k+1), which is of (4k+1) or (4k+3) form. Example is '9' and it is an interesting number, as it iterates into 7, 11, 17, 13, 5 and to '1'.

If 'n' is odd and A(n)=B(n), then A(4n+1)=B(4n+1)

The converse is also true.

If 'n' is odd and A(4n+1)=B(4n+1), then A(n) B(n)

Now, assume that, for n ≤ (4k+1), A(n)=B(n)

Consider n1=4(k+1)+1

If (k+r) is odd and (k+r) ≤ (4k+1), then

A(k+r)=b(k+r) and so A(n1)=B(n1)

If (k + r) is even and 3(k+r)+1 ≤ (4k+1), then A(n1)=B(n1)

That is, if, 3r ≤ k, A(n1)=b(n1)

'k' is odd, 'r' is even and so 'k+r' is odd.

1) k=1, therefore (k + r) ≤ 5

hence, (k + r)=3, 5 and so, A(3)=B(3), A(13)=B(13)

A(5)=B(5) and A(21)=B(21)

2) K=3, Therefore (k + r) ≤ 13. hence, (k + r)=5, 7, 9, 11, 13

3) K=5, therefore (k + r) ≤ 21

hence (k+ r) = 7, 9, 11, 13, 15, 17, 19, 21 and so on.

'k' is even, 'r' is odd and so 'k + r' is

odd. k=2, therefore (k + r) ≤ 9

hence, (k + r)=3, 5, 7, 9.2)

k=4, therefore (k + r) ≤ 17

hence, (k + r)=9, 11, 13, 15, 17

3) k=6, therefore (k + r) ≤ 25

hence (k + r) = 17, 19, 21, 23, 25

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4) k=8, there fore $(k + r) \leq 33$

hence, $(k + r)=25, 27, 29, 31, 33$ and so on.

These generates the set, k1={1, 3, 5, 7, − − − − − − − −}

k1={k: k=2s + 1, S∈ N}

'k' is odd, 'r' is odd & so 'k + r' is even

A(n1)=B(n1), if $3r \geq k$

Hence, for, k=3, 5, 7; r=1 and (k +r)=4, 6 ,8

for, k=9, 11, 13; r=1, 3 and (k + r)=10, 12, 14, 16. and so on

these generate the set K2={2S and S ∈ N}

(K=0, 2 is included, as it can be verified).

The set, k=k! u k2={K : K ∈ N}

This set 'k' generates the set,

N (2 2)={1, 5, 9, 13, − − −−}

={n2: n2=(4k+1), k ∈ N}

A(n2)=B(n2)

It may be noted that in the set 'K1', A(K)=B(k). Therefore, the set 'K1' itself proves Collatz's conjecture. But, however part 3 is important, as the necessary connectivity is provided by it [11,12].

Part 3: An analysis of n=(4k+3)

Though it is assumed, n=4k+1, is solved, 4k+1 and 4k+3 are interconnected. It will be shown how (4k+3) becomes (4k+1) before becoming '1'.

let, n=(4k+3); 'k' odd or even

let, k=2k3 or 2k3+1

therefore, n3=(8k3+3) or n31=(8k3+7)

(1) n3=(8k3+3) So, f (n3, 1)=f (2 3k3+2 2−1, 1)=3. 2 3k3 + 2 3+2 2−3+1. f (n3, 2)=3. 2 2k3+2 2+2−1=4(3k3+1)+1 therefore, A(n3)=B(n3)

thus the following set is generated.

Hence

N(2 3)={ 3, 11, 19, − − −−}={n3: n3=(2 3k+3), k ≥ 0 }

n31, can be odd or even

there fore let , n4=2 4k4 7 and n41=2 4k4+15

(2) n4=2 4k4+2 3−1.

f(n4, 1)=3. 2 4k4+2 4+2 3−3+1=2 4 (3k4+1)+2 3−2

f(n4, 2)=2 3 (3k4+1)+3 .

f(n4, 4)=2 2 (3 2k4+3+1)+1

Therefore, A(n4)=B(n4)

Thus the following set is generated

N(2 4)={7,23,39,~}={n4: n4=(2 4k4+7), k ≥ 0}

In, n41, k4 can be odd or even

Therefore, let, n5=25k5 + 15 and n51=2 5k5 + 31

(3) n5=2 5k5 + 2 4-1

f(n5,6)=4(3 3k5 + 3 2+3+1)+1

Therefore A(n5)=B(n5)

Thus the following set is generated. N (2 5)={15,47,79,~}={n5: n5=(2 5k=15,), k ≥ 0.}~~~~~~.

Let, nr=2 rkr + 2 r-1-1 and nr1=2 rkr + 2r -1

(r-2): - nr=2 rkr + 2 r-1 -1

F(nr, 2r -4)=4 (3 r-2kr + 3r- 4 +~+1)+1

Therefore A(nr)=b(nr)

Thus the following set is generated

N (2r)={ (2 r-1 - 1),~, (2 r-1-1 + k2 r),~}

N(2 r)={nr : nr=(2 rk + 2 r-1-1), k ≥ 0}

CONCLUSION

All these sets put together constitutes the master set N (2)

N (2) ≡ {N (2 2) UN (2 3) U~UN (2 r)~: r⟶∞}

The set, N (2)={1,3,5,~}={n2:n2= (2k+1), k ≥ 0}

And A (n2)=B (n2)

The above set represents all odd numbers. All even numbers tend to an odd number.

Therefore, A(n)=B(n) and n≡N

This, Collatz's conjecture, is proved for all positive integers.

REFERENCES

1. Nambu, U. Model and the factorization of the eneziano amplitude. Roen Symmetry: Selected papers of Nambu. 1999;1:2-8.
2. Nielsen. An almost physical interpretation of the dual N point function. Nordita report unpublished. 19:9.
3. Sussind. Armonic-oscillator analogy for the veneziano model. Phys Rev. 1992;5(10):4.
4. Sussind. Structure of hadrons implied by duality. Phys Rev D. 1901;4:1182.
5. Ramond P. Dual theory for free fermions. Phys Rev D. 1991;2(10):241.
6. Neveu A, Schwarz J. Tachyon-free dual model with a positive-intercept traectory. Phys Lett. 1999;4:1-8.
7. Scher J, Schwarz J. Dual models for non-hadrons. Nuclear Phys. 1994;81(1):118-144.
8. Oneya T. Connection of dual models to electrodynamics and gravidynamics. Prog Theor Phys. 1999;1:190 -192.
9. Das SR. Degrees of freedom in two dimensional string theory. Nuclear Phys -Proceed Suppl. 1999;4(2-6):224-228 .
10. Schwarz J. An S (2 Z) multiplet of type II superstrings. Phys Lett. 1991;0(1-2):1-8.
11. Palan P. Duality in statistical mechanics and string theory. Phys Rev Lett. 1998;8(1):19.
12. Itten E. Five-branes and M-theory on an orbifold. Nuclear Phys. 1999; 4(2):8-9.