A set of formulas for prime numbers

Daoudi Rédoane

Rédoane D. A set of formulas for prime numbers. J Pure Appl Math. 2023; 7(6):01-2.

ABSTRACT

Here I present several formulas and conjectures on prime numbers. I'm interested in studying prime numbers, Euler's totient function and sum of the divisors of natural numbers.

Key words: *Prime numbers; Sequence; Formula; Divisor function; Euler's totient*

INTRODUCTION

Let σ(*n*) denotes the divisor function which sums the divisors of *n*, an integer ≥ 1. We introduce the function *f* such that:

Formula 1

$$f(n) = 1 + (n!)^2 - \sigma(n!)(n!)^2 + 2\sum_{k=1}^{-1+\sigma(n!)} \left\lfloor \frac{k(1+(n!)^2)}{\sigma(n!)} \right\rfloor$$

When $f(n) = 2n + 1$, *is* $2n + 1$ always prime?

And for example for $n = 6$ we have the prime number 13 that is of the form 2(6) + 1.

The first examples are given by the following sequence:

1, 1, 1, 1, 1, 13, 1, 17, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 61, 1, 1, 1, 1, 1, 1, 1, 1,

61, 1, 1, 1, 193, 1, 1, 1, 757, 61, 1, 1, 1, 1, 1, 1,1, 1,1,1,1, 1, 1, 1, 109, 1, 1,

1, 181, 1,1, 1, 1,1, 1, 1, 1, 1, 1, 1, 1, 1,1, 1, 1, 1,1, 1, 1, 1, 1, 1, 1, 1, 1, 113...

Carl Schildkraut proved this property [1].

Let $\{x\} = x - \lfloor x \rfloor$, let $m = n!$, and let $t = \sigma(m)$. Then

$$2\sum_{k=1}^{t-1} \left\lfloor \frac{k(1+m^2)}{t} \right\rfloor = \sum_{k=0}^{t-1} \frac{2k(1+m^2)}{t} - 2\sum_{k=1}^{t-1} \left\{ \frac{k(1+m^2)}{t} \right\};$$

the first sum is $(1 + m^2)(t - 1)$, and so

$$f(n) = 1 + m^2 - m^2 t + (1 + m^2)(t - 1) - 2\sum_{k=0}^{t-1} \left\{ \frac{k(1+m^2)}{t} \right\}$$

$$= t - 2\sum_{k=0}^{t-1} \left\{ \frac{k(1+m^2)}{t} \right\}.$$

Let $u = gcd(1 + m^2, t)$. The t values $\{0, 1 + m^2, 2(1 + m^2), \ldots, (t - 1)(1 + m^2)\}$ modulo t consist of u copies of each multiple of u in $[0, t)$, and so

$$\sum_{k=0}^{t-1} \left\{ \frac{k(1+m^2)}{t} \right\} = u \sum_{j=0}^{\frac{t}{u}-1} \frac{uj}{t} = \frac{t-u}{2}.$$

This means

$$f(n) = t - 2\frac{t-u}{2} = \gcd(1 + (n!)^2, \sigma(n!)).$$

(In particular, if $1 + (n!)^2$ and $\sigma(n!)$ are coprime, $f(n) = 1$.)

With this knowledge about $f(n)$, we can tackle the problem at hand. If $f(n) = 2n + 1$, then, in particular, $2n + 1$ divides $1 + (n!)^2$. So, $2n + 1$ is relatively prime to n!. This means that $2n + 1$ cannot have any factors in the set $\{2, \ldots, n\}$. However, every number in $\{n + 1, \ldots, 2n\}$ is too large to be a factor of $2n + 1$. So, $2n + 1$ cannot have any factors strictly between 1 and $2n + 1$, and must be prime.

Formula 2

Let x denotes an integer such that $x > 1$. We define the function f such that:

$$f(x) = \frac{1}{\pi}\arctan(x)$$

We have:

$$f(x) = \cfrac{1}{a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d + \ldots}}}}$$

(a, b, c, d are integers ≥ 1) We have:

$$\lim_{x \to \infty} \frac{x}{b} = \frac{4}{\pi}$$

University of Caen Normandy, France

Correspondence: Daoudi Rédoane, University of Caen Normandy, France, e-mail: red.daoudi@laposte.net

Received: 12 Nov, 2023, Manuscript No. puljpam-24-6921, Editor Assigned: 15 Nov, 2023, PreQC No. puljpam-24-6921 (PQ), Reviewed: 18 Nov, 2023, QC No. puljpam-24-6921 (Q), Revised: 21 Nov,2023, Manuscript No. puljpam-24-6921 (R), Published: 30 Nov, 2023, DOI:-10.37532/2752-8081.23.7(6).01-2

Formula 3

Let k be a positive integer.

Let n be an integer such that $n = 6k - 1$

Let r be the remainder of the division of $(n - 1)! - n$ by $(n + 2)$

Property: if $6k + 1$ is prime $r = 3k + 2$

We define the prime $6k+1$ such that $6k+1 = r(n)+r(n-1)$ where $r(n)$ is the sequence of the successive remainders with $r(1) = 5$ and $n \geq 2$. We suppose $r(n) \neq 2$ and $r(n-1) \neq 2$.

For example the first 25 values of r are:

5, 8, 11, 2, 17, 20, 23, 2, 2, 32, 35, 38, 41, 2, 2, 50, 53, 56, 2, 2, 65, 2, 71, 2, 77

And we have:

8 + 5 = 13 = 6(2) + 1

11 + 8 = 19 = 6(3) + 1

20 + 17 = 37 = 6(6) + 1

23 + 20 = 43 = 6(7) + 1

35 + 32 = 67 = 6(11) + 1

38 + 35 = 73 = 6(12) + 1

41 + 38 = 79 = 6(13) + 1

53 + 50 = 103 = 6(17) + 1

56 + 53 = 109 = 6(18) + 1

Formula 4

Let a to be a natural number $(a \geq 1)$, $n = 4 * m$ where m is a natural number $\geq 1)$ and ϕ is the Euler's totient function such as:

$$\phi(n) = n\prod_{p|n}\left(1 - \frac{1}{p}\right)$$

Prove that if $\phi(a^n - 2) + 1 \equiv n - 1 \pmod n$ then $\phi(a^n - 2) + 1$ is always a prime number.

Max Alekseyev studied this conjecture but no proof has been found [2].

Formula 5

(a, b) is a couple of twin primes such that $b = a + 2$ and $a > 29$. Let $N = 4^b$ and q the quotient which results from the division of N by a and r is the remainder. We calculate $P = (q \bmod b)a + r - 1$ Below we prove that $P = 3(10b + 1)$ using Fermat's little theorem. $N = 16 \cdot 4^a \equiv 64 \pmod a$ then $r = 64$ $(a > 64)$ $4^b = (b-2)q + 64$ then $4 \equiv 64 - 2q \pmod b$ and $q \equiv 30 \pmod b$ Finally we have $P = 30a + 63 = 3(10b + 1)$

Formula 6

n is a natural number > 1, $\varphi(n)$ denotes the Euler's totient function, Pn is the n^{th} prime number and $\sigma(n)$ is the sum of the divisors of n. Consider the expression:

$F(n) = \varphi(|P_{n+2} - \sigma(n)|) + 1$

Conjecture: when $F(n) \equiv 3 \pmod{20}$ then this number is a prime or not. When the number is not a prime it can be a power of prime by calculating $|P_{n+2} - \sigma(n)| = p^k$ (p prime, k a natural number > 1).

Examples:

We have n = 680:

$F(680) = \varphi(|P_{682} - \sigma(680)|) + 1 = \varphi(5101 - 1620) + 1 = 3423$

which is not prime but we have $P_{n+2} - \sigma(n) = p^2$, more precisely it is the square of 59.

Interestingly for $n \leq 526388126$ (calculations with PARI/GP) all counterexamples are the power of prime.

Another example is found for k = 6, this is n = 526388126. In this case, we have:

$F(n) = 10549870323$

which is not prime and $|P_{n+2} - \sigma(n)| = 47^6$ (here k = 6).

The question is: "Are there only these two solutions? 1. A power of prime if the result is not a prime 2. Or the result is prime

REFERENCES

1. Carl Schildkraut (https://math.stackexchange.com/users/253966/carl-schildkraut), Primes of the form 2n +1, URL (version: 2022-06-23 https://math.stackexchange.com/q/4480961
2. Max Alekseyev (https://math.stackexchange.com/users/147470/max-alekseyev), Euler'stotient function and primes, URL (version: 2022-06-23): https://math.stackexchange.com/q/4478910