Classification of bifurcation diagrams for semilinear elliptic equations in the critical dimension

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Kumagai K. Classification of bifurcation diagrams for semilinear elliptic equations in the critical dimension. J Pure Appl Math. 2023; 7(5):306-311.

ABSTRACT

Until the discovery of the accelerated expansion of the universe in the late 1990s, the great majority of physicists were convinced that there was not any kind of sound in any kind of vacuum. The latter specification is necessary because nowadays we discuss whether there is the possibility that sound actually does propagate in space which is free of ordinary matter but filled with "dark energy". The term "vacuum" for such kind of empty space is avoided by most authors as it remains reserved for models of empty space in which the cosmological constant acts as a source of vacuum energy. Before this discussion started, evidence for vanishing propagation of sound in a vacuum apparently aroused from an experiment designed by Otto von Guericke in the 17th century although his intention was merely to show that a vacuum actually does exist. This article will show that the interpretation of this experiment as a proof for vanishing sound-propagation in a vacuum has been a fundamental philosophical mistake and furthermore that any theoretical investigations of sound in space which is free of any kind of matter may require a quantum-theoretical approach.

Key words: *Experimental epistemology; Sound waves; Dark energy; Elementary particles; Neutrino theory; Neutron stars*

INTRODUCTION

In the 17th century, Newton`s epoch, Otto von Guericke, famous constructor of the "Hemispheres of Magdeburg" showed another experiment to prove that it is possible to achieve a vacuum which was doubtful at that time. The historical setup was a chamber inside of which a source of sound was placed. On evacuation of that chamber the sound disappeared [1].

I want to clearly state that from the negative result of this experiment no conclusion can be drawn about the principal possibility of sound-propagation in a vacuum. Today one should complete the sentence by affirming that also about sound-propagation inside dark energy no conclusion can be drawn from this experiment because we also assume that dark energy is not removed by simple evacuation. Of course, the reverse conclusion is permitted. If sound does not propagate in a vacuum the experiment must yield a negative result. However, the experiment is often shown with the intention to demonstrate that sound does not propagate in a vacuum even by institutions like DESY (Deutsches Elektronen Synchrotron) which of course does not consider it as a part of their current research activities but rather as a didactic show-experiment.

The tentative disproval of phenomena by non-observation is always an epistemological contradiction although there are fortunate historical examples in favor of these attempts. A famous one is the octahedral structure of metal complexes with the formula Ma2b4 which was concluded from the number of obtained isomers despite the possibility that a third isomer could not have been isolated because the substance was not stable enough. A lack of evidence never can be interpreted as a proof for the non-existence of a phenomenon.

The same problem is encountered for the vanishing mass of zero rest mass particles like e.g. photons. An experimental proof that a particle´s mass exactly vanishes is impossible. One can only measure upper limits of masses which because of the experimental precision limit even may vanish exactly [2]. Thus the experimental finding of the neutrinos´ masses does of course show that there are neutrinos in certain circumstances which have mass, but it does not show that different circumstances do not even exist in which there are neutrinos that have no mass.

The historical experiment

In order to show rigorously that the negative result of Otto von Guericke´s experiment does not allow any conclusion in respect to the non-propagation of sound in a vacuum let me first investigate this experiment. Of course, if the experiment had given a positive result, i.e. if outside the evacuated chamber the sound could be heard, this

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Received: *8 August, 2023, Manuscript No. puljpam-23-6724,* ***Editor Assigned:*** *9 August, 2023, PreQC No. puljpam-23-6724 (PQ),* ***Reviewed:*** *19 August, 2023, QC No. puljpam-23-6724 (Q),* ***Revised:*** *20 August,2023, Manuscript No. puljpam-23-6724 (R),* ***Published:*** *30 September, 2023, DOI:-10.37532/2752-8081.23.7(5).306-311*

would have been strong evidence for an actual propagation of sound in a vacuum. However, can we expect to recognize this positive effect from what we know about sound in general?

Firstly, I want to emphasize that there is no need to distinguish between a vacuum and dark energy if we want to investigate the negative result of the historical experiment because we have to assume the most optimistic case in order to show that even in that case the chance of yielding a positive effect is negligible. Nowadays the idea of sound-propagation inside dark energy is widely accepted but I did not find any documented effort to investigate this kind of sound experimentally by direct observation [3].

Although sound waves inside dark energy are supposed to be not only adiabatic changes of state but also non-adiabatic changes of state, for practical purposes I want to restrict the investigation of the historical experiment to the adiabatic case [4].

Adiabatic sound waves correspond to zero rest mass particles which necessarily move with c in a vacuum and several dark energy models assume as well that sound waves propagate with the speed of light inside dark energy [5]. Therefore we also will do so for the investigation of the historical experiment. Finally, I will design a Gedankenexperiment that permits a quantitative evaluation by a slight modification of the historical setup.

Imagine that inside the evacuated chamber is another chamber containing air, inside of which the source of sound is placed. Thus, we can neglect any problem that arises from the question how sound is at all excited inside the evacuated space by the employed source. Instead, we can focus our investigation on the problem of whether a transfer of sound waves across the phase boundary from a material medium into a vacuum, or dark energy respectively, is possible.

The most optimistic assumption for our investigation is now that we consider the vacuum or dark energy respectively, as transparent for sound waves. Absorption as well as diffraction is neglected, and the sum of the reflected and transmitted intensities equals the incident intensity. These propositions allow us to apply the usual equations in order to calculate the sound´s intensity which is transmitted through the evacuated chamber.

These equations distinguish three cases. The first case is that of transmission and reflection, the second case is that of total reflection and the third one is the case of negligible reflection but total transmission [6-8]. The transmitted intensity could be detected outside the evacuated chamber; the reflected intensity heats the content of the inner chamber.

The speed of sound in air is 343 ms-1 for standard conditions and the speed of sound in a vacuum, or the dark energy respectively, is assumed to be the speed of light. The angle of total reflection is given by the arc sine of the ratio of the velocities of propagation in both media, supposing the medium of incidence is that of smaller velocity of propagation:

$$\Theta_t = \arcsin \frac{c_1}{c_2} \tag{1}$$

With c_1 being the speed of sound in air and $c_2 = 3 \cdot 10^8\ \mathrm{ms^{-1}}$ the speed of sound in a vacuum, or inside dark energy respectively, we obtain an angle of $6.5 \cdot 10^{-5}\,°$ or 0.23´´ relative to the vertical onto the phase boundary.

Total transmission is achieved in the case that $c_2 > c_1$ and $\rho_1 c_1 > \rho_2 c_2$ when the angle of incidence fulfils the condition:

$$\Theta_B = \arctan \sqrt{\frac{\rho_2^2 c_2^2 - \rho_1^2 c_1^2}{\rho_1^2 \left(c_1^2 - c_2^2\right)}} \tag{2}$$

With ρ_1 =density of air and ρ_2 =mass density of vacuum energy; The first condition, $c_2 > c_1$, is fulfilled for the values given above. So, now we have to investigate the second condition, $\rho_1 c_1 > \rho_2 c_2$. According to Ostriker and Steinhardt the energy density of the vacuum is about $4\ \mathrm{eV \cdot mm^{-3}}$ [9]. The corresponding mass density calculated by Einstein ´s mass-energy-relation is $\rho_2 = 7.1 \cdot 10^{-27}\ \mathrm{kg \cdot m^{-3}}$. From this value one obtains an acoustic resistance of $\rho_2 c_2 = 2.13 \cdot 10^{-18}\ \mathrm{kgs^{-1}m^{-2}}$ and from the density of air, $1.29\ \mathrm{kg \cdot m^{-3}}$, and the above given value of its speed of sound, the acoustic resistance of air is $\rho_1 c_1 = 428\ \mathrm{kgs^{-1}m^{-2}}$. The second condition, $\rho_1 c_1 > \rho_2 c_2$, is fulfilled, too. Equation two can be simplified considerably because $c_2 >> c_1$ and also $\rho_1 c_1 >> \rho_2 c_2$. We finally obtain:

$$\Theta_B = \arctan \frac{c_1}{c_2} \tag{3}$$

In the optical case this is called the Brewster angle, therefore it is indicated here by the index B. One should also remember that in the optical case this angle allows for total transmission only of the component of radiation with a polarization parallel to the plane of incidence. In air, sound waves are longitudinal waves whose vector of displacement is always oriented parallel to the direction of propagation, i.e. parallel to this plane. For zero approximating ratios of the velocities, the arc tangent approaches, but always remains smaller than the arc sine.

Behind the phase boundary this totally transmitted intensity is refracted according to the condition of the Brewster angle, which means that the sum of the angle of incidence and the angle of refraction gives 90°. The incident and the refracted beam are orthogonal to each other, and the refraction angle is 90° -0.23´´, agreeing very well with perpendicular refraction relative to the vertical. One also should keep in mind that in the optical case an advantage is taken from the application of lasers which provide for beams of very small angular divergence in order to show this very subtle effect.

Now we are left with the first possibility above mentioned, i.e. the angle of incidence relative to vertical is small enough to allow for transmission and reflection. As we have seen, even the slightest deviation from orthogonal incidence onto the phase boundary will lead to total reflection. The only chance to achieve transmission can be the orthogonal incidence itself onto the interface. In practice this means that the diameter of the chamber has to be bigger than the wavelength which is emitted from the source, in air about 34 cm for sound waves in the kHz-region. In this case only the velocity of the wave alters at transmission, not the direction of propagation. The coefficient of transmission is given by the following equation:

$$T = \frac{4\rho_1 c_1 \rho_2 c_2}{(\rho_1 c_1 + \rho_2 c_2)^2} \quad (4)$$

With the above given values one obtains T=1.9·10^{-20} for a single penetration of the phase boundary in either direction as the equation for the coefficient of transmission is invariant under exchange of indices.

The human voice has a frequency in the kHz-region. Those audible waves have a wavelength of about 300 km inside a vacuum due to the ratio of the velocities of propagation inside air and inside a vacuum or dark energy. The corresponding phonons have a particle energy of E = 4.1·10^{-12} eV. This is an extraordinarily small value compared to optical photons of the wavelength λ =500 nm and their energy of 2.5 eV. We neglected here the problem that for the outer chamber the conditions of perpendicular incidence which we assumed for the inner chamber are much harder to be fulfilled. Furthermore, we remember that the original experiment was controlled by the ear and renounce any further discussion of detection.

The quantitative evaluation of the historical experiment shows that even under the most optimistic assumptions we cannot expect any observable effect.

Sound waves in empty space

Thermodynamic considerations**:** Since the discovery of the accelerated expansion of the universe one assumes a non-vanishing energy-density inside free space. The reader may still have recognized that whenever we talk about “empty space” in this article, the author carefully tries to choose his expressions in agreement with current attitudes of the scientific community. However, this is not always an easy task as e.g. the “Report of the Dark Energy Task Force” (DETF) shows [10]. Therefore I want to cite literally the passage which explains in the most compact form how this problem is currently discussed. “One possible explanation for dark energy may be Einstein´s famous cosmological constant. Alternatively, dark energy may be an exotic form of matter called quintessence, or the acceleration of the Universe may even signify the breakdown of Einstein´s Theory of General Relativity.”

In the cosmological constant case the equation of state for empty space (=the vacuum) has the fixed value $w = p{\cdot}\varepsilon^{-1} = -1$ (p = pressure, ε =total energy density). As the energy density has a positive value the pressure is negative. A negative pressure is required for the explanation of cosmic acceleration as it leads to gravitational repulsion. In this case empty space is called a “vacuum” and the dark energy is the vacuum´s energy. It is not believed that sonic perturbations are able to propagate.

Quintessence or also k-essence is a class of models which assume an energy density which varies with time. The equation of state, w, is allowed to admit any value. The accelerated expansion is explained by values of w between minus one third and minus one. An interesting question is if this energy is a property of space itself or if the energy is the property of something distinct from it, according to the DETF some exotic kind of matter. Propagation of sonic perturbations are widely discussed [11]. The properties of sound waves depend on the special assumptions which are made for a concrete model. Most models assume a scalar field with potential as well as kinetic energy. Models with a standard kinetic energy term admit the speed of light as the speed of sound inside dark energy [12]. If dark energy can be regarded as a single perfect fluid, sonic perturbations are adiabatic changes of state, otherwise not [13]. Models which differ in the value of their equation of state and/or their speed of sound are distinguished by the comparison of calculations which predict properties of observable phenomena like e.g. the anisotropy of the cosmic microwave background, the distribution of galaxies and the relative brightness of supernovae of type Ia as a function of redshift.

It seems remarkable that even in models in which the propagation of sound is non-adiabatic, the speed of sound is assumed to be constant although the authors explicitly point out that corresponding changes of state are not isentropic. In general this means that non-adiabatic sound waves suffer from dispersion, i.e. that the velocity of propagation is a function of the wave´s frequency.

Quantum theory: The possibility of propagation of sonic perturbations in a vacuum is generally refused [14]. As the vacuum we understand here only the cosmological constant case in which the equation of state has everywhere and at any time the fixed value w =1.

A quantum theoretical approach to sound regards all the quasiparticle excitations as corresponding to equivalent waves [15]. These quantum states are called phonons in analogy to photons which represent the electromagnetic field. Phonons have energies E = h·f, where h is Planck´s constant and f the frequency of the sound wave.

As the following circumstance seems to be of great importance for the question if phonons can be regarded as real particles, I want to cite the decisive passage literally: “We emphasize that the momentum of an elementary excitation in a microscopic homogeneous system – the fluid – is a true momentum and not, like in the periodic field of a crystal lattice, a quasi -momentum” [16]. This means that phonons in a microscopic homogeneous system, in the referred case a superfluid, have momenta according to de Broglie´s relation $p = h{\cdot}\lambda^{-1}$. Here λ is the wavelength of the sound wave.

We postulate that also the vacuum can be regarded as a microscopic homogeneous system and phonons inside are real particles. This leads to the crucial question if it is possible to regard certain elementary particles or their compositions as phonons of the vacuum. In other words, is it possible to take advantage from our knowledge of particles which we call elementary and whose corresponding material waves are supposed to propagate in a vacuum? Are there any which fit the properties of sound waves?

Flowers, Ruderman and Sutherland investigated the neutrino pair emission from a finite-temperature neutron superfluid [17]. “The neutrons inside neutron stars are almost certainly superfluid below a critical temperature $T_c \sim 10^{10}$ K. below T_c, pairs of excited neutron quasiparticles may recombine, resulting, if weak neutral currents exist, in the emission of neutrino-antineutrino pairs.” “At nonzero temperature $T<T_c$ the neutron fluid has two components: a superfluid condensate and quasiparticle excitations (broken “Cooper pairs”). We calculate the neutrino pair emissivity due to the recombination of broken pairs that then join the condensate.” “Similar calculations have been performed for conventional superconductors where the recombination takes place through phonon and photon emission.”

Neutrino-antineutrino pairs which are generated in superfluid neutron matter can be regarded as phonons. One should recall that in a relativistic theory there is no difference of principle between elementary and composed particles [18]. As Flowers, Ruderman and Sutherland calculate explicitly the neutrino emissivity due to the described process, they assume that all the neutrino-antineutrino pairs are emitted into the vacuum after generation inside the superfluid neutron matter. Can we regard them as phonons as well when they continue propagating inside a vacuum after emission from the neutron star? If the neutron superfluid was able to sing, could the song be heard by another neutron star after passage of the neutrino-antineutrino pairs through the vacuum?

DISCUSSION

More precisely the above question should be asked in form of another Gedankenexperiment. Consider a technical loudspeaker which is constructed of superfluid neutron matter and a microphone, also constructed of superfluid neutron matter. Could an acoustic signal be transmitted from the speaker to the microphone through a vacuum?

This Gedankenexperiment shows twice. The historical experiment would be completely naïve. Therefore, I renounced the investigation of the effect of frustrated total reflection and the Goos – Hähnchen (displacement of a small beam between the gaps of two total reflecting prisms) effect as well. The demonstration of both effects requires a very special setup and could not have enhanced the probability of obtaining a signal from the evacuated chamber. Secondly it shows that the design of an experiment for direct observation of sound inside evacuated space would be a technically impossible task at the time. Finally, even the DETF did not plan amongst more than 40 experiments, even one for directly measuring the transmission of sound through evacuated space, although the members of this group ought to consider sound inside dark energy as an existing phenomenon.

Now I want to insist on the signal-character of sound. This is only fulfilled if the sound waves which carry the information correspond to adiabatic changes of state because otherwise the entropy increases and the signal is obstructed. Sound waves which correspond to non-adiabatic changes of state suffer from dispersion, as already mentioned. This means that different tones, i.e. frequencies, have different velocities of propagation. The original information would be distorted if it was transmitted by non-adiabatic waves of sound. Different tones would arrive at different times at the audience and not in the same sequence of emission from the source. Human acoustic communication would be impossible even inside air if sound waves did not correspond to adiabatic changes of state. This implies that phonons which correspond to acoustic signals are massless particles whereas those phonons which correspond to non-adiabatic changes of state must yield a rest mass. One should keep in mind that also in the optical case messages are obstructed by dispersion through chromatic aberration of light in optical lenses. Dispersion blurs the images taken with white light because different colors do not meet in the same focus. It is remarkable that in this case photons yield a rest mass, too [19]. On the other hand even material waves can produce an image if dispersion is prevented by using mono frequent waves. Consequently, the images of an electron microscope are one-colored.
Let us now investigate the question if we can distinguish phonons which correspond to neutrino-antineutrino pairs inside superfluid neutron matter from the same particles inside the vacuum. The speed of sound inside superfluid neutron matter approaches the speed of light [20]. For the same velocity inside a vacuum we obtain the same wavelength from the equation $c_s = \lambda \cdot f$ and the same momentum of the particles by the de Broglie relation. The energy of the particles is also the same. If their rest mass vanishes exactly then the neutrinos´ helicity equals -1 and the helicity of the antineutrinos equals +1 [21]. This implies that neutrino-antineutrino pairs can be regarded as composed bosons with spin 0, as their spin is always opposed to each other. Sound waves inside liquids are longitudinal waves. In the case of electromagnetic waves and photons we demand that the internal symmetry of the wave coincides with the internal symmetry of the particles. The longitudinal polarization of neutrinos and antineutrinos may also permit the propagation of longitudinal sound waves in a vacuum [22-25].

CONCLUSION

A quantum theory of a vacuum´s sound contradicts the idea that any special properties of these sound waves are of minor interest. If dark energy is a consequence of a cosmological constant it does not define a preferred frame. Issues of preferred frame arise if dark energy were an imperfect fluid or when its distribution is not homogeneous.

If exact homogeneity is not demanded for dark energy it has to be regarded as something distinct from empty space, not as a property of space itself. Consequently, the DETF regards dark energy as an exotic kind of matter.

One should remember, however, that any kind of matter always defines a preferred frame. For sound waves which propagate inside this matter it must be regarded as their rest frame. Relative to this, one is always able to decide whether an observer or a source of sound waves is moving by determination of the Doppler-shift which in both cases yields different results. This is not possible in the case of light which propagates in a vacuum.

The idea that adiabatic sound waves propagate in the form of zero rest mass particles with the speed of light in a vacuum signifies that their velocity is not only constant with respect to their frequency but also with respect to different inertial frames.

The identification of adiabatic phonons with neutrino-antineutrino pairs requires an exactly vanishing rest mass for at least one mass eigenstate of neutrinos inside a vacuum. Several possibilities regarding the other mass eigenstates may be distinguished. Neutrino oscillations may be possible only inside matter due to the MSW (Mikheyev-Smirnov-Wolfenstein)-effect or also inside a vacuum. The latter case requires different mass eigenstates of neutrinos also inside a vacuum. One may ask if massive neutrinos in a vacuum could be regarded as a kind of non-adiabatic sound.

Two other tasks which may be affected are the extraordinarily high value of a cosmological constant if one considers all of the virtual particles which are generated due to the action of various fields in vacuum and a quantized version of the Theory of General Relativity. Here is not the place to discuss these issues but I want to mention that the hypothesis of quintessence seems not be capable of solving these problems. A convenient hypothesis on the idea of phonons of the vacuum was that it may introduce yet unconsidered symmetry principles into physical theories.

One may wonder if there is any theory which is able to solve these problems all together.

Albert Einstein suggested to extend the idea of an "ether" comprehending it not as a kind of substance but as the essence of physical parameters of state inside space which is free of ordinary matter. Wolfgang Pauli concedes this attitude that of course there is ether in this extended sense, but one must keep in mind that it has no mechanical properties. This means that we cannot address spatial coordinates and velocities to its physical parameters.

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