EDITORIAL

Computational Chebyshev technology and Approximation Theory

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ABSTRACT

Everything is practical and fast, so we will routinely compute polynomial interpolation/Gauss quadrature weights for tens of thousands of points. In fact, each chapter of this book is a single Matlab Mfile, and the book has been produced by executing these files with the Matlab "publish" facility. There are quite a number of excellent books on approximation theory.

EDITORIAL

Three classics are [Cheney 1966], [Davis 1975], and [Meinardus 1967], and a slightly more recent computationally oriented classic is. Perhaps the first approximation theory text was.A good deal of my emphasis will be on ideas related to Chebyshev points and polynomials, whose origins go back more than a century to mathematicians including Chebyshev, de la Vall ee Poussin, Bernstein, and Jackson. In the computer era, some of the early figures who developed "Chebyshev technology," in approximately chronological order, were Lanczos, Clenshaw, Babenko, Good, Fox, Elliott, Mason, Orszag, and V. I. Lebedev. Books on Chebyshev polynomials have been published by Snyder, Fox and Parker, Paszkowski, Rivlin, and Mason and Handscomb.

One reason we emphasize Chebyshev technology so much is that in practice, for working with functions on intervals, these methods are unbeatable. For example, we shall see in Chapter 16 that the difference in approximation power between Chebyshev and "optimal" interpolation points is utterly negligible. Another reason is that if you know the Chebyshev material well, this is the best possible foundation for work on other approximation topics,and for understanding the links with Fourier analysis.

Theorems are stated and proofs are given, often rather tersely, without all the details spelled out. It is assumed that the reader is comfortable with rigorous mathematical arguments and familiar with ideas like continuous functions on compact sets, continuity, integrals in the complex plane, and norms of operators. I have taken pleasure in trying to cite the originator of each of the main ideas. The entries in the References stretch back several centuries, and each has an editorial comment attached. Often the original papers are surprisingly readable and insightful, at least if you are comfortable with French or German, and in any case, it seems particularly important to pay heed to original sources in a book like this that aims to re-examine material that has grown too standardized in the textbooks. A reason for looking at original sources is that in the last few years it has become far easier to track them down, thanks to the digitization of journals, though there are always difficult special cases likewise.

Why is approximation theory useful? The answer goes much further than the rather tired old fact that your computer relies on approximations to evaluate functions like sin(x) and exp(x). There are also many other fascinating and important topics of approximation theory not touched upon in thisvolume, including splines, wavelets, radial basis functions, compressed sensing,and multivariate approximations of all kinds.

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