RESEARCH

Enumeration of measurable functions on finite sigma algebras on sets with at most ten elements

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Hildbrand M, Kivunge B. Enumeration of measurable functions on finite sigma algebras on sets with at most ten elements. J Pure Appl Math. 2023; 7(5):326-327.

ABSTRACT

Sigma algebras plays an integral role in explaining the entire concept of measure theory, majorly because it's a collection of subsets of a given set, whereby the subsets can be finite or infinite intriguingly, measurable functions are derived by this concept of sigma algebras. This paper provides a formidable guide towards deriving the respective measurable functions using finite sigma algebras by examining sigma algebras associated to a given set X and their corresponding measurable functions. Finite $\delta -$ algebras will be constructed by ensuring all the axioms of $\delta -$ algebras are satisfied.

Key words: *Measurable functions on finite sigma algebras; Algebras; Enumeration of measurable; Functions*

INTRODUCTION

In measure theory, $\delta -$algebra is a non-empty subsets of a set X satisfying axioms below;

i. $(X, \varnothing) \in \wp$

ii. If $Y \in \wp$ then $Y^C \in \wp$

iii. Take $W_1, W_2, W_3, ..., \in \wp$ then $\bigcup_{i=1}^{\infty} W_i \in \wp$

According to Axler, 2020, $\delta -$ algebra may be finite or infinite where the former is a $\delta -$ algebra containing finite non-null sets and the later contains non-empty infinite subsets. They play a crucial role in the enumeration of measurable functions [1-7].

For instance, in the proof of measurability of functions, Bogachev, 2007 states that, Suppose a collection $\{N_j\}$ of non-null sets and h a measurable functions such that $h^{-1}(N_j) \in J$ then N a $\delta -$ algebra [8-17].

If $N_j \in K$ then $h^{-1}(\bigcup_{j=1}^{m} N_j) = \bigcup_{j=1}^{m} h^{-1}(N_j) \in J$, where both J and K are $\delta -$ algebras. This area of study has not been focused particularly in generation of $\delta -$ algebras and their counterpart measurable functions. Particularly, Sigma algebras generated by finite collection of subsets say $\varphi = \{A_1, A_2, A_3, ...A_n\}$, where the finite subsets are contained in the set X, There exists the smallest unique $\delta -$ algebra $\{\varnothing, X\}$ and the largest $\delta -$ algebra $P(X)$ containing φ . This knowledge of sigma algebra is very useful in measure theory. Moreover, in the study of measurable functions since it aids in the generation of measurable functions associated to a given $\delta -$ algebras of a given set. This area of study has remains untapped in the recent past since researchers have not given much focus on it, by and large; researchers have generally drawn their attention on the wider study of measure theory.

This work shall investigate $\delta -$ algebras associated to a given set X and their corresponding measurable functions. Finite $\delta -$ algebras will be constructed by either ensuring all the axioms of $\delta -$ algebras are satisfied.

<u>**Proposition:**</u> There is no sigma algebra with odd number of elements

Proof

Suppose by contradiction there exists a sigma algebra say B, with five elements, such that;

$B = \{\emptyset, X, \{A\}, \{B\}, \{C\}\}$, since B is sigma algebra, we show whether it satisfies the axioms of a sigma algebra;

$\{\emptyset, X\} \in B$ The property is satisfied
If $A \in B$ $then\ A^c = \{B\}$, but since $C \in B$, C^c $does\ not\ exist$ thus the compliment property is not satisfied
$\{A\} \cap \{B\} = \{B\} \cap \{C\} = \emptyset \cap X = \emptyset$ The property is satisfied since $\{A\} \in B, \{B\} \in B, \{C\} \in B$ but $\{A, B, C\} \notin B$ hence the property is not satisfied.

Since all the axioms are not satisfied then B is not sigma algebra thus there is no sigma algebra with five elements.

Further to that, below table shows distribution of elements in each sigma algebra (Table 1).

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Received: 29 Sep, 2023, Manuscript No. puljpam-23-6812, Editor Assigned: 1 Oct, 2023, PreQC No. puljpam-23-6812 (PQ), Reviewed: 7 Oct, 2023, QC No. puljpam-23-6812 (Q), Revised: 12 Oct,2023, Manuscript No. puljpam-23-6812 (R), Published: 31 Oct, 2023, DOI:-10.37532/2752-8081.23.7(5).326-327

Table 1
Distribution of elements in each sigma algebra

|X|	|P(X)|	No. of	Distribution of in ascending order
1	2	1	1 with 2elements
2	4	2	1 each with 2 and 4 elements
3	8	5	1 each with 2 and 8 elements; 3 with 4 elements
4	16	15	1 each with 2 and 16 elements;
			7 with 4 elements;
			6 with 8 elements
5	32	52	1 each with 2 and 32 elements;
			15 with 4 elements,
			25 with 8 elements;
			10 with 16 elements
6	64	113	1 each with 2 and 64 elements;
			31 with 4 elements;
			45 with 8 elements;
			20 with 16 elements;
			15 with 32 elements
7	128	237	1 each with 2 and 128 elements;
			63 with 4 elements;
			81 with 8 elements;
			35 with 16 elements;
			35 with 32 elements;
			21 with 56 elements

The above table illustrates the distribution of sigma algebra for each set X, with a sample size of 5 elements in X, it can be deduced that no sigma algebra with 5 elements, moreover, its notably seen that, all the number of elements of sigma algebras are power sets of 2. For instance, $2,4,8,16\ and\ 32$ are all powers of 2, hence, any sigma algebra has $2^k elements\ \forall k \in N,$

Therefore, it can be concluded that there is no sigma algebra with an odd number of elements.

CONCLUSION

The paper clearly tabulates the findings and concludes that there are no odd number of sigma algebras given any size of the set X.

It is clearly observed that; as the number of elements of the set generating sigma algebra increases, the number of sigma algebras generated also increases thus there exists a direct proportionality.

ACKNOWLEDGMENT

Firstly, I thank almighty God for his providence of life, resonating mind and the fortuity to compile this work. Moreover, My earnest gratitude extends to my supervisor Prof. Benard Kivunge for his inestimable sacrifice on time despite his ever-strained schedule, his irreplaceable tolerance and invaluable advice during the research period gave me the tenacity to get things done right.

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