Fundamentals of Gyroscope Theory

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Since the Industrial Revolution gyroscopic phenomena represent the analytically unsolvable problem. The famous mathematician L. Euler expressed only one precession torque generated by the rotating body that is the change in the angular momentum. The known analytical and numerical modeling for gyroscopic phenomena did not correspond to actual reality. In fact, the movable rotating objects manifest the action of the undiscovered inertial torques. The latest studies of gyroscopic properties show their physics are compound in origin. Inertial torques acting on the spinning body contain eight interacted components operating by the centrifugal, common inertial, Coriolis forces, and the change in the angular momentum that entailed by the ratio of the angular velocities of the rotating bodies around their axes. These system of inertial torques and the ratio of the angular velocities construct the fundamentals of gyroscope theory.

Rotating objects in engineering constitute a vast group of machine components of different forms (disc, cylinder, ring, sphere, cone, paraboloid, propeller, etc.). The movable rotating parts manifest the gyroscopic phenomena, which dynamics remain not adequately explained. The motions of the rotating components of different mechanisms are described analytically by the action of their center mass. Such models do not well match with practice tests and do not satisfy industrial requirements. Researchers did not examine the action of the distributed rotating masses that create several inertial forces. The new studies have shown the origin of gyroscope phenomena are compound. Gyroscopic manifestations are a result of the activity of the interacted inertial torques carry out by the rotating mass of bodies. These inertial torques are based on the activity of the centrifugal, common inertial, and Coriolis forces and the change in the angular momentum. The ratio of the angular velocities of the rotating body around the axes of motions is linked to all inertial torques. The pointed inertial torques and the ratio of the angular velocities construct the basis of gyroscopic effects. The method for formulating of inertial torques created by masses of rotating bodies is applicable to any object. It can be axial symmetric as a spinning disc, cylinder, ring, cone, spheres, paraboloid, propeller, etc., and nonsymmetrical rotating objects. This work contains the mathematical models and ratio of the angular velocities of the spinning disc that examined by tests. The new analytical formulation for the gyroscope phenomena solves all gyroscopic problems and discovers new approaches in classical mechanics.

New studies of the inertial torques produced by the spinning disc discovered the interaction of eight inertial torques around two axes. The basic components of inertial torques are demonstrated in Table 1. All inertial torques are used for the formulation of the motions of the rotating objects around three axes of the 3D coordinate system.

Type of the torque generated by	Equation
Centrifugal forces	$T_{ct} = T_{in} = \frac{2}{9}\pi^2 J\omega_i$
Inertial forces	
Coriolis forces	$T_{cr} = (8/9)J\omega_i$
Change in angular momentum	$T_{am} = J\omega_i$

where ω_i is the angular velocity of the spinning disc around axis *i*; ω is the angular velocity of the spinning disc around axis oz; J is the mass moment of inertia of the spinning disc.

The action of the system of the interacted inertial torques described in several publications and demonstrated in Fig. 1.

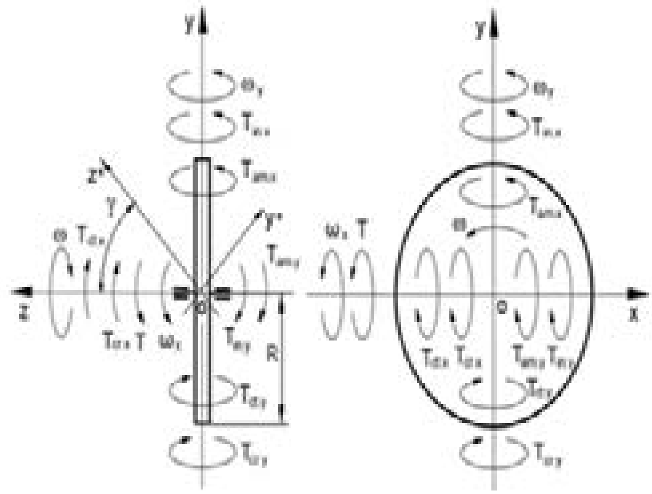

Figure 1. The system of interacted inertial torques of the spinning disc

The inertial torques of each axis express the kinetic energies of the spinning disc that are equal by the principle of the conservation of mechanical energy. The equality of the kinetic energies is represented by the equality of the inertial torques around their axes. These inertial torques linked by the ratio of the angular velocities of the spinning disc. For the spinning disc with the inclined axis on the angle γ (Fig. 1), the equality of the inertial torques is represented by the following equation:

$$-\left(\frac{2\pi^2+8}{9}\right)J\omega_x-\left(\frac{2\pi^2+9}{9}\right)J\omega_y=\left(\frac{2\pi^2+9}{9}\right)J\omega_x\cos\gamma-\left(\frac{2\pi^2+8}{9}\right)J\omega_y\cos\gamma$$

where ω_x and ω_y are the angular velocities of the spinning disc around axis ox and oy respectively.

$$\omega_y=\left[\frac{2\pi^2+8+(2\pi^2+9)\cos\gamma}{2\pi^2+9-(2\pi^2+8)\cos\gamma}\right]\omega_x$$

Equation (2) represents the variable-ratio ω_y/ω_x of the angular velocities of the gyroscope around axes that depends on the angle γ of the axis of the spinning disc location. For the horizontal location of the spinning disc (γ = 0), the ratio ω_y/ω_x = 4π2 + 17 is maximal. The ratio ω_y/ω_x = 0 is null for the vertical location of the spinning disc(γ = 90o), where the precession torques are absent, and the gyroscope turns on the angle . For the horizontal location of the gyroscope and its turn around the vertical axis and the maximal turn around another axis, the ratioω_y/ω_x is active until the turn of the gyroscope on the angle γ = 164,841101855o/(4π2 + 17) = 2.918656521o . These angles are maximal for the gyroscope simultaneous turn around two axes that maintain their ratio.

The internal torques and the ratio ω_y/ω_x represent the fundamentals of the gyroscope theory that enable formulating the motions of spinning objects around axes of the accepted 3D coordinate system.

In engineering, spinning objects are represented by numerous designs of the movable machines. Most of the spinning parts of mechanisms are represented by the simple and standard geometry. The geometry of these

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types of spinning objects is described by simple equations that enable computing their inertial torques by known analytical methods. Their inertial torques depend on the location of distributed masses along two axes and expressed by different equations.

Analytical modeling for gyroscopic phenomena was problematic until new breakthrough solution for the inertial torques of spinning bodies. Obtained equations for the system inertial and the ratio of the angular velocities of the gyroscope around axes of motions validated by the practical tests. These components constitute the fundamentals of the gyroscope theory that can be used for solving any gyroscopic phenomena. New studies of dynamics of rotating objects represent a new direction in the science of classical mechanics, which will be presented in textbooks, and handbooks.