# Linear map and spin II. Kummer surface and focal error 

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polynomial. Whereas coordinates $\mathrm{ds}^{2}$ are norms of cubic number field a spinor is a sextic number field having a cubic subfield. Despite a linear least squares algorithm $K(X)$ yield a systematic focal error $\varepsilon_{\text {focal }}$.

Key Words: Kummer surface; focal tensor; Riemann surfaces; hyperelliptic theta functions; Weber invariant; sextic number field; cubic number subfield

## INTRODUCTION

Arbitrary world point shifts $\delta \mathrm{X}=\mathrm{a} \mu \mu^{\prime} \mathrm{X}_{\mu^{\prime}}-\mathrm{b}_{\mu} \mu^{\prime} \mathrm{X}_{\mu^{\prime}}$ require 256 components of a $2^{4} \cdot 2^{4}$ matrix $\mathrm{a} \otimes \mathrm{b}$. Matrix b measures an arbitrary coordinate change. A quadrifocal tensor $Q$ has $3^{4}=81$ components in space [1]. A camera matrix $\mathrm{P}=\mathrm{K}[I, 0]$ maps between the 3D world $\mathrm{X}, \mathrm{Y}$ and a 2D image $\mathrm{x}, \mathrm{y}$.
A 3.3 calibration matrix $K$ depends on five parameters. Trifocal or quadrifocal geometry allows image calibration. In practice, the overconstrained system generates a focal error $\boldsymbol{\varepsilon}_{\text {focal }}$ within a Linear East Squares algorithm (LLS). The present calculation shows that arbitrary changes $\delta X$ yield a systematic focal error $\varepsilon_{\text {focal }}$ which serves as a basis for our understanding of spinor matter.

According to part I a spinor state is connected with a Poncelet involution in space of elliptic curves. Information about $\delta X$ is limited by the quadrifocal tensor Q having the highest rank in space, introduced only recently. A quadrifocal tensor $\mathrm{Q}=\operatorname{det} \mathrm{K}(\mathrm{X})$ can be arranged as a singular matrix determinant $\operatorname{det} K(X)=0$ being a Kummer surface [2-4]. Sixteen vanishing first minor of $K(X)$ stand for sixteen focal points in between four images x . Shifts $\delta \mathrm{X}$ can be detected via image cycles in x . A relation between projective space $\mathbb{P}^{3} \rightarrow \mathbb{P}^{2}$ and Minkowski space is drawn as cycles of a world point $\delta \mathrm{X}$ scanning map $\gamma\left(\phi_{3}(t)\right)$.

Another interesting equivalence concerns the fundamental or essential tensor $\mathrm{F}=\mathrm{uv}$ '-u'v in epipolar geometry and a hyperbolic form $\alpha(\mathrm{u}, \mathrm{v})=\operatorname{det}$ $(u, v)$ of abelian hyperelliptic surfaces [5]. Here $u$ and $v$ are threecomponent image coordinates or three-component number fields, respectively. Despite investigation of Jacobi inversion, the direct problem of determining period elliptic lattices $\omega_{\mathrm{I}}(\mathrm{I}=0,1)$ via iterating a cubic number field $\mathbb{K}[\partial]$ has not been paid much attention.

In the following calculation hyperelliptic period lattices $\omega$ enter as unknown variables $\omega\left(P X=x, u, v,\left[\begin{array}{c}\varepsilon \\ \varepsilon^{\varepsilon}\end{array}\right]\right.$ with 4.3 projector P from world point X to image point x . Hyperelliptic characteristics $\left[\begin{array}{c}\varepsilon \\ \varepsilon^{\varepsilon}\end{array}\right]$ are a maximally accessible detail of a more general Riemann surface $\mathbb{X}_{\mathrm{g}}$. This can be shown by orthogonal substitutions or the existence of 16 linear forms of quaternions. Then world points $X=X\left[\begin{array}{c}\varepsilon \\ \varepsilon^{\prime}\end{array}\right]$ depend on sixteen hyperelliptic half-periods. A regular scan of hyperbolic $K(X)$ space of an open universe is defined as a map in flat space, i.e. of elliptic line bundles.

Complexity predicts that hyperelliptic details $\left[\begin{array}{c}\varepsilon \\ \varepsilon^{\prime}\end{array}\right]$ are a maximal and minimal detail of a general Riemann surface $\mathbb{X}_{\mathrm{g}}$ for obtaining a high composite algorithm. Hyperelliptic theta functions $\vartheta\left(\mathrm{u}_{ \pm}\right)$are Euler parameters of exact dynamical equations with uniformization $u_{ \pm}$as rigid body time or fluid time. Jacobi theta functions $\vartheta_{1}(\mathrm{u})$ are Euler parameters in a Cayley-Klein precession of a spinning top.

In this paper a bi spinor $\psi_{s}$ is extracted from the exact classical solution $\vartheta\left(u_{ \pm}\right)$by an earlier approach [6]. In distinction, the Euler parameter lives in a sextic number field $\mathbb{K}\left[\partial^{1 / 2}\right]$ with a cubic subfield $[\partial]$. A bi spinor $\psi s$ is defined as the simplest cyclic map in $\mathbb{K}\left[\partial^{1 / 2}\right]$ with symmetry $k, k+1$ and $\mathrm{k}+2, \mathrm{k}+3$ in $\mathbb{K}[\partial]$.

An iterate $k$ corresponds to a Poncelet involution $i_{k}(\mathrm{u})$ where $\mathrm{i}_{k}(\mathrm{u}) \circ_{\mathrm{i}_{k+1}}(\mathrm{u}){ }^{\circ} \mathrm{i}_{\mathrm{k}+2}(\mathrm{u}){ }^{\circ} \mathrm{i}_{\mathrm{k}+3}(\mathrm{u})=$ identity. A bi spinor results from a square root of a hyperelliptic theta quadrate for a product of elliptic curves. The binary form of the Kummer surface is investigated by transforming roots of a cubic invariant equation $\phi_{3}$ [2].

[^0]Having obtained the Dirac equation, the most general form of a hyperelliptic detail of a general Riemann surface generalizes the Eddington equation for the electron-to-proton mass ratio.

## Invariant surface as one-dimensional map

The relation drawn between the quadrifocal tensor $\mathrm{Q}\left(\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}, \mathrm{x}_{4}\right)$, the Kummer surface $K(X)$ and a binary map $\gamma\left(\phi_{3}\right)$ relies on a parametrization of $K(X)$ by a twisted cubic $\mathrm{C}_{\mathrm{tw}}$ [2]. On $\mathrm{C}_{\mathrm{tw}}$ a world point $X=\left(1,-\theta, \theta^{2},-\theta^{3}\right)$ is parametrized by one-dimensional complex parameter $\theta \in \mathbb{C}$.

Here $\theta$ is identified with algebraic units of a cubic field solving a cubic invariant equation $\phi(\theta)=0$. Then 16 focal points of $K(X)$ correspond to vanishing first minors. A vanishing three-dimensional determinant corresponds to inflection points on elliptic curves E E .

Then focal points of $Q\left(x_{1}, x_{2}, x_{3}, x_{4}\right)=0$ span an elliptic line bundle $L[E \lambda]$ or an hyperelliptic line bundle parametrized by products of elliptic curves. Epipolar lines get inflection tangents $\mathrm{F}_{\mu \mu}(\mathrm{X}, \mathrm{Y})$ 'as vanishing 2.2 minors $\delta \zeta_{21}^{\prime} \delta \zeta_{31}^{\prime}=\delta \zeta_{31}^{\prime} \delta \wp_{21}^{\prime}$ of four points $u_{\mu}(\mu=0,1,2,3)$ measured from uo on $\mathrm{E}_{\lambda}$. Flex tangents appear for class number one $\mathrm{h}_{\Delta}=1$ fields where $\phi_{3}(\theta)=0$ is an integer polynomial.
The Homogeneity of $F_{\mu \mu^{\prime}}=(M, u, \zeta(u, \omega), \wp(u, \omega))$ via $\zeta(\theta u, \theta \omega)$ and $\wp(\theta u, \theta \omega)$ of Weierstrass zeta $\zeta(u, \omega)$ and Weierstrass $\wp-$ functions $\wp(\mathrm{u}, \omega)$ ) of $\mathrm{L}\left[E_{\lambda}\right]$ scales $F(X, Y)=|M, u, \zeta, \wp|$ as $\left(M, \theta, \theta^{-1}, \theta^{-}\right.$ ${ }^{2}$ ).

For an integer polynomial $\phi_{3}(\theta)=0$ a Hermite map $\gamma\left(\phi_{3}(t)\right)$ of a subfield $\mathbb{K}[\partial]$ covers an infinity of roots of $\mathbb{K}[\sqrt{ } \partial]$ if $t$ is viewed as a complex root. Inflection tangents scan X-Y geodesics of $K(X)$ starting from integer values of $\zeta(u, \omega)$ and $\wp(u, \omega)$.
According to a Hermite map $\gamma_{3}\left(\phi_{3}\right)$ transforms cubic roots to cubic roots of $\phi_{3}$ for $h \Delta=1$ [7]. Especially values of the Weber invariant $f(\omega)=1 \frac{-1}{48} \frac{\eta\left(\frac{\omega+1}{2}\right)}{\eta \omega}$ are real where $\eta(\omega)$ is the Dedekind eta function $\eta(\omega)$ getting a simple norm $\operatorname{Nm}(f(\sqrt{ } \Delta))=2$ for $\omega=\sqrt{ } \Delta$. Real $f(\omega)$ corresponds to real algebraic units of a simple real cubic field. Complex iterates $\mathrm{f}(\sqrt{ } \Delta)$ denote complex conjugated roots of a cubic or a sextic. One gets the remarkable result that powers $f^{e}(\sqrt{\Delta}) e=1,2,3 \ldots$ remain irrational within $[\delta]$ allowing a one-dimensional chaotic map on $K(X)$. Equivalences between Kummer surfaces $K(X)$, a quadrifocal configuration $Q\left(x_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}, \mathrm{x}_{4}\right)$ and a twisted cubic curve $\mathrm{C}_{\mathrm{tw}}$ in space raises the question about calibration [8].

On a curve with 'double curvature' $\mathrm{C}_{\mathrm{tw}}$ dangerous projective configurations appear. Formulating $K(X)$ and $Q\left(x_{1}, x_{2}, x_{3}, x_{4}\right)$ uniformized by a generalized Riemann surface $\mathrm{u}_{ \pm}$lines bifurcate on flex tangents allowing only a local calibration. In the present paper, a conclusion is drawn that a systematic focal error is related to the partition function in statistics.

## Connection between invariant theory and one-dimensional maps

It turns out that a bi spinor is determined by one-dimensional cyclic iterates of a sextic invariant polynomial having cubic subfields as extracting the square root $\sqrt{ } \mathrm{K}(\mathrm{X})$ or $\sqrt{ } \mathrm{d}$.

A one-dimensional fractional map as a map of equivalent lattice periods $\omega$ yields exact solutions, e.g. Jacobi theta functions $\vartheta_{1}(u, \omega)$ which can be classified as an exactly solvable chaos [9].

A one-dimensional fractional map $\gamma(\phi)$ as a map of complex roots of invariant cubic polynomials is classified as a quadratic map. Mapping variables of elliptic functions and not periods, Complex Multiplication (CM) and modular units can appear. Here bifurcation of lines transmits to periods and variables which is classified as a hyperellipticelliptic transition of a generalized Riemann surface. Then binary invariant theory concerns different points as subsequent iterates $\theta_{k+3}=\gamma(\phi)^{\circ} \theta_{k+2}, \theta_{k+2}=\gamma(\phi)^{\circ} \theta_{k+1}$ and $\theta_{k+1}=\gamma(\phi)^{\circ} \theta_{k}$
Compared to Lattés maps $u \rightarrow 2 \mathrm{u}$ as a quartic map a Hermite map $\gamma(\phi)$ as a quadratic map is much simpler. However, the determination of periods of lattices is left open.

The surface $\mathrm{K}(\mathrm{X})$ of world points
$X \simeq\left(b^{2},-b, 1,1\right)$ and, $Y \simeq\left(\theta^{3},-\theta^{2}, \theta,-1\right),\left(\phi^{3},-\phi^{2}, \phi,-1\right.$ yields the invariant equation.
$2(\theta \phi)^{2}(b \theta)(b \phi)-a_{\theta}^{3} a_{\phi}^{3}=0 \quad$ (1)
in terms of binary $(a \theta)=a_{1} \theta_{2}-a_{2} \theta_{1}, a_{i}=a_{1}^{n-1} a_{2}^{i}, a_{\theta}=a_{1} \theta_{2}+$ $a_{2} \theta_{1}$.
The generating polynomial $\Phi_{n}(\theta)=\sum_{i=0, \ldots, n} a_{1} \theta^{n-1}$ is written as $\Phi_{6}(\theta, \Phi)=a_{\theta}^{3} a_{\phi}^{3}$. In case of a cubic invariant, one gets two symbolic cubic invariant polynomials $a_{\theta}^{3}=0, a_{\phi}^{3}=0$ leading to $a_{\theta}^{3} a_{\phi}^{3}=0$ and $(\theta \phi)^{2}(b \theta)(b \phi)=0$.

Kummer surfaces $\mathrm{K}(\mathrm{X})$ and Weddle surfaces $\mathrm{W}(\mathrm{Y})$ are written in terms of second and third $\mathrm{u}_{ \pm}-$derivatives $X=\left(\wp_{ \pm \pm}, 1\right)$ and $Y=\left(\wp_{ \pm \pm \pm}\right)$in $\mathrm{P}^{3}$ which are simply $\left(b^{2},-b, 1,1\right)$ and $\left(\theta^{3},-\theta^{2}, \theta,-1\right)$. A filter of projective line bundles can be given a quantitative form to define massive and inert points. First $\mathrm{K}(\mathrm{X})-\mathrm{W}(\mathrm{Y})$ relations consist of inflection points $\mathrm{F}_{\mu \mu}$. Coordinates of Kummer and Weddle surfaces are second and third $u_{ \pm}$ - derivatives. An alternate view of $K(X)$ coordinates are characteristics dependent squared theta functions $\vartheta^{2}\left[\begin{array}{c}\varepsilon \\ \varepsilon^{\varepsilon}\end{array}\right]$. Degrees of freedom of orthogonal substitutions are dealt with in the following section. Fortunately, the second and fourth derivatives of hyperelliptic sigma functions $\sigma(\mathrm{u})$ obey a differential invariant.
$\frac{\operatorname{det}^{2} \gamma(\phi)}{3} D_{c}^{4} \sigma(u) \sigma\left(u^{\prime}\right)=\left[(a b)^{4} a_{c}^{2} b_{c}^{2}-\operatorname{det}^{2} \gamma(\phi) a_{c}^{4}(a D)^{2}\right] \sigma(u) \sigma\left(u^{\prime}\right)$ (2)
where $(\mathrm{aD})=\mathrm{a}_{1} \mathrm{D}_{-} \mathrm{a}_{2} \mathrm{D}_{+}$and $D_{ \pm}=\frac{\partial}{\partial_{u \pm}}-\frac{\partial}{\partial_{u \pm}}$. A form capable of (1) appears for cubic invariants $a_{\theta}^{3}=0, a_{\phi}^{3}=0$. In this case the solution is $D_{c}^{4} \sigma(u) \sigma\left(u^{\prime}\right)=0$ and $(a b)^{2} a_{c} b_{c}=0$.
Next the differential invariant of the hyperelliptic sigma function $\sigma(\mathrm{u})$ simplifies in case of cubic invariants on $\mathrm{K}(\mathrm{X})$ as $(a b)^{2} a_{c} b_{c}=0$ for a point $a=\left(-h_{2}, h_{1}\right)$ to
$\frac{1}{3} D_{c}^{4} \sigma(u) \sigma\left(u^{\prime}\right)=a_{c}^{4}(a D)^{2} \sigma(u) \sigma\left(u^{\prime}\right)=(c h)^{4} D_{h}^{2} \sigma(u) \sigma\left(u^{\prime}\right)^{(3)}$

Extracting a square root of an invariant form introduces an error. However, the next section supports (3) for second derivatives $d^{2}$. The invariant (ch) can be understood as one of four basis vectors in determining a discriminant of a 2.2 matrix as detq where a four-
dimensional matrix $q$ is defined on skew basis vectors $\Psi_{s}=(c h)[9-$ 10].
Owing that 16 linear forms $(\mathrm{ab})$ are equivalent to half-periods and characteristics $\left[\begin{array}{c}\varepsilon \\ \varepsilon^{\prime}\end{array}\right] 16 \Psi$-values determine the vicinity of tangent cones $\frac{d_{u_{+}}}{d_{u_{-}}}=1$.

The aim is to find a representation $D_{h}^{2}=\left(\gamma^{\mu}\left(\partial_{\mu}-A_{\mu}\right)\right)^{2}$ connecting differences of $u_{ \pm}$in the vicinity of half-periods. As known $K(X)$ is independent on the differentials du $\pm$ in the vicinity of half-periods. The surface $K(X)$ is equivalent to binary invariants $(a b)^{2}(a c)(b c)$ where $\mathrm{a}, \mathrm{b}$ and c are three points subjected to a cycle of point h . Points $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{h}$ in binary invariants are viewed as a subsequent fractional substitution $\theta_{k+3}=\gamma(\phi) \circ \theta_{k+2}, \theta_{k+2}=\gamma(\phi) \circ \theta_{k+1}$ and $\theta_{k+1}=\gamma(\phi) \circ \theta_{k}$ for an interval $\theta_{k} \theta_{k}$.

A fractional substitution as a linear map is a trivial iterated function. However, as a Hermite map, $\theta_{\mathrm{k}}$ exhibits Sharkovskii's ordering. The point $a=\left(-h_{2}, h_{1}\right)$ represents a rotation of an interval by $\pi$ leading to $\theta_{\mathrm{k}+3}$. Accordingly, one gets four basis vectors $\psi_{\mathrm{k}}, \psi_{\mathrm{k}+1}, \psi_{\mathrm{k}+2}$ and $\psi_{\mathrm{k}+3}$. An invariant (3) implies the existence of the simplest cycle of four points $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{h}$ with rotated $\mathrm{h}=\left(-\mathrm{a}_{2}, \mathrm{a}_{1}\right)$ of four iterates $\mathrm{k}, \mathrm{k}+1, \mathrm{k}+2, \mathrm{k}+3$. Iterates k are reflected as a path $\mathrm{u}\left\{\mathrm{k}_{0}, \ldots, \mathrm{k}_{\mathrm{N}}\right\}<\omega$

Then an invariant equation of four points $a, b, c, h$ can describe $a$ bifurcating chaotic one-dimensional map as the simplest cycle of iterates. In simplest cycles, Hermite maps $\gamma\left(\phi_{3}\right)$ of an integer invariant cubic polynomial $\phi_{3}$ and a cubic number field $\mathbb{K}[\partial]$ are connected. The map $\gamma\left(\phi_{3}\right)$ extended to $\mathbb{C}$ describes algebraic units of a cubic number field $\mathbb{K}[\partial]$ and leaves $(\mathrm{ab})^{2}(\mathrm{ac})(\mathrm{bc})=0$ and $D_{h}^{2}$ invariant. Its square rootthe invariant differential $D_{h} \sim \gamma\left(\phi_{1}\right)+\gamma\left(\phi_{2}\right)$ lives in a sextic number field $\mathbb{K}\left[\partial^{1 / 2}\right]$ of a cubic subfield $[\partial]$. The simplest case $\partial=2^{1 / 3}$ and $\operatorname{Nm}(f(\sqrt{ } \Delta))=2$ is able to generate powers of $f^{e}(\sqrt{\Delta}) e=1,2,3, .$. within $\mathbb{K}\left[\delta^{1 / 2}\right]$.

## Differentiable surface of orthogonal substitutions

Coordinates are defined as indices of smooth fields as infinitely differentiable manifolds. For the time being times $u_{ \pm}$of exact $\vartheta$ dynamics suffering discrete iterations $k$ are not differentiable on path $\mathrm{u}\left\{\mathrm{k}_{0}, \ldots, \mathrm{k}_{\mathrm{N}}\right\}$.

Uniformization differentials $\frac{d_{u+}}{d_{u-}}=\theta$ [2] suffer steps at $\theta_{\mathrm{k}}=\mathrm{f}\left(\sqrt{ } \Delta_{\mathrm{k}}\right)$ where

$$
\begin{equation*}
\left.\operatorname{Inf}(\sqrt{\Delta})=1 / 3 \int_{0}^{1} d z[\zeta(z \sqrt{\Delta}+3) / 18)-\zeta(v, 1 / 9 \sqrt{\Delta})\right] \tag{4}
\end{equation*}
$$

depends on iterated sequences of vertical lattice axes $\sqrt{ } \Delta_{k}$. Formerly vpath independent on the interval $[0,1]$ iterated discriminants $\Delta_{k} \operatorname{set} \zeta(\mathrm{u})$ singularities. Thus, iteration changes allowed paths which must surround singularities. Mathematically the task is unusual but develops a low-complexity algorithm.

Despite $\frac{d_{u+}}{d_{u-}}$ jumps the whole system is differentiable. Unfortunately, the differential properties of a Kummer surface $\mathrm{K}(\mathrm{X})$ as characteristics
$\left[\begin{array}{c}\varepsilon \\ \varepsilon^{\prime}\end{array}\right]$ dependent surfaces of squared hyperelliptic theta functions $\vartheta^{2}\left(u_{ \pm}\right)$ are rarely investigated.
The present paper aims to unify $\frac{d_{u_{+}}}{d_{u_{-}}}$tangent planes and orthogonal substitutions. We aim to show that $\mathbb{P}^{3}$ lines in tangent planes of Kummer surface $K(X)$ consist of $\infty$ differentiable image points which are not reducible to one-dimensional differentials [3] [11].

Uniformization parameter $u_{ \pm}$of $K(X)$ - tangents satisfy $\frac{d_{u+}}{d_{u-}}=\theta=$ $f(\sqrt{\Delta})$ but $\mathrm{K}(\mathrm{X})$ is $\mathrm{d}^{2-}$ differentiable. A world point X indexed rational by $X \simeq\left(\theta^{2},-\theta, 1,1\right)$ suffers discontinuities. A differential dv of elliptic uniformization in (4) exists for piecewise continuous paths. A v-path surrounds $\zeta(v, \omega)$ - lattice singularities. Then the Weber- Schlaefli invariant $f(\sqrt{ } \Delta)$ is well defined because $\eta(\omega)=\frac{1}{\sqrt{3}} \vartheta_{1}\left(\frac{1}{3}, \frac{\omega}{3}\right)$ with Jacobi function $\vartheta_{1}$. A discovery [3] that all 2 n -th order differential $d^{2 n} \vartheta\left[\begin{array}{c}\varepsilon \\ \varepsilon^{\prime}\end{array}\right](u+v) \vartheta\left[\begin{array}{c}\varepsilon \\ \varepsilon^{\prime}\end{array}\right](u-v) \quad$ constitute orthogonal substitutions $g\left(\mathrm{e}, \mathrm{e}^{\prime}\right)$ remains valid if $\vartheta\left[\begin{array}{c}\varepsilon \\ \varepsilon^{\prime}\end{array}\right](u \mp v)$ is replaced by an arbitrary sum of theta functions. A system of four characteristics constitutes an Euler parameter $\mathrm{e}_{\mu}[3]$. Via Cayley's map $g\left(e, e^{\prime}\right)=\frac{1+7}{1-7}$ an orthogonal substitution $\mathrm{g}(\mathrm{e}, \mathrm{e}$ ') corresponds to a four-dimensional rotation with skew matrix $\Gamma$ if a local scale change by the factor $1 / \mathrm{e}_{0}$ with $\operatorname{detg}\left(\mathrm{e}, \mathrm{e}^{ }\right)=1$ corresponds to a local rotation in space with Euler parameter $\mathrm{e}_{\mu} / \mathrm{e}_{0}$.

Now we built an exponential map $\exp \left(\mathrm{d}^{2}\right)$ subjected to an orthogonal substitution $g\left(e, e^{\prime}\right)$. One has $g(e, e)=a(e)$ and $g\left(e^{\prime}, e^{\prime}\right)=a\left(e^{\prime}\right)$ showing that $\mathrm{SO}(4)$ is isomorphic to $\mathrm{SO}(3) \times \mathrm{SO}(3)$ where the three-dimensional substitution corresponds to rotation matrix R for $\mathrm{e}_{\mu} / \mathrm{e}_{0}$.

The two-dimensional differential $\mathrm{d}^{2}$ should correspond to the invariant differential $(\text { ch })^{4} D_{h}^{2}$. If $\mathrm{d}^{2}$ acts on one factor $\vartheta\left[\begin{array}{c}\varepsilon \\ \varepsilon^{\prime}\end{array}\right](u+v)$
all even derivatives $\mathrm{d}^{2 \mathrm{n}}$ and $\exp \left(\mathrm{d}^{2}\right)$ are again orthogonal substitutions [3]. Rational points on $\mathrm{K}(\mathrm{X})$ with
$X^{t} j X=0, j=\sigma_{0} \otimes \sigma_{3}$
are related to discrete $\mathrm{SE}(3)$ steps $k$ for rotation $\mathrm{R}=\mathrm{R}_{\mathrm{k}}$. Rational points obey congruences with respect to an invariant absolute quadric $Q_{a b s}=x_{1}^{2}+x_{2}^{2}+x_{3}^{2}$ establishing the correspondence to n -focal geometry. The absolute quadric $\mathrm{Q}_{\text {abs }}$ relates to kinetic energy of precession of a one spinning top
$T=1 / 2 \operatorname{mdet} \frac{d}{d t} \gamma_{c k}^{2} \rightarrow 1 / 2 m \sum_{\mu \nu} \frac{d}{d t}\left(e_{f v}^{2}\right) \rightarrow m \sum_{\mu \nu} \dot{e}_{f u} \dot{e}_{f v} g^{\mu \nu}$

If precession determining Cayley-Klein parameter $\gamma_{\text {СK }}$ obey fixed points where $e_{f \mu}=\operatorname{vec} \gamma_{C K}=\left(e_{f 1}, 0, e_{f 3}, 0\right)$ a quadrifocal image ( $\mu=1,2,3,4$ ) sum behaves relativistic. If, fixed points eff of Euler parameter allow a radix-4 discrete Fourier transform (DFT) the Euclidean vierbein $\mathrm{e}_{\mathrm{f} \mu}$ being an Euler parameter is multiplied by a twiddle factor $1^{1 / 4}$ giving a metric tensor $g^{\mu \nu}=e_{f \mu} e_{f v}$.

As a result Minkowski metric $\operatorname{diag}(1,1,1,-1)=\operatorname{diag}\left(1^{1 / 4} \mu\right)^{2}$ is set in context to a squared tensor of twiddle factors of DFT-4. Then the apparent relativistic kinetic energy $1 / 2 m \sum_{\mu v} \frac{d}{d t}\left(e_{f u}^{2}\right) \frac{d}{d t}\left(e_{f v}^{2}\right)$ depends on the differentiated square of a Jacobi theta function $\mathrm{e}_{\mathrm{f} \mu}=\mathrm{vec} \gamma_{\mathrm{CK}}$.

The linear forms $\psi_{\mathrm{s}}=(\mathrm{ch})$ of four points $\mathrm{k}, \mathrm{k}+1, \mathrm{k}+3, \mathrm{k}+4$ are related to half-periods and to 5 linear forms ( $\xi \eta$ ) expressing a general theta function [12].
The invariant expression $(c h)^{4} D_{h}^{2}=\bar{\psi}_{s_{1}} \bar{\psi}_{s_{1}} \Gamma_{s_{1} s_{2} s_{1}{ }^{\prime} s_{2}} \bar{\psi}_{s_{1}}, \bar{\psi}_{s_{2}{ }^{\prime}}$ is approximated by a sextic norm $\mathbb{K}\left[\partial^{1 / 2}\right]$ where
$\Gamma_{s_{1} s_{2} s_{1} s^{\prime} s_{2}}=\gamma^{\mu} \gamma^{v}\left(A_{\mu}-\frac{\partial}{\partial x_{\mu}}\right)\left(A_{v}-\frac{\partial}{\partial x_{v}}\right)$ carries real $\mathbb{K}\left[\partial^{1 / 2}\right]$ conjugates.

Owing to s-(ch)- $\left[\begin{array}{c}\varepsilon \\ \varepsilon^{\prime}\end{array}\right]$ relations $\Gamma$ is formulated in terms of 15 syzygetic (Göpel) quadruples which are vectorized two dimensional minors of g(e,e'). Additionally $\Gamma$ depends on $80(120)$ azygetic (Rosenhain) quadruples which are columns or rows of $g(e, e$ '). Here coordinates $x \mu$ are understood as images $(\mathrm{K}(\mathrm{X})$ roots) $\mu=1,2,3,4$. Rational $\mathrm{K}(\mathrm{X})$ points are SE (3) steps
$X=M(A, a) X=e^{S(A, a)} X$. where i-components are diagonalized by
$S_{\mu, i i,}(A, a) \gamma_{s s^{\prime}}^{\mu} \psi_{i^{\prime} s^{\prime}}=m \delta_{i i} \psi_{i s}$
where the mass m is proportional to algebraic units of $\mathbb{K}[\partial]$ and depends on $\mathrm{f}(\sqrt{ } \Delta)$ fluctuations.

Accordingly rational points of $\mathrm{K}(\mathrm{X})$ should result in rational points of $\mathrm{W}(\mathrm{Y})$ which requires.
$d F_{\mu i i^{\prime}} \gamma_{s s^{\prime}}^{\mu} \psi_{i^{\prime} s^{\prime}}=m \delta_{i i^{\prime}}, \psi_{i s}$
with inflection tangents as four first minors of $\mathrm{F}_{\mathrm{Hi}_{\mathrm{ii}}}=(\mathrm{M}, \mathrm{u}, \zeta(\mathrm{u}), \wp(\mathrm{u}))$ with $\mathbb{P}^{3}$ index $i$, image index $\mu$ and cyclic iteration index s.

## Statistical properties of $K(X)$

Iterates $\mathrm{u}\{. \mathrm{k}\}$ on $\mathrm{K}(\mathrm{X})$ satisfy the hyperelliptic addition theorem
$\frac{\sigma_{(u+v)} \sigma_{(u-v)}}{\sigma_{u}^{2} \sigma_{v}^{2}}=X(u) j X(v)$
Where $\mathrm{X}(\mathrm{u})=\mathrm{d}^{2} \sigma(\mathrm{u})$ is defined up to an orthogonal substitution $e^{d^{2}} X(u)$.
An addition step is defined as orthogonalizing $\mathrm{d}^{2}$ which yields $\sigma\left(\mathrm{u}^{+} \mathrm{v}\right)$ $\sigma(u-v)=0$ whereas $X=M(A, a) X$ is a $\operatorname{SE}(3)$ joint in $x, y, z$ space surrounded by complex itera-tions. The vanishing of $\sigma(\mathrm{u})$ defines the Kummer surface $K(X)$ by a point $\vartheta^{\prime}(u) \vartheta(v)$ as [4]
$\left[\varepsilon_{2} \varepsilon_{3}\right] \vartheta_{\varepsilon_{1}}+\left[\varepsilon_{3} \varepsilon_{1}\right] \vartheta_{\varepsilon_{2}}+\left[\varepsilon_{1} \varepsilon_{2}\right] \vartheta_{\varepsilon_{3}}=0$

Where $\varepsilon \triangleq\left[\begin{array}{c}\varepsilon \\ \varepsilon^{\prime}\end{array}\right]$ and $[\varepsilon \varepsilon]=\frac{\partial}{\partial_{\mu}} \vartheta \Lambda \frac{\partial}{\partial_{\mu}} \vartheta$. Starting from a point $\mathrm{x}_{0}, \mathrm{y}_{0} \mathrm{z}_{0}$ addition is mainly performed on $\mathrm{x}, \mathrm{y}$ plane within a quasi-two dimensional configuration where one has $\frac{d u_{+}}{d u_{-}}=\theta=f(\sqrt{\Delta})$. The bitangent condition (9) is viewed as a detail of a sixth degree product of a complex function $\phi_{3}{ }^{k}(z)$ of roots $\left(z-z_{1}\right)\left(z-z_{2}\right)$ viewed as a Kummer surface.

The number of combinations $\left[\varepsilon_{1} \varepsilon_{2}\right]$ of nontrivial even functions is $1,10,10 \cdot 10+6 \cdot 6$ for a sixth-degree polynomial and $1,10,2^{2 \mathrm{~g}-1}\left(2^{2 \mathrm{~g}}+1\right)$ for a polynomial of genus $g$ having $2^{\mathrm{g}-1}\left(2^{\mathrm{g}}+1\right)$ even and $2^{\mathrm{g}-1}\left(2^{\mathrm{g}}-1\right)$ odd theta functions. Cycle relations (6) and (7) for two $\operatorname{SE}(3)$ joints with masses $\mathrm{m}_{1}$ and $\mathrm{m}_{2}$ inserted into (4.2) yield a quadratic equation $(\alpha \beta \gamma) \mathrm{m}^{2}=0$ where coefficients are proportional to the number of even function combinations. For $(1,10,136)$ one recovers Eddington's equation of the electron-to-proton- mass ratio $[13,14]$.

## CONCLUSION

Lattice periods $\omega$ of hyperelliptic and elliptic theta functions $\vartheta(u, \omega)$ are determined by an inverse process via iterates k of algebraic units with $f(\sqrt{\Delta})$. As a result, a line bundle appears around a discriminant
$\Delta$. An algorithm of fractional substitution $\gamma\left(\phi_{3}\right)$ of binary invariants greatly simplifies if iterates are accompanied by simplest cycles $k, k+1$, $\mathrm{k}+3, \mathrm{k}+4 . \Delta_{\mathrm{k}}$ values are determined by a six-component and threecomponent number field.

Number- theoretically a bi spinor $\psi_{\text {s }}$ is a sextic number field $\mathbb{K}\left[\partial^{1 / 2}\right]$ with cubic sub-field $[\partial$ ] where $\partial$ is a simple cubic irrationality. Algorithmically a bi spinor $\Psi_{\mathrm{s}}$ reflects the simplest cycle of a perioddoubling bifurcation process.
Observables ar field-norms $\operatorname{Nm}\left(\mathbb{K}\left[\partial^{1 / 2}\right]\right)$ and $\operatorname{Nm}(\mathbb{K}[\partial])$ as a sum over all iterates. Thus, a normal field $\mathbb{N}[\sqrt{ } \Delta]=\mathbb{K}[\partial] \mathbb{K} ’[\partial] \mathbb{K} ’[\partial]$ is a field extension in quantum statistical equations which has been so far neglected.

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[^0]:    Independent Researcher, Berlin, Germany
    Correspondence: Otto Ziep, Independent Researcher, Berlin, Germany. Telephone +491785574250 , e-mail: ottoziep@gmail.com
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