RESEARCH

Physics in the vicinity of SAGITTARIUS A, gauss curvature values of spacetime in its vicinity

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Iza F. S. Physics in the vicinity of SAGITTARIUS A, gauss curvature values of spacetime in its vicinity. J Mod Appl Phys. 2023; 6(4), 01-03.

ABSTRACT

We are working on concrete applications of the relativistic Schwarzschild model to the cosmos. As first results of this work we present some values of space-time curvature in the vicinity of SAGITTARIUS A*. The relativistic Schwarzschild model solves Einstein's equations exactly assuming a point gravitational mass and empty space in its vicinity. This model leads to a static and symmetric solution to the mathematical equation of space-time that allows its Gaussian curvature to be calculated at each point. We have calculated some curvature values and found an equation to calculate them that allows us to extend the results to a wider range of distances. Finally, we have applied these results to the real case of the supermassive black hole of our galaxy SAGITTARIUS A*, obtaining values of Gaussian curvature of the space-time in its vicinity, valid if the Schwarzschild model is applicable.

Key Words: *Schwarzschild model; Curvature of space-time; Black holes; SAGITTARIUS A**

INTRODUCTION

The physical problem at hand is the calculation of the curvature of space-time caused by a spherical and static black hole at a point located at a distance "r" from the centre of the black hole. This point will always be further away from the event horizon or Schwarzschild radius, "Rs". Schwarzschild solves the equations of the general theory of relativity for an assumption of a point gravitational mass and a surrounding space, establishing a metric and an equation for space-time that turns out to be stationary in time and with spherical symmetry, resulting in a 2D surface, (the Flamm paraboloid), which is represented in (Figure 1 (a and b)) [1].

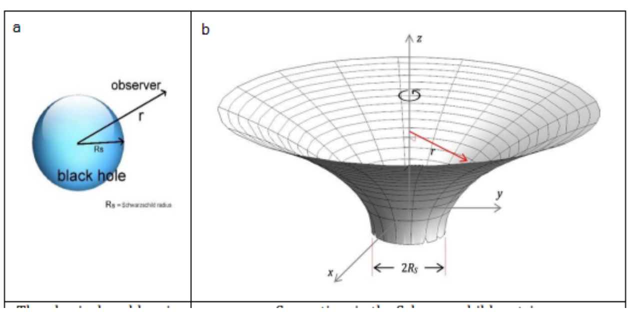

Figure 1) *a) The physical problem in Euclidean space b) Space-time in the Schwarzschild metric Flamm paraboloid*

Resolution of the mathematical problem

Flamm's paraboloid, mathematical solution to the Schwarzschild model, is a 2D surface inserted in a space R3. Its geometry allows us to parameterize the paraboloid as a function of the observer's distance from the point mass "r" and the azimuth angle "φ". The problem admits a mathematical treatment of differential geometry of surfaces, and with it we are going to calculate values of Gaussian Curvature [2].

Surface parameters (r, φ)

$0 \le r < \infty,\ 0 \le \varphi < 2\pi$

which has this parametric equation:

$x = r\ cos\varphi$

$y = r\ sen\varphi$

$z = 2(Rs(r - Rs))^{1/2}$

and by vector equation:

$f(x, y, z) = (r\ cos\varphi, r\ sen\varphi, 2(Rs(r - Rs))^{\frac{1}{2}})$

Determination of velocity, acceleration, and normal vectors to the surface

$\partial f/\partial\varphi = (-r\ sen\varphi, rcos\varphi, 0)$

$\partial^2 f/\partial\varphi^2 = (-r\ cos\varphi, -r\ sen\varphi, 0)$

$\partial f/\partial = (cos\varphi, sen\varphi, (r/Rs - 1)^{-1/2})$

$\partial^2 f/\partial r^2 = (0, 0, (-1/(2R_s)).(\frac{r}{Rs} - 1)^{-3/2})$

$\partial f/\partial\varphi\partial r = (-sen\varphi, cos\varphi, 0)$

$n = (\partial f/\partial\varphi \times \partial f/\partial r) = (rcos\varphi/(\frac{r}{Rs} - 1)^{-1/2}, rsen\varphi/(r/Rs - 1)^{-1/2}, -r)$

$[n] = r((1/(\frac{r}{Rs} - 1)) + 1)^{-1/2}$

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Received:- 25 Oct, 2023, Manuscript No. puljmap-23-6839; Editor assigned:- 26 Oct, 2023, Pre-QC No. puljmap-23-6839 (PQ); Reviewed:- 30 Oct, 2023, QC No. puljmap-23-6839 (Q); Revised:- 3 Nov, 2023, Manuscript No. puljmap-23-6839 (R); Published:- 24 Nov 2023, DOI: 10.37532.2023.6.4.01-03

$n = n/[n]$

Curvature and curvature parameters

Gauss curvature $K = LN - M^2/EG - F^2$

$L = ð^2 f/ð\varphi^2 . n$

$E = ðf/ð\varphi . ðf/ð\varphi$

$N = ð^2 f/ðr^2 .n$

$G = ðf/ðr . ðf/ðr$

$M = (ðf/ð\varphi ðr). n$

$F = ðf/ð\varphi . ðf/ðr$

Completing a previous work of ours, we have particularized the equations in 20 points between 1 and 1400 Schwarzschild radii, $Rs = 2GM/c^2$, calculating the corresponding curvatures as shown in the results (Table 1) [3].

Thus, although in the metric there is a singularity at the point 1Rs, the value of the Gaussian curvature for the singularity is resolved mathematically by calculating a limit. We have calculated that limit for Gauss curvature (Figure 2).

Results of curvature values

Table 1) GAUSSIAN CURVATURE VALUES ACCORDING TO THE SCHWARZSCHILD MODEL

Distance to the point mass	Value of Gauss Curvature k	Distance to the point mass	Value of Gauss Curvature k
1Rs	$-0{,}5000 \times Rs^{-2}$	60Rs	$-2{,}325.10^{-6} \times Rs^{-2}$
1,2Rs	$-0{,}2873 \times Rs^{-2}$	80Rs	$-9{,}596.10^{-7} \times Rs^{-2}$
1,4Rs	$-0{,}1821 \times Rs^{-2}$	100Rs	$-4{,}925.10^{-7} \times Rs^{-2}$
1,6Rs	$-0{,}1220 \times Rs^{-2}$	200Rs	$-5{,}963.10^{-8} \times Rs^{-2}$
1,8 Rs	$-0{,}0790 \times Rs^{-2}$	400Rs	$-4{,}800.10^{-9} \times Rs^{-2}$
2Rs	$-0{,}0625 \times Rs^{-2}$	600Rs	$-2{,}376.10^{-9} \times Rs^{-2}$
3Rs	$-0{,}0186 \times Rs^{-2}$	800Rs	$-9{,}710.10^{-10} \times Rs^{-2}$
4Rs	$-0{,}0078 \times Rs^{-2}$	1000Rs	$-5{,}059.10^{-10} \times Rs^{-2}$
5Rs	$-0{,}0030 \times Rs^{-2}$	1200Rs	$-2{,}883.10^{-10} \times Rs^{-2}$
6Rs	$-0{,}0023 \times Rs^{-2}$	1400Rs	$-1{,}810.10^{-10} \times Rs^{-2}$

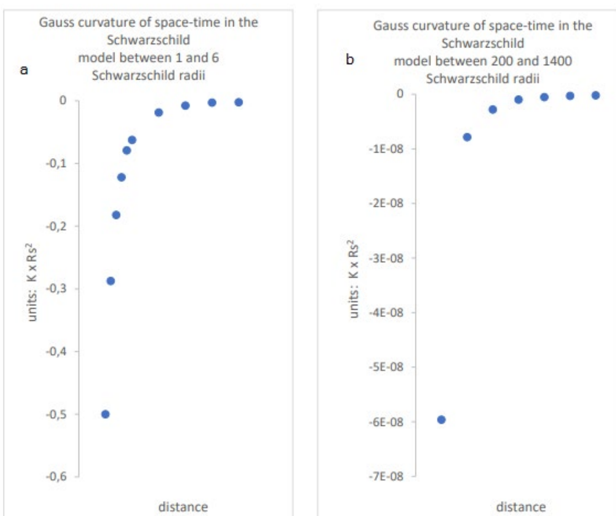

Figure 2) *Gauss curvature of spacetime in the Schwarzschild model a) 1 and 6 b) 200 and 1400*

An equation to calculate the curvature of space-time according to the Schwarzschild model

An adjustment equation has been obtained using an Excel program by regression methods throughout this wide range of distances. The degree of quality of the fit obtained by calculating the parameter R^2 is very high, 0.9999. For this reason, it is to be expected that this equation allows interpolating the calculation of Gaussian curvature values, in this wide range of distances, with high accuracy without the need to carry out the laborious calculations that would otherwise have to be done.

Fit equation between 1 and 1400 Schwarzschild radii

Gaussian curvature: $k = -0{,}5268\ (r/Rs)^{-3{,}054} \times Rs^{-2}$

Fit quality $R^2 = 0{,}9999$

Rounding decimals and according to the definition of Schwarzschild radium, Rs

$Rs = 2GM/c^2$

where G is the universal gravitation constant, and M is the mass of the black hole, we can express the adjustment equation we have found as the following approximate equation:

the simple equation found: $k = -GM/c^2 r^3$

where k is the Gaussian curvature of space-time according to the Schwarzschild model.

Application to the calculation of the curvatures in the vicinity of SAGITTARIUS A*

We assume that Sagittarius A* behaves like a symmetric and static black hole, that is, the Schwarzschild model is assumable for the calculation of curvatures. Sagittarius A* is the supermassive black hole at the galactic centre of the Milky Way [4-6]. Like the nuclei of most spiral and elliptical galaxies, the Milky Way contains a black hole at its centre.

The mass of SAGITTARIUS A* is estimated at 3,6 million suns if the mass of the sun is $1.989.10^{30}$ Kg, the mass of Sagittarius A* is estimated at $7.16.10^{36}$ Kg, its Schwarzschild radius, Rs, thus turns out to be $10{,}61.10^{9}$ meters [7]. According to these data, the values of space-time curvature in its vicinity are those expressed in (Table 2).

Table 2) VALUES OF CURVATURES OF SPACETIME IN THE VICINITY OF SAGITTARIUS A*

Distance in Schwarzschild radii	Distance in meters	Gauss curvature, model Schwarzschild (m^{-2})	Gauss curvature, approximate equation found (m^{-2})
1Rs	10.10^9	$-0,44.10^{20}$	$-0,53.10^{-20}$
1,2Rs	12.10^9	$-0,25.10^{20}$	$-0,31.10^{-20}$
1,4Rs	14.10^9	$-0,16.10^{-20}$	$-0,19.10^{-20}$
1,6Rs	16.10^9	$-0,11.10^{-20}$	$-0,13.10^{-20}$
1,8 Rs	18.10^9	$-0,70.10^{-21}$	$-0,91.10^{-21}$
2Rs	20.10^9	$-0,55.10^{-21}$	$-0,66.10^{-21}$
3Rs	30.10^9	$-0,17.10^{-21}$	$-0,20.10^{-21}$
4Rs	40.10^9	$-0,69.10^{-22}$	$-0,83.10^{-22}$
5Rs	50.10^9	$-0,27.10^{-22}$	$-0,42.10^{-22}$
6Rs	60.10^9	$-0,20.10^{-22}$	$-0,24.10^{-22}$
60 Rs	600.10^9	$-0,21.10^{-25}$	$-0,24.10^{-25}$
80 Rs	800.10^9	$-0,85.10^{-26}$	$0,10.10^{-25}$
100 Rs	1000.10^9	$-0,44.10^{-26}$	$-0,53.10^{-26}$
200 Rs	2000.10^9	$-0,53.10^{-27}$	$-0,66.10^{-27}$
400 Rs	4000.10^9	$-0,43.10^{-28}$	$-0,83.10^{-28}$
600 Rs	6000.10^9	$-0,21.10^{-28}$	$-0,24.10^{-28}$
800 Rs	8000.10^9	$-0,86.10^{-29}$	$-0,10.10^{-28}$
1000 Rs	10000.10^9	$-0,45.10^{-29}$	$-0,53.10^{-29}$
1200 Rs	10200.10^9	$0,26.10^{-29}$	$-0,31.10^{-29}$
1400 Rs	10400.10^9	$-0,16.10^{-29}$	$-0,19.10^{-29}$

From the results obtained, we can see that our equation slightly overestimates the curvature values at all points. In any case, the order of magnitude in each of the points is of the same degree in both calculations, therefore, our simple equation does seem to be valid with a reasonable degree of accuracy in the wide range of distances between 1Rs and 1400 Rs and, there is no objection to extending it to greater distances according to the form of Schwarzschild space-time, Flamm's paraboloid, (Figure 3).

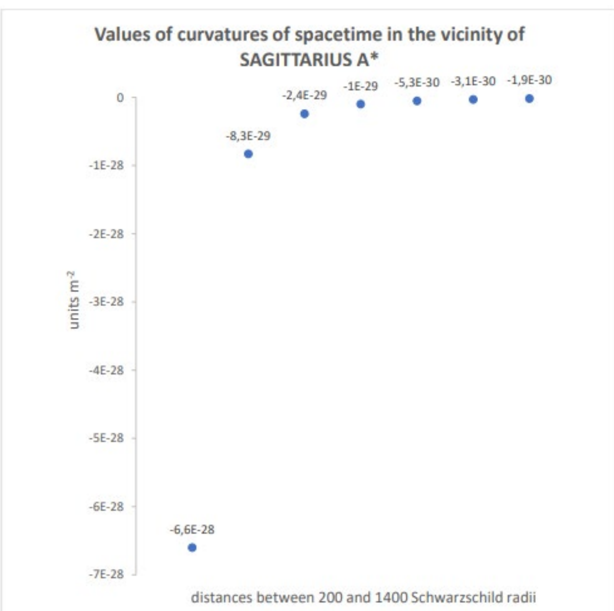

Figure 3) *Values of curvature of spacetime in the vicinity of Sagittarius A**

CONCLUSION

First, we have calculated some values of the Gaussian curvature of space-time according to the Schwarzschild model. As the calculations are laborious, from the results obtained we have established an equation that allows us to interpolate and extrapolate these curvature values. SAGITTARIUS A* in some of its aspects and in a first approximation can be treated as a static and symmetric black hole and based on this assumption we have applied the Schwarzschild model by calculating some values of the Gaussian curvature in its vicinity.

We have compared the values that result from the Schwarzschild model with those that result from applying our equation. The result indicates that our approximate equation is valid, at least, in the range of distances studied.

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