

Premises and simple estimation on proof of the positive mass theorem, and negative matter as unified dark matter and dark energy

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ABSTRACT

First, we found that some proofs of the positive mass theorem have all certain premises: an isolated gravitational system and infinite space, but both are all impossible. Therefore, the negative matter cannot be restricted. Second, we discuss some simple estimations of the positive matter theorem, which agree from astronomy to particles. Third, based on Dirac negative energy, Einstein mass-energy relation and principle of equivalence, we propose the negative matter as the simplest model of unified dark matter and dark energy. All theories are known, only mass includes positive and negative. Because there is repulsion between positive matter and negative matter, so which is invisible dark matter, and repulsion as dark energy. It may explain many phenomena of dark matter and dark energy. Bondi's results are wrong. Fourth, we

derive that the rotational velocity of galaxy is approximate constant. Assume that dark matter is completely the negative matter, so we may calculate an evolutionary ratio between total matter and usual matter from 1 to present 11.82 or 7.88. Fifth, we propose the mechanism of inflation as origin of positive-negative matters created from nothing, whose expansion is exponential due to strong interactions at small microscopic scales. Moreover, we research some theories of the negative matter, and obtain the quantitative relation of the negative mass and the cosmological constant, and predict a judgment test and other possible tests. The negative matter as a candidate of dark matter and dark energy is not only the simplest, and is calculable and testable, and may be changed and developed

Key Words: Positive mass theorem; Negative matter; Repulsion; Dark matter; Dark energy; Calculation; Inflation; Test

INTRODUCTION

The positive mass (energy) theorem is an important conjecture in mathematical physics, from which the negative matter seem to be not existence. But, we study carefully some proofs of the positive mass (energy) theorem and found that these proof processes all have certain premises. In 1979 Schoen and Yau proved first the positive mass conjecture in general relativity. Let M be a space-time whose local mass density is non-negative everywhere. Then they prove that the total mass of M as viewed from spatial infinity must be positive unless M is the flat Minkowski space-time. In general relativity, they provided that an isolated gravitating system having non-negative local mass density must have non-negative total mass, measured gravitationally at spatial infinity. But, two premises are all physically impossible: gravitation forms an isolated system, and space is infinity. In fact, this is proof that an isolated system of general relativity with a positive density must have a total positive mass-energy, or a proof that an isolated gravitational system guarantees that the positive density becomes a total positive mass-energy.

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$$g_{ij} = \left(1 + \frac{M}{2r}\right)^4 \delta_{ij} + h_{ij} \quad (1)$$

holds and has zero total mass. The positive mean curvature is related to the metric

$$d\bar{s}^2 = \phi^4 ds^2 \text{ which should be } \geq 0.$$

The positive-mass conjecture states that for a nontrivial isolated physical system, the total energy, which includes contributions from both matter and gravitation, is positive. This assertion has been demonstrated in the important case when space-time admits a maximal slice.

Further, they extended this proof to the positive-action conjecture of Hawking for asymptotically Euclidean metric and the path integral convergent in the Euclidean quantum gravity theory. For an isolated physical system, an initial data set for space-time consists of a three-dimensional manifold N , a positive definite metric, a symmetric tensor, a local mass density

$$\mu = \frac{1}{2} \left[R - \sum_{i,j} h^{ij} h_{ij} + \left(\sum_i h_i^i \right)^2 \right] \quad (2)$$

and a local current density J . When $M=0$, the space-time is the Minkowski space-time. It is usual result of general relativity.

Witten proposed a new proof of the positive energy theorem of

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classical general relativity. If the positive energy theorem was false and a state of negative energy existed in general relativity, Minkowski space would presumably be unstable and would decay into the negative energy state. The main importance of the positive energy theorem is that it is related to the stability of Minkowski space as the ground state of general relativity. Parker and Taubes discussed Witten's proof of the positive energy theorem [1-7]. But, according to the gravitational force:

$$F = -\frac{G}{r^2} M_1 M_2 \tag{3}$$

there is still a gravitational force between negative-negative matters, it is a universal repulsive force between the positive and negative matters. Therefore, the positive and negative matters are two regions of topological separation in the general case by different interactions (Figure 1) [8-10].

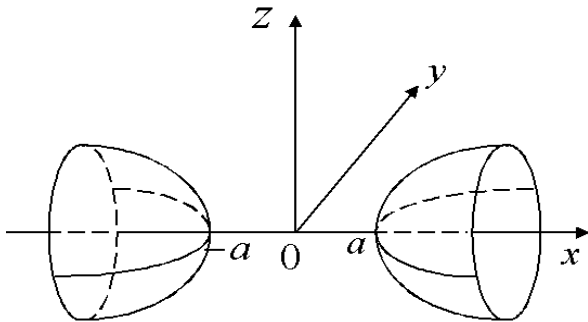


Figure 1) Positive and negative matters as two topological separation regions

Moreover, Townsend considered, for spacetimes of arbitrary dimension, the general two-derivative gravity plus scalar field theory allowing anti-de Sitter space as a solution, and studied the restrictions on the scalar field potential required by the positivity of the energy. In particular, he rederived the perturbative stability criterion and applied it to higher dimensional supergravity theory. Recently, Wang and Zhang established the positive energy theorem for weak asymptotically anti-de Sitter spacetime with distributional curvature under the weak dominant energy condition.

In a word, the positive energy theorem states that the total energy of a gravitating system, including the energy of the matter and also the energy of the field, is always positive, if the matter contribution is positive. But, it is quite like a circular argument, which is admitted in Yau's book. Conversely, if the matter contribution is negative, this proof will not hold [11-13].

Some Simple Estimations of the Positive Mass Theorem

In this paper, we propose some simple estimation of the positive matter theorem.

In the modern cosmology, for a radiation-dominated universe of the big-bang cosmology, the total energy of usual matter is mainly the positive energy of photon [14, 15]. When the evolutional process from inflation and radiation-dominated universe to the matter-dominated universe, the known total energy of usual baryon matter of non-relativity is:

$$Mc^2 - \frac{GmM}{r} \tag{4}$$

The positive mass theorem is simplified to $Mc^2 > GmM/r$ i.e., $c^2 > Gm/r$ This constant $c^2 / G = 1.35 \times 10^{27} \text{ kgm}^{-1}$ is very big. In astronomy, Solar mass: Solar radius is

$M / R = 2.848 \times 10^{21} \text{ kgm}^{-1}$ For the solar system R is bigger, M/R is smaller. For black hole, $r = 2Gm/c^2$ so $r > 0.5$ hold always. For galactic System

$$M / R \approx 4 \times 10^{42} \text{ kg} / 2.85 \times 10^{21} \text{ m} = 1.4 \times 10^{21} \text{ kgm}^{-1}$$

For the particles and nuclei,

$$M / R = n(1.67 \times 10^{-27} \text{ kg}) / 10^{-15} \text{ m} = 1.67n \times 10^{-12} \text{ kgm}^{-1}$$

These all comply with the positive mass theorem.

But, the known total mass of Universe is, and the corresponding scale is

$M = 2 \times 10^{53} \text{ kg}$ and the corresponding scale is, so. Both are comparable, and M/R is slightly big. It corresponds just to the acceleration of the universe and dark energy [16,17].

Dark matter and Dark energy

Now dark matter and dark energy are always basic focus in astronomy, and total physics. They are also the greatest mysteries in physics. The general hypothesis is that dark matter and dark energy are two different concepts.

Since 1970 to 1978, Rubin, et al., confirmed the existence of dark matter for ten spiral galaxies. Further, the dark matter in the Galaxy, in group of galaxies and cluster of galaxies, in the universe, is confirmed by the mass-to-light ratio and the galactic rotational curves, etc [18,19].

The dark matter may become huge conglomeration, and is possibly the Weakly Interacting Massive Particle (WIMP), neutrino with mass, baryonic dark matter and nonbaryonic dark matter, monopole, supersymmetric dark matter, axion [20-23], etc.

The simplest kind of dark matter model is to add phenomenally a real scalar field as the dark matter field in the standard model. Clowe, et al., proved a direct empirical proof of the existence of dark matter. Gianfranco, et al., discussed history of dark matter [24-27].

Recently, in Nature there are two reports: "Tighter limits on dark matter" and "Dark-matter evidence weakens". Both are based on search for ultralight scalar dark matter with atomic spectroscopy, and 7.1 keV sterile neutrino constraints from X-ray observations of 33 clusters of galaxies with Chandra ACIS. Vermeulen, et al., discussed direct limits for scalar field dark matter from a gravitational-wave detector. Arguelles, et al., studied dark matter annihilation to neutrinos. Allali, et al. discussed general relativistic decoherence with applications to dark matter detection. Arcadi, et al., searched muon g-2 and B anomalies from dark matter [28-33]. Bringmann, et al., discussed dark matter from exponential growth. Holst, et al., proposed simplest and most predictive model of muon g-2 and thermal dark matter [34, 35].

Dark energy as a huge repulsive force is proposed to explain the acceleration of inflation in the universe, and may unify many different results of observations. The dark energy seems to be the energy of a vacuum, and be zero mass. Usually assume that the dark energy connects with the cosmological constant, and is a dynamical scalar field (quintessence model). Scherrer proposed a new k-essence models unified dark matter and dark energy. Caldwell, et al., discussed observations that continue to indicate that the Universe is dominated by invisible dark matter and dark energy [36, 37].

Recently Foster, et al., searched decaying dark matter with XMM-Newton Blank-Sky Observations. PandaX Collaboration, et al., searched some experiments on dark matter. But, so far many models on dark matter and dark energy are not testability [38-42].

For new data, usual visible matter is 4.84%, dark matter is 25.96%, and dark energy is 69.2%. Some believe that dark energy distributes uniformly in the whole space, and its interactions are repulsive. But, in the solar system dark energy and dark energy cannot exist, since in which general relativity is very exact. The negative matter should be grouped, so complete uniformity is impossible. Recently, astronomers find more galaxies in the Universe without any dark matter. Guo, et

al. reported 19 dwarf galaxies that could consist mainly of baryons, and provided observational evidence that could challenge the formation theory of low-mass galaxies within the framework of standard cosmology [43-45].

Three Basic Principles of Negative Matter as Unified Dark Matter-Energy

In 1928 Dirac predicted anti-particles and the negative energy state from his equation, and he emphasized: "we cannot ignore the negative energy states". In order to prevent jumping continuously from a positive energy state to a negative energy state in the quantum theories, and to keep the stability of the world, Dirac proposed that as long as suppose that all the states of negative energy are occupied except perhaps a few of small velocity. The vacuum of the realistic world has already been filling with all negative energy state, such as the Pauli exclusion principle will come into play and prevent more than one electron going into any one state, and avoid this jumping difficulty. It is namely the well-known Dirac negative energy sea whose vacancy or hole is an anti-particle (or opposite particle). From this, the annihilation and creation between positive and opposite particles may be predicted. There is exact description in (The Principles of Quantum Mechanics). But, it prevent only jump of fermions, but cannot be applied to bosons. Therefore, the stability problem exists still. In fact, the negative energy state appears in all relativity theories as $E = \pm\sqrt{p^2c^2 + m^2c^4}$, even also in the classical theory.

From 2007 we proposed that the negative matter developed the Dirac negative energy may unify dark matter and dark energy [46-52], in which the anti-(opposite) matter and the negative matter are different. The anti-matter is that some properties of matter are opposite, for instance, charge, baryon number, lepton number, strangeness number, and so on, but their masses and total energy are still positive. These particles include positrons and various anti-particles[46-52]. The existence of these particles is already verified. Both positive and opposite matters meet to annihilate photons with conservation of energy and zero-charge. The negative matter has a negative mass and total energy. The creation of negative matter is difficult, but its existence will be stable.

2.1. Based on Dirac's negative energy state, we proposed the negative matter, whose key is negative mass. Based on $M \rightarrow M_+ - M_-$ and Eq.(3), all theories are all known. The positive and negative matters are two regions of topological separation in general case by different interactions (Figure 1), so the negative matter is invisible dark matter. The negative matter should be a necessary development of Dirac theory.

2.2. According to the principle of equivalence in general relativity, inertial mass, and gravitational mass must be equal always. Based on Eq.(3), there are only three cases: positive and positive matters, positive and negative matters, negative and negative matters. But, for negative mass, Bondi proposed three kinds of mass: inertial, passive gravitational, and active gravitational mass, and there are four cases. Such Bondi believes that the positive body will attract the negative one (since all bodies are attracted by it!), etc. It is a fallacy with contradictions.

2.3. According to Einstein's mass-energy relation $E = mc^2$, dark matter and dark energy should be unified, and it agrees with Occam's Razor. The repulsion between positive matter and negative matter shows dark energy. This is the simplest candidate of dark matter, and can be unified dark energy, and may explain many phenomena of dark matter and dark energy.

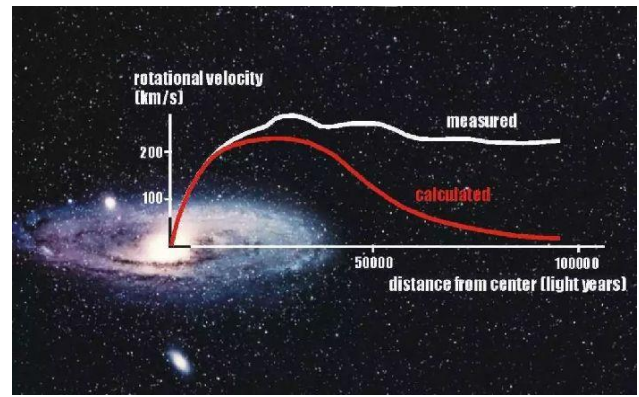


Figure 2) In Andromeda Galaxy the curve of the rotational velocity with distance. The red curve is the theoretical calculated prediction of the no-dark matter, and the white curve is the measured curve.

In Figure 2, the difference between the two curves is one of the key evidence for the existence of dark matter. According to classical mechanics, the rotational velocity is:

$$\frac{1}{R}V^2 = \frac{GM}{R^2} \tag{5}$$

The measured curve for large scale $R > r$ is:

$$V^2 = \frac{GM}{R} \approx \text{constant} \tag{6}$$

If the negative matter is introduced $M \rightarrow M_+ - M_-$, the equation (5) will become:

$$\frac{G}{R^2}(M_+ - M_-) = \frac{1}{R}V^2 \tag{7}$$

The total mass of the spherality galaxy inside radius R is:

$$M(R) = M_+ - M_- = \int_0^R (\rho_+ - \rho_-) dV = (\rho_+ - \rho_-) \int_0^R 4\pi r^2 dr = \frac{4\pi}{3} R^3 (\rho_+ - \rho_-) \tag{8}$$

$$\frac{dV}{dt} = -\frac{G}{R^2}(M_+ - M_-) = -\frac{4\pi}{3} GR(\rho_+ - \rho_-) \tag{9}$$

For the plane disk-like galaxy

$$M(R) = (\rho_+ - \rho_-) \int_0^R 2\pi r dr = \pi R^2 (\rho_+ - \rho_-) \tag{10}$$

$$\frac{dV}{dt} = -\pi G(\rho_+ - \rho_-) \tag{11}$$

If $\rho_+ \approx \rho_-$ and $M_+ \approx M_-$ $dV/dt \approx 0$, integral derive V and are approximate constants.

Basic Calculations of Negative Matter as Unified Dark Matter-Energy

For a radiation-dominated universe [14, 15] the total energy of positive and negative matters should be:

$$M_+c^2 - M_-c^2 \tag{12}$$

Because inflation is the origin of nothing, the total energy should be zero, i.e., $M_+ = M_-$

When the evolutionary process from inflation and radiation-dominated universe to the matter-dominated universe, the known total energy of usual baryon matter of non-relativity is:

$$M_+c^2 - \frac{GM_+^2}{R_+} \tag{13}$$

Assume that dark matter and dark energy are completely negative matter, so the total energy includes three parts: one of the positive

matter, one of negative matter, and their repulsion force:

$$E_{\text{Both ratio}} = M_+ c^2 - \frac{GM_+^2}{R_+} + (-M_- c^2 - \frac{GM_-^2}{R_-}) + \frac{GM_+ M_-}{R_{\pm}} \quad (14)$$

$$\frac{M_+ c^2 - \frac{GM_+^2}{R_+} + (-M_- c^2 - \frac{GM_-^2}{R_-}) + \frac{GM_+ M_-}{R_{\pm}}}{M_+ c^2 - \frac{GM_+^2}{R_+}} \quad (15)$$

We suppose that for early inflation cosmology the positive matter and the negative matter have the same mass $M_+ = M_- = M$ and both are separated. In order to simply assume that positive and negative matters form two identical spheres, respectively, so $R_+ = R_- = R, R_{\pm} = 2R$ Such Eq.(15) is simplified to:

$$\frac{-\frac{3GM^2}{2R}}{Mc^2 - \frac{GM^2}{R}} = \frac{3}{2} \frac{GM}{GM - Rc^2} \quad (16)$$

Based on the known total mass of Universe and corresponding scale [16], so

$$Rc^2 = 1.17 \times 10^{43} m^3 s^{-2} \text{ and } GM = 1.34 \times 10^{43} m^3 s^{-2} \quad (17)$$

A simple calculation obtains $\frac{3}{2} \frac{GM}{GM - Rc^2} = 11.82$

Of course, the actual situations are more complicated. But, this is a model that can be computed and compared, and may also be developed.

If $R_{\pm} = R$, positive matter and negative matter are mixed together in the same volume, so Eq.(15) is simplified to:

$$\frac{-\frac{GM^2}{R}}{Mc^2 - \frac{GM^2}{R}} = \frac{GM}{GM - Rc^2} = 7.88 \quad (18)$$

According to new data, ratio between usual matter, total matter and dark energy is 4.84:30.8:69.2 [43,44], so $30.8/4.84=6.36$, and $69.2/4.84=14.3$. Since G, M, c is invariance, R is bigger, and this ratio is bigger

When $\rho_+ = \rho_- , M_+ = M_-$, the expansion rate is unchanged.

~~$\rho_+ / \rho_- < 0, M_+ - M_- < 0, dV/dt > 0$~~ , the expansion rate is acceleration.

Other Deductions and Possible Tests on Negative Matter

It is known that the gravitational field equations with the cosmological constant are:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi k T_{\mu\nu} \quad (19)$$

We proposed the field equations of general relativity on the negative matter [48-50]:

$$G_{\mu\nu} = 8\pi k (T_{\mu\nu} - T'_{\mu\nu}) \quad (20)$$

So the cosmological constant corresponds to the negative matter, i.e., $\Lambda = 8\pi k T'_{\mu\nu} / g_{\mu\nu}$

Here $\Lambda g_{\mu\nu}$ corresponds to the negative energy state and vacuum energy (Dirac sea), and is consistent with conformal gravity theory Caldwell proposed phantom as cosmological consequences of a dark energy component with the super-negative equation of state, whose cosmic energy density has negative pressure. The total energy is negative, so it is namely a type of negative matter. Then phantom becomes an important dark energy model.

In cosmology, it is important progress that Guth proposed inflation

whose time origin is from s, and cosmic scale factor exponential expansion . Then Linde and Albrecht, et al. [53-57], proposed chaotic inflation. We proposed the mechanism of inflation as the origin of positive-negative matters created from nothing at the same time, whose quantum fluctuations correspond just to the chaotic inflation. It is a Planck time s and length cm. At this very small space the positive matter and negative matter are the very strong repulsive interaction, and exponential inflation is just a form of the strong interaction:

$$(21)$$

Here the positive matter is g, and the negative matter is-g, so $F > 0$ is a huge strong repulsive force for the length inside cm. When the scale is bigger than one of the strong interactions, the inflation finishes, and the positive matter and opposite matter will form two regions of topological separation repulsed each other (Figure 1).

Based on observations of a remarkable cosmic structure called the bullet cluster, Bradac, et al., discovered that this structure is actually two clusters of galaxies passing through one another. The new research is the first to detect luminous matter and dark matter independent of one another, with the luminous matter clumped together in one region and the dark matter clumped together in another. These observations demonstrate that there are two types of matter: one visible and one invisible. In 2007, COSMOS obtained the first three-dimensional distribution map of dark matter in the world. We researched the most complete theory of negative matter, its quantum theory, and corresponding Lobachevskian geometry. We proposed a judgment test for the negative matter as dark matter, and other possible tests [58,59].

Various positive matter and black hole exhibit the gravitational lensing effect. The negative matter will be the repulsive lensing (Figure 3), and will form the bigger Einstein ring. Both should be different in observations.

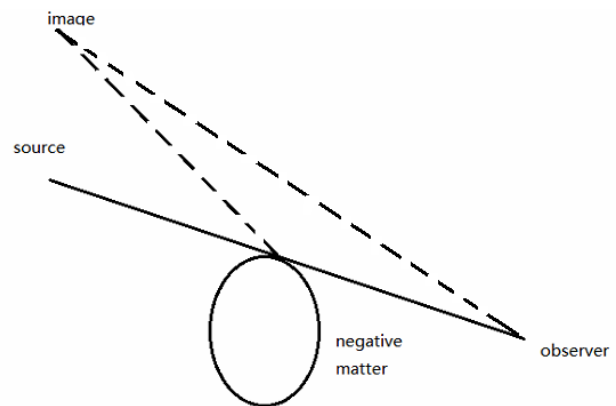


Figure 3) Repulsive lensing

Recently, astronomers observed a super-huge hole about 2 billion light-years scale (Figure 4). This is most likely an invisible region of the negative matter, because it is not the spherical symmetry, and is not found intense activity if as a huge black hole.



Figure 4) A super-huge hole in Universe

CONCLUSION

It is a notable process that the positive energy from a conjecture became to a theorem, and then to a restriction for the negative matter. But, the premises on proof of the positive energy conjecture cannot rule out Dirac's genius prophecy on the negative energy state. Further, a completely similar approach can prove that the negative matter region is always the total negative energy, and gravity is also negative energy, so the total negative energy is greater. Usually, the positive matter and the negative matter are two regions separated each other (Figure 1), and both are stable. When they annihilate, both are certainly unstable.

Generally, the whole matter space is divided into:

- Positive matter region, the gravitational energy is negative, and the total energy is positive.
- In the negative matter region, the gravitational energy is negative, and the total energy is greater negative energy.
- In a region between positive and negative matters, the repulsion energy is positive.

The total region of the three is mixed, the positive matter and negative matter are equal, the double gravity is greater than the repulsion, and the total energy should be negative, so that the whole universe accelerates the expansion, and corresponds to the dark energy as a huge repulsive force.

In fact, some physicists researched already the negative energy and various related problems.

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