

# Principles of prime numbers - Part IV - Twin prime & Polignac conjectures – Conservation of gaps

James M McCanney\*

McCanney JM. Principles of prime numbers - Part IV - Twin prime & Polignac conjectures – Conservation of gaps. J Pure Appl Math. 2026; 10(1):01-20.

## ABSTRACT

The prior papers “Principles of Prime Numbers - Part I - New Definition of Prime Numbers with ModNt Number System & Induction”, “Principles of Prime Numbers - Part II - Anatomy of  $Spp_n$  &  $Rpp_n$  Tables with Conversion from mod10 to modNt Number System” and “Principles of Prime Numbers - Part III -  $Spp_n$  &  $Rpp_n$  Table Parameters with Advanced Topics and Twin Prime Conjecture” must be read to put this paper into perspective. Those papers are a summary of three books “Calculate Primes” (2007), “Principles of Prime Numbers – Volume I” (2010) and “Breaking RSA Codes” (2014). Another peer reviewed paper can be included in the list of prior papers “New Definition of Prime Numbers with  $Spp_n$  Tables and Proofs by Induction”.

In the prior books and papers, the concept of direct calculation of prime numbers was presented using the McCanney Generator Function and a sole boundary condition. It showed that the prime numbers constitute a

complete number system modNt based on sequential prime products  $Spp_n$  with visual representation in the form of “ $Spp_n$  and  $Rpp_n$  tables”, creating the basis for proofs by induction for the Twin Prime Conjecture and other problems. The groups of prime numbers form algebraic groups. Groups of prime numbers generate larger groups beginning with Peano’s Postulates. The new mathematical system has properties of closure, completeness, reciprocity, symmetry and generates the wave equations “comb<sub>n</sub>” that predict all future primes to infinity. Each prime number or series of prime numbers has a unique ancestry. This is demonstrated in the digits of the modNt number system. Likewise, each prime or group of primes has an infinite number of descendants. Composite numbers that are relatively prime to the values of  $Spp_n$  are important in the development of real primes, twin primes and larger groupings of real primes. Without these relative primes, the process would not generate all real primes and would not be complete. Prime numbers belong to families with ancestors, descendants as well as siblings.

**Key words:** Prime numbers; Twin prime conjecture; Polignac conjecture; Calculate primes

## INTRODUCTION

Prior papers Parts I, II and III [1-3] presented many concepts with basic examples including the basis for the proof of the Twin Prime & Polignac Conjectures. They demonstrated the more than exponential growth of the twin prime only  $Spp_n$  tables discovering real twin primes. The formulae which generate future  $Spp_n$  tables show why this occurs. The current paper Part IV presents a proof for the Twin Prime Conjecture and arrives at the conclusion that there is an infinite number of twin primes. It advances that there is another way of looking at this infinity based on the work on transfinite numbers by Georg Cantor. This new view states that “every relative twin prime pair generates an infinite number of future real twin primes”. Thus, starting at any point in any  $Spp_n$  twin prime only table, every relative twin prime pair will generate an infinite number of real twin primes and thus is not a simple countable infinity. Relative primes are needed to generate the entire complete list of true primes, true twin primes or prime pairs of any gap size.

Also formalized is the concept that extends to gaps of 4 and larger showing that there is an infinite number of subsequent gap sizes and refines the concept of “Conservation of Gaps” introduced in prior papers. It is demonstrated that once a gap of size  $k$  is encountered ( $k=2, 4, 6 \dots$ ), it continues to generate relative prime gaps of the same size in all future  $Spp_n$  Tables and therefore will continue to generate real prime pairs of this gap size to infinity (presenting a solution to the Polignac Conjecture of which the Twin Prime Conjecture is a special case).

It is a result of the fact that only one “white cell” (products  $p_n \times p_i$  where  $p_n$  is the  $n^{\text{th}}$  prime number and  $p_i$  is the list of comb<sub>n-1</sub> elements) per column does not transfer to the subsequent table  $Spp_{n+1}$ . It also gives understanding

to the Goldbach Conjecture which will be the topic of the next Part V paper, showing that the criteria for failure of the Goldbach Conjecture can be understood and tested using induction. As with the Twin Prime Conjecture, it depends on the well-organized  $Spp_n$  tables building subsequent tables  $Spp_{n+1}$  from prior tables. The problem is understood in pieces. It also shows that the Goldbach solutions come in families and the solutions are related to ancestors and descendants as are all primes. The comb<sub>n-1</sub> formulae are generalized to predict twin primes with repeating wavelength of  $Spp_{n+1}$  to infinity. The ratio of  $(\sqrt{Spp_n})/Spp_{n-1}$  is shown to be the basis for understanding the growth of twin primes as well as primes in general as demonstrated in prior papers.

Before presenting the proposed solution to the Twin Prime Conjecture, the discussion will review concepts built in prior papers. This will be followed by the proposed proof of the Twin Prime Conjecture concentrating on the following:

- “Conservation of Prime Gaps”
- importance of the expression  $(\sqrt{Spp_n})/Spp_{n-1}$  (the  $E=mc^2$  of prime numbers)
- $Rpp_n$  tables
- the concept of “duplicate products”
- the effects of gap size on the final solution

## MATERIALS AND METHODS

### Review and summary of prior papers

Use the following two diagrams of “Anatomy of an  $Spp_n$  table” and “Anatomy of  $Rpp_n$  table” to help interpret the text below (Figures 1 and 2).

Department of Mathematics, Encryption-Jmccanneyscience, LLC, United States

**Correspondence:** James M McCanney, Department of Mathematics, Encryption-Jmccanneyscience, LLC, United States; Email: jmccanney@usinternet.com

**Received:** 29-Jul-2026, Manuscript No. PULJPAM-26-7757; **Editor assigned:** 31-Jul-2026, PreQC No. PULJPAM-26-7757 (PQ); **Reviewed:** 14-Aug-2026, QC No. PULJPAM-26-7757; **Revised:** 27-Aug-2026, Manuscript No. PULJPAM-26-7757 (R); **Published:** 03-Sep-2026, DOI: 10.37532/2752-8081.2026.10(1).1-20



This open-access article is distributed under the terms of the Creative Commons Attribution Non-Commercial License (CC BY-NC) (<http://creativecommons.org/licenses/by-nc/4.0/>), which permits reuse, distribution and reproduction of the article, provided that the original work is properly cited and the reuse is restricted to noncommercial purposes. For commercial reuse, contact [reprints@pulsus.com](mailto:reprints@pulsus.com)

Reprint permissions: Most of the materials in this paper are exactly copied from two previously copyrighted sources “Calculate Primes” (c 2007) and “Principles of Prime Numbers – Volume I” (c 2010) author J M McCanney publisher jmccanneyscience.com press. Permission is granted to Journal of Pure and Applied Mathematics to reprint as open source as long as the paper and author are cited.

## Anatomy of $Spp_n$ Table

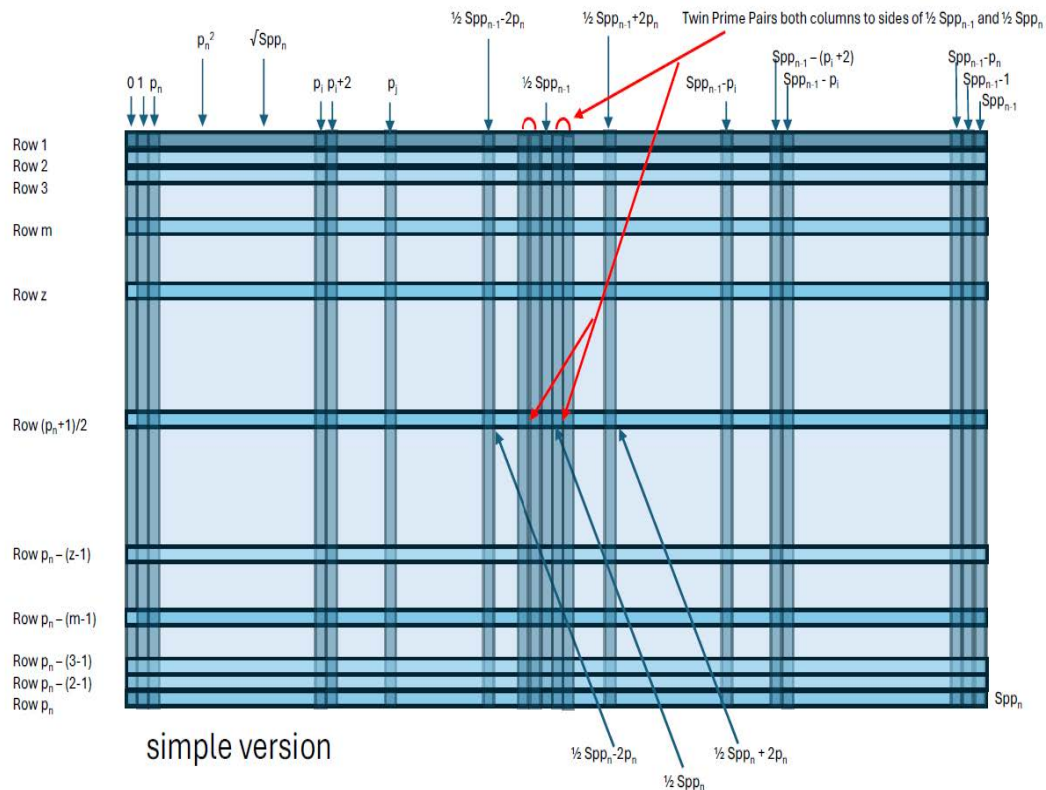


Figure 1) Anatomy of an  $Spp_n$  table

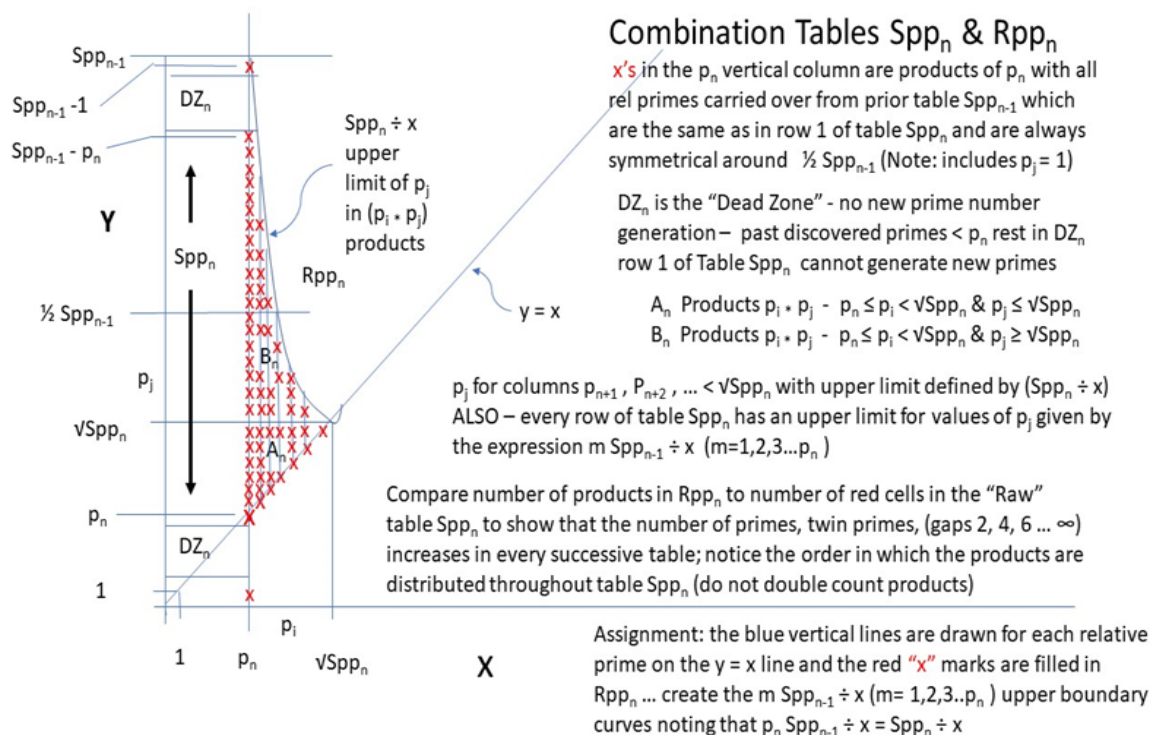


Figure 2) Anatomy of a  $Rpp_n$  table

All elements of row 1 of an  $Spp_n$  table are defined as the  $comb_{n-1}$  of wavelength  $Spp_{n-1}$  which repeats to infinity and contains all future relative primes including all real prime numbers using the expression  $=mSpp_{n-1} + comb_{n-1}$  ( $m=1, 2, 3 \dots$ ). All relative primes in table  $Spp_n$  that are not real primes are eliminated using the  $Rpp_n$  table which is the mapping of all the products  $p_i \times p_j$  where  $p_i$  are all numbers from row 1 such that  $p_i = p_n, p_{n+1}, p_{n+2}, \dots, p_{max} < \sqrt{Spp_n}$  and  $p_j$  are all elements of row 1 such that  $p_j = 1$  and  $p_i \leq p_j \leq (Spp_n \div p_i)$ . The first series of these products have a special purpose ( $p_n \times p_j$ ) and are labeled “white cells” since they are not relatively prime to  $Spp_n$ . Duplicate products arise and there is a process for eliminating duplicate products. Each future table has a larger percentage of duplicate products, the effect of which will be explained. It is the  $Rpp_n$  table that is used to determine the number of real primes in each  $Spp_n$  table by eliminating composite numbers.

A subsequent table  $Spp_{n+1}$  is formed by first stringing rows 1 through  $p_n$  of table  $Spp_n$  in sequence (after removing the “white cells”) to form row 1 of table  $Spp_{n+1}$ . Complete rows 2 through  $p_{n+1}$  by adding multiples of  $Spp_n$  to the numbers in row 1. It will be shown that every subsequent  $Spp_{n+1}$  table has more primes, twin primes, prime pairs of gap  $k$ , or gap patterns than in the prior table. There will not only be more, but more than exponentially more in each subsequent  $Spp_{n+1}$  table. It is the essential piece of the puzzle needed to prove the Twin Prime Conjecture. The ability to divide the direct calculation of prime numbers into groups (the  $Spp_n$  tables) allows for generalized equations to be built which leads to the proof by induction.

The Twin Prime Conjecture is a special case of the general case defined by the conservation of gaps which states that every pair of relative primes of gap  $k$  in each table  $Spp_n$  will produce an infinite number of future prime pairs of gap  $k$ . As the tables are developed, the future combs are modifications of prior combs by reducing the number of elements and therefore puts an upper limit on the counting function. This is an important development over prior counting functions in that it: 1) Predicts exactly where to find primes, twin primes or prime pairs of any gap size or gap pattern and 2) Sets the upper limit on the location of primes (the prior methods only could give a local average leaving the possibility of rogue gaps or rogue primes) and 3) It creates a family structure of the primes having ancestors, descendants, cousins and distant relatives. This is why twin primes continue to occur no matter how far out the number line you go and no matter how far apart the prime numbers become. With each subsequent  $Spp_n$  table, the predicting equation ( $comb_{n-1}$ ) is refined to be more precise than the prior table.

A new number system results directly from the McCanney Generator Function called Nature's Number System or modNt. As opposed to mod10 or other modulo number systems, it uses the  $Spp_n$  values 2, 6, 30, 210, 2310, ... for digits (rather than for example 10, 100, 1000 ... in mod10 system). It is the natural number system of prime numbers. It is one of major discoveries of this series of papers first presented in the 2007 book “Calculate Primes” [4]. It was shown in prior papers that the modulo M number systems were a hindrance to understanding prime numbers. When used in conjunction with the  $Spp_n$  tables, prime numbers are noted by sequences based on their modNt ancestry. Every number has an ancestral path going back to the alpha prime  $p_a=0$ . Every digit of the modNt number represents a generation ancestor. The ancestral path going back to the alpha prime  $p_a=0$  involves removing modNt digits requiring no calculations, unlike mod10 numbers all of which show no ancestral heritage or patterns, and for which extensive calculations are required for each one. In the  $Spp_n$  table using the modNt number system, the value of a given cell derives its higher order digit from the left column and lower order digits from the value of the cell at the top row 1 of the  $Spp_n$  table. For this reason, all the values of a given  $Spp_n$  table need not be filled in since all you need is the value of the left column and top row (unlike mod10 numbers which require extensive calculations and which are unrelated by any patterns).

The  $Spp_n$  tables contain only real and relatively prime numbers and form a complete isolated number system. Since the cells are colored red, they are termed as “red cell only”. The red cell only  $Spp_n$  table can be further reduced to tables specific to certain gap sizes such as the twin prime only tables which has its own  $comb_{n-1}$  predictive equation. This comb is the same as the comb for the complete red cell only table (which contains all relative primes including all real primes from  $p_n$  to  $Spp_n$ ) but with only the relative twin primes of row 1 and their respective columns included. When working with the twin prime only table (or any table containing only row 1 relative prime pairs of a given gap size  $k$ ), all real twin prime pairs from  $p_n$  to  $Spp_n$

are contained in the columns of this table. All future twin primes (or pairs of primes of gap  $k$ ) are predicted by the  $comb_{n-1}$  equation, being directly calculated in groups by addition only. This is the fundamental concept of the new definition of primes numbers and directly calculating prime numbers (no long division or other numbers are necessary).

The structure of “white cells” is a major discovery. It has been established that there is one and only one white cell in a given column in the  $Spp_n$  table, therefore any relative prime pair of gap  $k$  that exists in row 1 of any  $Spp_n$  table will have  $p_n - 2$  pairs with gap  $k$  in its columns to carry forward to form row 1 of the subsequent table  $Spp_{n+1}$ . By induction, this process continues to infinity, guaranteeing the propagation of all gap sizes once they are formed in the parent  $Spp_n$  table. The other part of the solution shows that there is: 1) a greater than exponential increase in red cells in each subsequent table, 2) while there is an exponentially diminishing number of composite numbers (non-primes) products of  $p_i \times p_j$  in each subsequent table. The result is a more than exponential growth of primes, twin primes or prime pairs of gap  $k$  in each subsequent table. Examples were given in the previous papers of this series. It cannot be stressed enough, the importance of the concept that relative primes are as much a prime number as real prime numbers in a given  $Spp_n$  table and are instrumental in generating real primes in future tables. These are not counted as real primes; they are only used in the generation process of calculating real primes. This is one of the pieces of the puzzle that has been missing in the understanding of prime numbers.

Once a gap of length  $k$  has been generated, it will continue to generate gaps of length  $k$  in all future tables to infinity. A comb equation can be written from any of the tables to predict all descendant gaps, and all future gaps  $k$  will have ancestry leading back to the original prime pair which will be reflected in the digits of the modNt number. There is an aspect to the concept of “Conservation of Gaps” in that, if a large gap occurs in the list of prime numbers, then there will be small gaps nearby to compensate for the large gap. Look at the prime number table to verify this is true. Likewise, if there is a small gap in a region or series of small gaps, there will be a large gap to balance. The principle of “Conservation of Gaps” is a formulated as a theorem. This is because there is a fixed number of products  $p_i \times p_j$  that “cancel cells” in a given  $Spp_n$  table (produce non-primes), so if they tend to cluster in one area of the  $Spp_n$  table (creating a large gap), then that leaves fewer products to cancel primes in nearby areas of the same table. This is observed in the scope of analyzing the  $Spp_n$  tables.

It was also established in the prior papers that if a large gap occurs in a given area of the prime number table, the large gap does NOT carry to future  $Spp_n$  tables or affect the future distribution of gaps in any way, since the large gap is a local event in the  $Rpp_n$  product table caused by a clustering of products  $p_i \times p_j$ . Since there is a set number of products that identify composite numbers, this means that near this clustering there will be a lack composite numbers (causing the gaps of smaller sizes). These two statements may seem to be contradictory but are not. This is because the real prime gaps themselves do not carry to generate future tables, but the entire red table (all rows and columns) carry forward to the next  $Spp_{n+1}$  table except for the evenly (and symmetrically) distributed white cells for which one and only one occurs in the columns of the prior table to build row 1 of the subsequent table. An example is the largely out of place gap 34 between 1327 and 1361. It is due to a clustering of products in the  $Spp_n$  table but does not carry to future  $Spp_{n+1}$  tables. As prime numbers get larger, the max gaps have relatively smaller deviations from the average gap of the region. This is another result of the  $comb_{n-1}$  equations that predict all prime numbers preventing “rogue gaps”. The following is a summary of progression of tables from  $n=a$  ( $p_a=0$ ) to  $n=6$  ( $p_6=13$ ) developed in prior papers of this series to review the process of building tables.

**Review of prior work – Gap  $k=2$  (Twin primes):** Relative twin prime gaps carrying to future tables and generate real twin primes. Look in the  $Spp_n$  tables below and follow a relative twin prime pair into subsequent tables. For this example, chose (17, 19) found in row 1 of table  $n=4$  with  $p_4=7$  and  $Spp_4=210$  (Figures 3 and 4). Looking down the columns below 17 and 19, locate the generated pair (167, 169\*). 169 is marked with a star (\*) denoting a relative prime. 167 and 169 are generated by adding multiples of  $Spp_{n-1}=30$  to 17 and 19. 169 has its lowest prime factor 13 which is larger than  $p_4=7$ . It is relatively prime to  $210=2 \times 3 \times 5 \times 7$ . 169 is as much of a prime number in this table as any real prime. Although it is not itself a prime number, it is needed to generate future real prime numbers and therefore real twin prime

in this table as any real prime. Although it is not itself a prime number, it is needed to generate future real prime numbers and therefore real twin prime pairs. It is this understanding of the roll of relative prime numbers in the generation of real prime numbers that has been missing in the standard definition and view of prime numbers. See all the twin primes (relative and real) generated in row 2 to  $p_n=7$  in table Spp<sub>4</sub> below. Figure 3 shows the sequence of Spp<sub>n</sub> tables from the  $n=\alpha$  (alpha) table and Peano's Postulates

to table n=4. Figure 4 shows the n=4 table with all prime and non-prime numbers color coded. Figure 5 removes all other non-relative prime columns showing the progression to “red cell only” table n=4. Figure 6 shows only the twin prime columns in the “red twin prime only” table (one could also select a gap k=4 table that would create a gap 4 only table) [4]. These are the ancestors of all future twin primes using the McCanney Generator Function which is visualized in the Spp<sub>n</sub> tables.

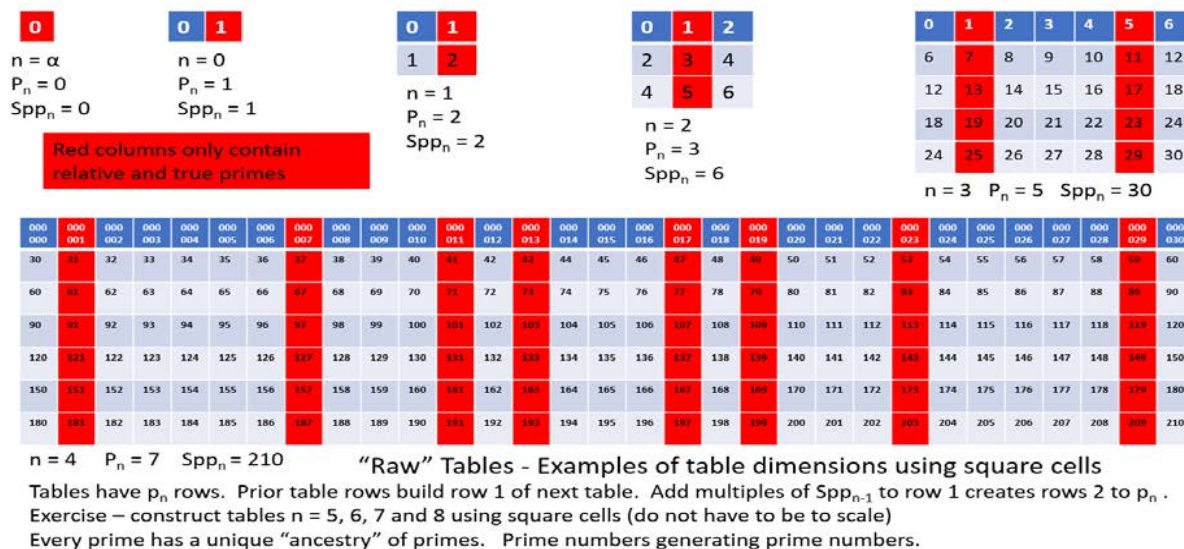


Figure 3)  $S_{p|pn}$  tables  $n=a, 0, 1, 2, 3$  and 4

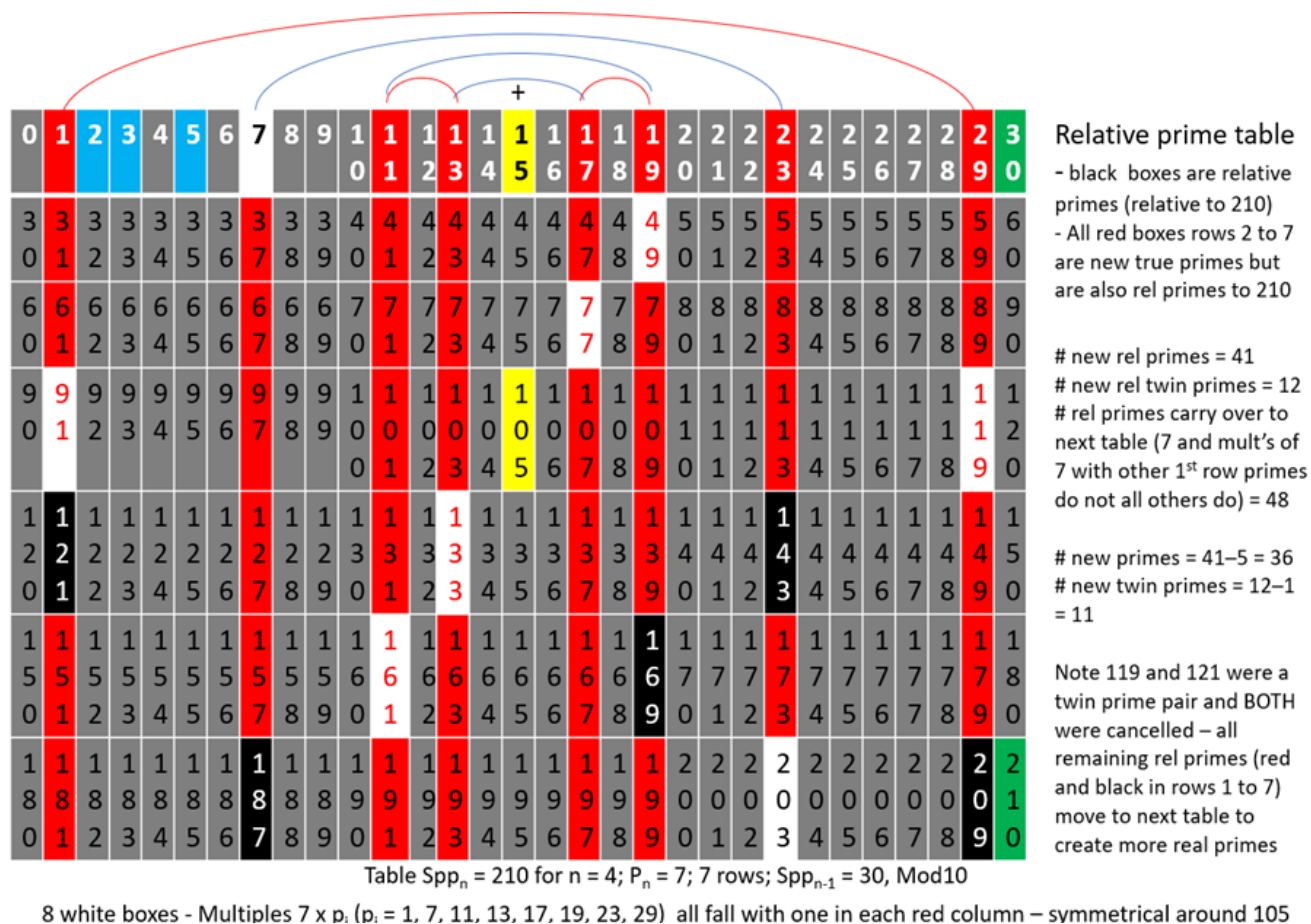


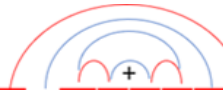
Figure 4)  $Spp_4$  "raw" table includes all numbers  $< 210$ ; relative primes in red columns

White boxes (multiples of 7) are symmetrical around center point 105 (between 103 and 107). White boxes have “complements”  
 $7+203=49+161=77+133=91+119=210$

There is one and only one white box in each column.

Black boxes are symmetrical; multiples of 11 are symmetrical around  $1/2Spp_5 = \frac{1}{2} \times 2310 = 1155$  and of 13 around  $1/2Spp_6 = \frac{1}{2} \times 30030 = 15015$ .

Notice this is a mathematical closed group with 209 and 1 being a “twin prime pair” having the properties of closure, reciprocity, symmetry, completeness and forms a wave of wavelength 210 that repeats to  $\infty$  where all future prime numbers are defined by this repeating wave (see text for further details)



|   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 7 | 1 | 1 | 1 | 1 | 2 | 2 | 3 |
| 3 | 3 | 3 | 4 | 4 | 4 | 4 | 5 | 5 | 6 |
| 0 | 1 | 7 | 1 | 3 | 7 | 9 | 3 | 9 | 0 |
| 6 | 6 | 6 | 7 | 7 | 7 | 7 | 8 | 8 | 9 |
| 0 | 1 | 7 | 1 | 3 | 7 | 9 | 3 | 9 | 0 |
| 9 | 9 | 9 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 7 | 0 | 0 | 0 | 0 | 1 | 1 | 2 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 3 | 3 | 3 | 3 | 4 | 4 | 5 |
| 0 | 1 | 7 | 1 | 3 | 7 | 9 | 3 | 9 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 5 | 5 | 6 | 6 | 6 | 6 | 7 | 7 | 8 |
| 0 | 1 | 7 | 1 | 3 | 7 | 9 | 3 | 9 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 8 | 8 | 8 | 9 | 9 | 9 | 9 | 0 | 0 | 1 |
| 0 | 1 | 7 | 1 | 3 | 7 | 9 | 3 | 9 | 0 |

Primes Only Table  $Spp_n = 210$  for  $n = 4$ ;  
 $P_n = 7$ ; 7 rows;  $Spp_{n-1} = 30$ ; mod10

Red arcs indicate twin primes

#### Table 4 – Primes Only

Multiples of prime numbers in row 1 removed (white multiples of 7 and black multiples of higher order primes 11 & 13). Note 7 generates new primes.

White Product boxes:  
 $7 \times 1 = 7$ ;  $7 \times 7 = 49$ ;  $7 \times 11 = 77$ ;  
 $7 \times 13 = 91$ ;  $7 \times 17 = 119$ ;  
 $7 \times 19 = 133$ ;  $7 \times 23 = 161$ ;  $7 \times 29 = 203$

Black Product Boxes

$11 \times 11 = 121$ ;  $11 \times 13 = 143$ ;  
 $11 \times 17 = 187$ ;  $11 \times 19 = 209$ ;  
 $13 \times 13 = 169$

This is Table 3 with all the gray columns removed

white boxes will NOT carry to next table; red and black will carry to create the next table. Prime numbers are generating new primes (twin primes are generating new twin primes)

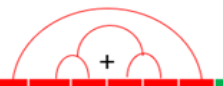
Figure 5)  $Spp_4$  relative primes only table with white and colored cells

White boxes (multiples of 7) are symmetrical around center point 105 (between 103 and 107). White boxes have “complements”  
 $49+161=77+133=91+119=210$

There is one and only one white box in each column.

Black boxes are symmetrical; multiples of 11 are symmetrical around  $1/2Spp_5 = \frac{1}{2} \times 2310 = 1155$  and of 13 around  $1/2Spp_6 = \frac{1}{2} \times 30030 = 15015$ .

Notice this is a mathematical closed group with 209 and 1 being a “twin prime pair” having the properties of closure, reciprocity, symmetry, completeness and forms a wave of wavelength 210 that repeats to  $\infty$  where all future twin prime numbers are defined by this repeating wave (see text for further details)



|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 2 | 3 |
| 3 | 3 | 4 | 4 | 4 | 4 | 5 | 6 |
| 0 | 1 | 1 | 3 | 7 | 9 | 9 | 0 |
| 6 | 6 | 7 | 7 | 7 | 7 | 8 | 9 |
| 0 | 1 | 1 | 3 | 7 | 9 | 9 | 0 |
| 9 | 9 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 2 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 3 | 3 | 3 | 3 | 4 | 5 |
| 0 | 1 | 1 | 3 | 7 | 9 | 9 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 5 | 6 | 6 | 6 | 6 | 7 | 8 |
| 0 | 1 | 1 | 3 | 7 | 9 | 9 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 |
| 8 | 8 | 9 | 9 | 9 | 9 | 0 | 1 |
| 0 | 1 | 1 | 3 | 7 | 9 | 9 | 0 |

Twin Primes Only Table  $Spp_n = 210$  for  $n = 4$ ;  
 $P_n = 7$ ; 7 rows;  $Spp_{n-1} = 30$ ; mod10

Red arcs indicate twin primes

#### Twin Primes Only

Multiples of prime numbers in row 1 removed (white multiples of 7 and black multiples of higher order primes 11 & 13).

White Product boxes:  
 $7 \times 7 = 49$ ;  $7 \times 11 = 77$ ;  $7 \times 13 = 91$ ;  
 $7 \times 17 = 119$ ;  $7 \times 19 = 133$ ;  
 $7 \times 23 = 161$

Black Product Boxes

$11 \times 11 = 121$ ;  $11 \times 19 = 209$ ;  
 $13 \times 13 = 169$

white boxes will NOT carry to next table; red and black will carry to create the next table. Twin Primes are generating new twin primes and will carry to next table to generate more twin primes

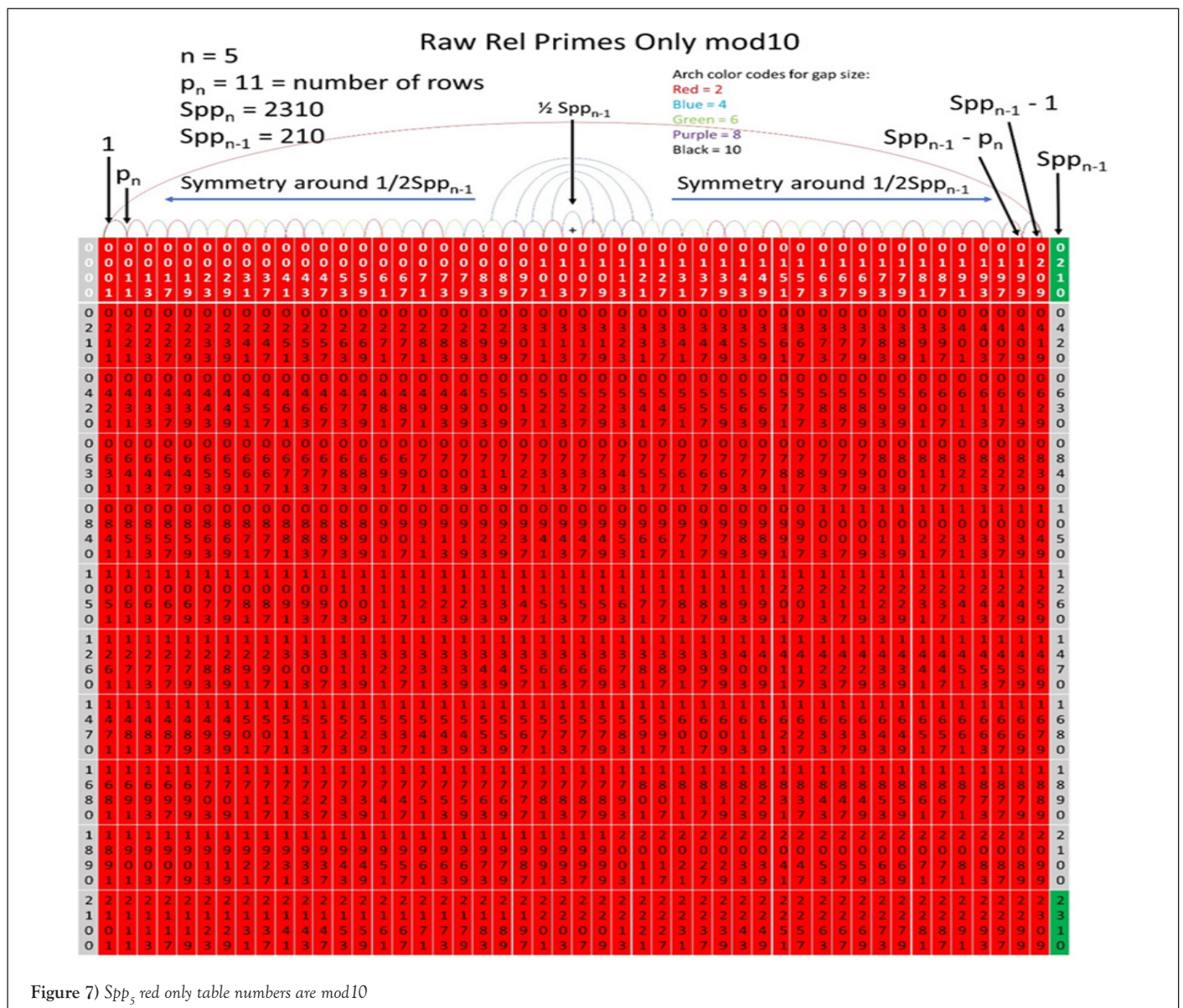
Figure 6)  $Spp_4$  table - twin primes only with white and black cells

The  $n=4$  "red cell only" Figure 5 is shown above. All the prime numbers (relative and real) are found in the red columns, being generated by the prior table group of prime numbers. This is an autonomous set of numbers closed under addition. This is the basis of "Calculate Primes". It is a self-contained mathematical system of prime numbers (prime numbers generating prime numbers in groups using only the operation of addition).

Use the standard method to create table  $Spp_5$  from table  $Spp_4$  (see red only  $Spp_3$  table Figure 7 below). Find (167,169\*) in row 1 of table  $Spp_5$  and see all the generated relative prime pairs of gap  $k=2$  (twin prime pairs) in the columns below (many will be discovered to be real twin primes). Then find all the other twin prime pairs in row 1 and all of the generated twin prime pairs in the columns below these (locate twin primes using the red arcs above the pairs). Gap sizes are identified by color coded arcs connecting adjacent columns. After including the white cells, for each relative twin prime pair in row 1, there will be  $(p_5-2)=11-2=9$  relative twin prime pairs that proceed to row 1 of the next table  $Spp_6$ . This is a multiplication of 9 and this number increases in the next table by another factor of  $(p_6-2)=13-2=11$ . Be sure to include the twin prime pair (1, 29) (Figure 6) which in the columns below generates (29,31), (59,61), (149,151), (179,181) and (209,1) which will generate more twin primes in the subsequent table. As explained in prior papers of this series, addition is closed as in an Abelian group (the rational for grouping 29 with 1 and 209 with 1). Follow these to row 1 of the following table. Select the gap 4 pairs and do the same as they propagate into subsequent tables.

Below is the series of  $Spp_5$  tables generated from the  $n=4$  tables above. Figure

7 below is the "raw"  $Spp_5$  red cell only table using mod10 numbers with no white cells or other colored cells noted. It contains all real primes and cells whose values are relatively prime to  $Spp_5=2310$ . It is closed in that all factors of composite numbers in this table are contained in row 1. It is also closed under group addition. It is complete in that all numbers that are not relatively prime to  $Spp_5$  are not found in this table and all numbers that are relatively prime to 2310 (including all real primes) are found in this table. There is reciprocity such that all cells of the prior table  $Spp_4$  can be calculated from all cells in this table. There are numerous symmetries including but not limited to 1) row 1 symmetry around midpoint  $\frac{1}{2} Spp_4=105$ , 2) symmetry all cells around midpoint of table  $Spp_5=2310/2=1155$  and 3) there is symmetry in  $comb_{5,1}$  when added to and subtracted from  $Spp_{5,1}=210$  to predict all future relative prime numbers to infinity. Figure 8 is the definition of modNt number system to 510,509. Figure 9 is the same  $Spp_5$  table but converted to modNt numbering. Figure 10 is the same  $n=5$  table with white and black colored cells (products of 13). Figure 11 is table  $n=5$  with blue columns marked to note columns that have no twin primes (to be removed in creating the Figure 12). Figure 12 is the twin prime red cell only table. Figure 13 is the twin prime only table with white cells marked. Figure 14 is the same table with white cells and all other products from the  $Rpp_5$  table applied (black cells). Figure 15 is the isolated rows of table  $Spp_5$  ready to be placed to form row 1 of table  $Spp_6$ . These tables were developed in prior papers with the intent of using them for the final proof of the Twin Prime Conjecture using induction.



| Digit in column                                                                          | 7                                                                                                                                                                                                                  | 6                                                                                                                                                     | 5                                                                                                                      | 4                                                                  | 3                                         | 2                          | 1              |
|------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------|-------------------------------------------|----------------------------|----------------|
| $n \bmod 10$                                                                             | 6                                                                                                                                                                                                                  | 5                                                                                                                                                     | 4                                                                                                                      | 3                                                                  | 2                                         | 1                          | 0              |
| $p_n \bmod 10$                                                                           | N/A                                                                                                                                                                                                                | N/A                                                                                                                                                   | N/A                                                                                                                    | N/A                                                                | N/A                                       | N/A                        | N/A            |
| $P_n \bmod 10$                                                                           | 13                                                                                                                                                                                                                 | 11                                                                                                                                                    | 7                                                                                                                      | 5                                                                  | 3                                         | 2                          | 1              |
| $P_n \bmod Nt$                                                                           | 201                                                                                                                                                                                                                | 121                                                                                                                                                   | 101                                                                                                                    | 21                                                                 | 11                                        | 10                         | 1              |
| $Spp_n \bmod 10$                                                                         | 30030                                                                                                                                                                                                              | 2310                                                                                                                                                  | 210                                                                                                                    | 30                                                                 | 6                                         | 2                          | 1              |
| $Spp_n \bmod Nt = 10^n$                                                                  | 1,000,000<br>– $10^6$                                                                                                                                                                                              | 100,000<br>– $10^5$                                                                                                                                   | 10,000<br>– $10^4$                                                                                                     | 1000<br>– $10^3$                                                   | 100<br>– $10^2$                           | 10<br>– $10^1$             | 1<br>– $10^0$  |
| Max digit Value in $\bmod 10 = p_{n+1} - 1$                                              | 16                                                                                                                                                                                                                 | 12                                                                                                                                                    | 10                                                                                                                     | 6                                                                  | 4                                         | 2                          | 1              |
| Pattern each digit repeats the given pattern over and over (values given in $\bmod 10$ ) | 0 x 30030<br>1 x 30030<br>2 x 30030<br>3 x 30030<br>4 x 30030<br>5 x 30030<br>6 x 30030<br>7 x 30030<br>8 x 30030<br>9 x 30030<br>10x30030<br>11x30030<br>12x30030<br>13x30030<br>14x30030<br>15x30030<br>16x30030 | 0 x 2310<br>1 x 2310<br>2 x 2310<br>3 x 2310<br>4 x 2310<br>5 x 2310<br>6 x 2310<br>7 x 2310<br>8 x 2310<br>9 x 2310<br>10x2310<br>11x2310<br>12x2310 | 0 x 210<br>1 x 210<br>2 x 210<br>3 x 210<br>4 x 210<br>5 x 210<br>6 x 210<br>7 x 210<br>8 x 210<br>9 x 210<br>10 x 210 | 0 x 30<br>1 x 30<br>2 x 30<br>3 x 30<br>4 x 30<br>5 x 30<br>6 x 30 | 0 x 6<br>1 x 6<br>2 x 6<br>3 x 6<br>4 x 6 | 00=0x2<br>11=1x2<br>22=2x2 | 0=0x1<br>1=1x1 |
| Max number in pattern $\bmod 10$                                                         | 510509                                                                                                                                                                                                             | 30029                                                                                                                                                 | 2309                                                                                                                   | 209                                                                | 29                                        | 5                          | 1              |
| Max number pattern $\bmod Nt$                                                            | 16.12.10.6.4.2.1                                                                                                                                                                                                   | 12.10.6.4.2.1                                                                                                                                         | 10.6.4.2.1                                                                                                             | 6.4.2.1                                                            | 4.2.1                                     | 2.1                        | 1              |

Figure 8) Definition of  $\bmod Nt$  numbers to 510509



Figure 9)  $Spp_5$  red only table -  $\bmod Nt$  number

Twin Primes columns noted with red arcs above the columns – note symmetry around  $\frac{1}{2} Spp_{n-1}$   
 Red arc from 000001 to 006421 forms twin prime that generates future twin primes  
 To create twin prime only tables - select only these columns  
 Fill in blank spaces – combine left column higher order digit with top row lower order digits  
 White double headed arrows indicate patterns carried from rows of  $Spp_{n-1}$  Table to build row 1  
 Note symmetry around  $\frac{1}{2} Spp_n$  of white boxes (products of  $p_n$  with all row 1 relative primes)  
 One white box per column.  
 Black boxes are products of 13 with row 1 relative primes. All products to complete table see  
 $Rpp_n$  Table discussion in next section.

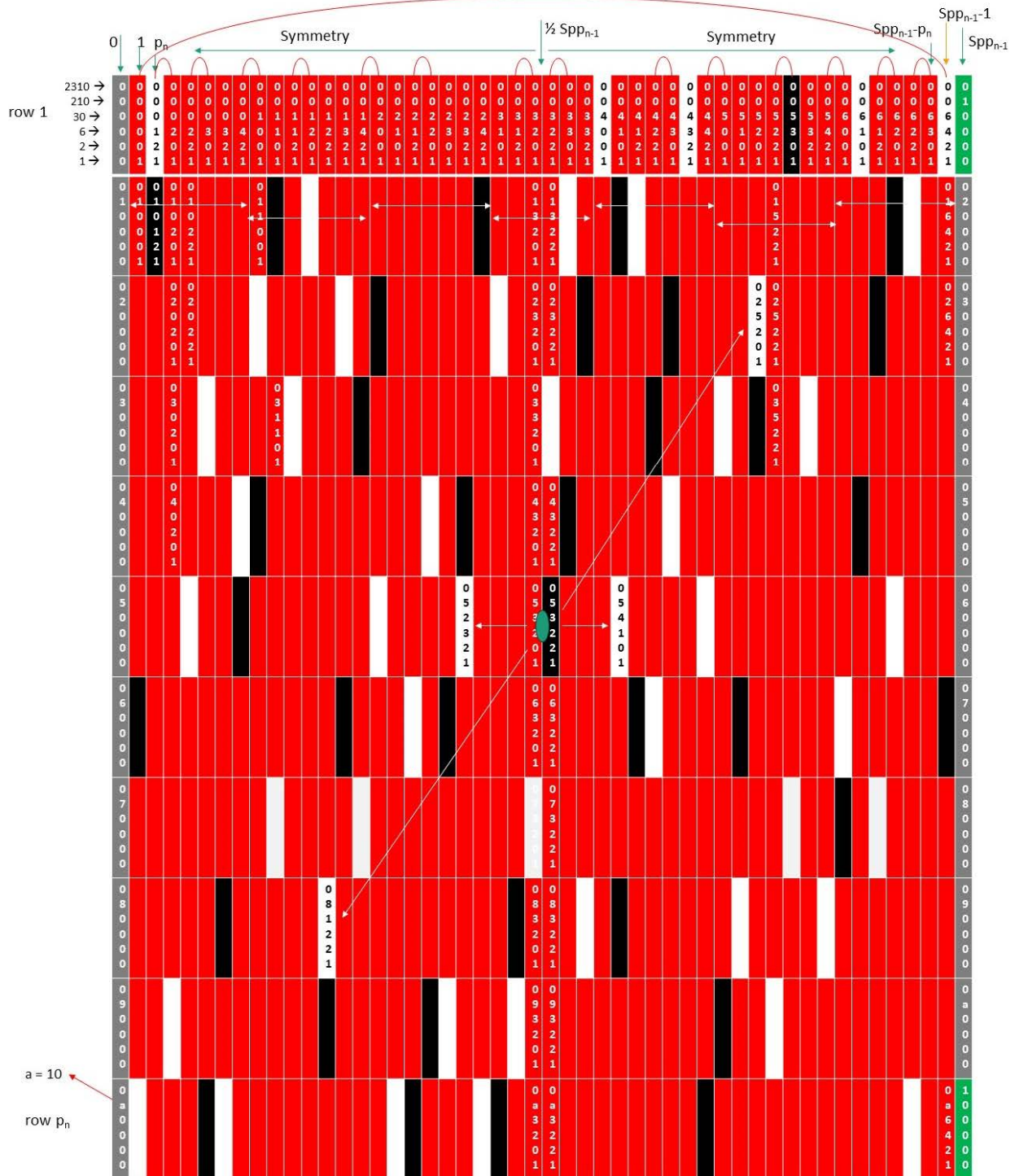


Figure 10)  $Spp_5$  red only table with white cells (products of 11) and black cells (products of 13)

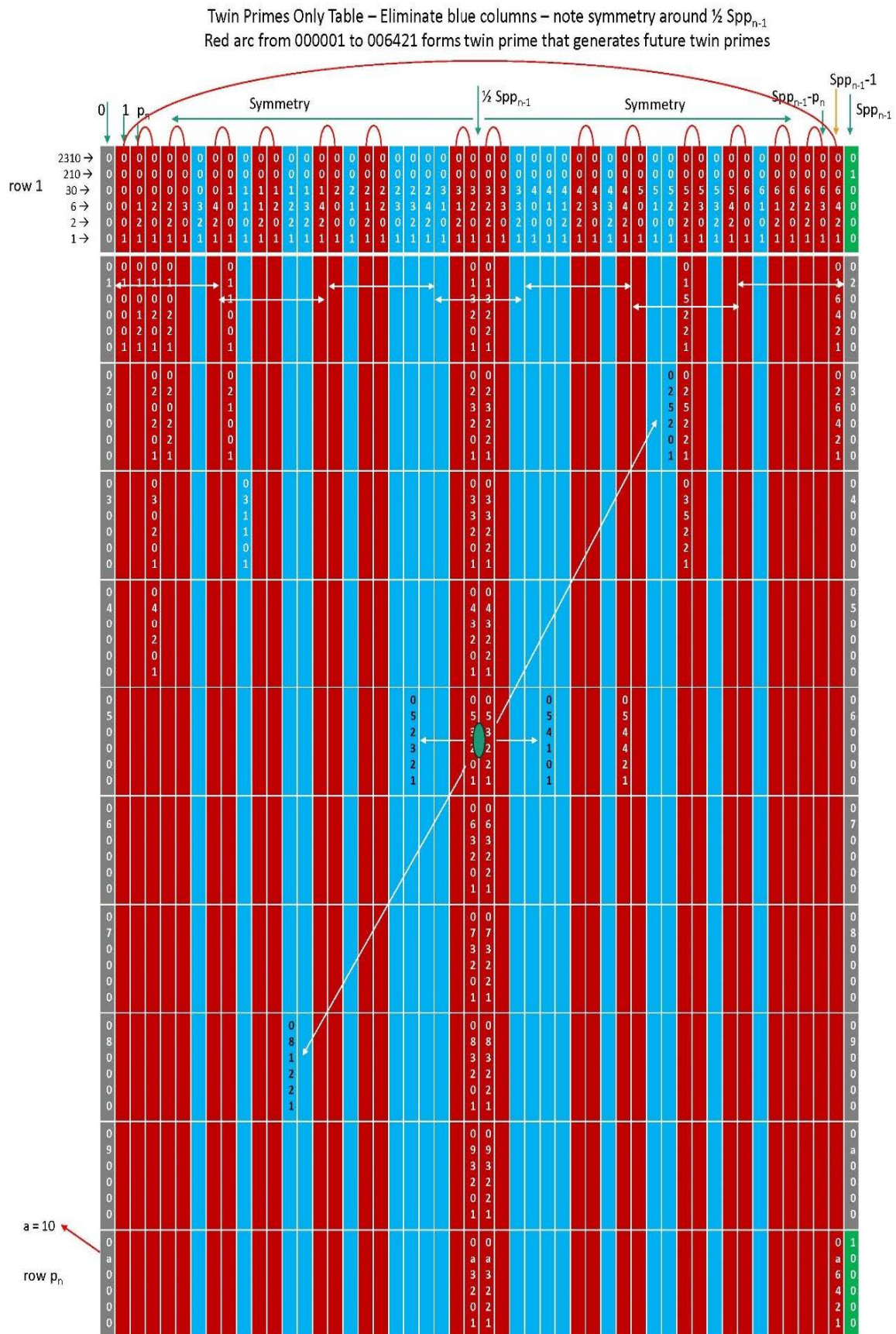


Figure 11) Blue columns are non-twin primes to be removed

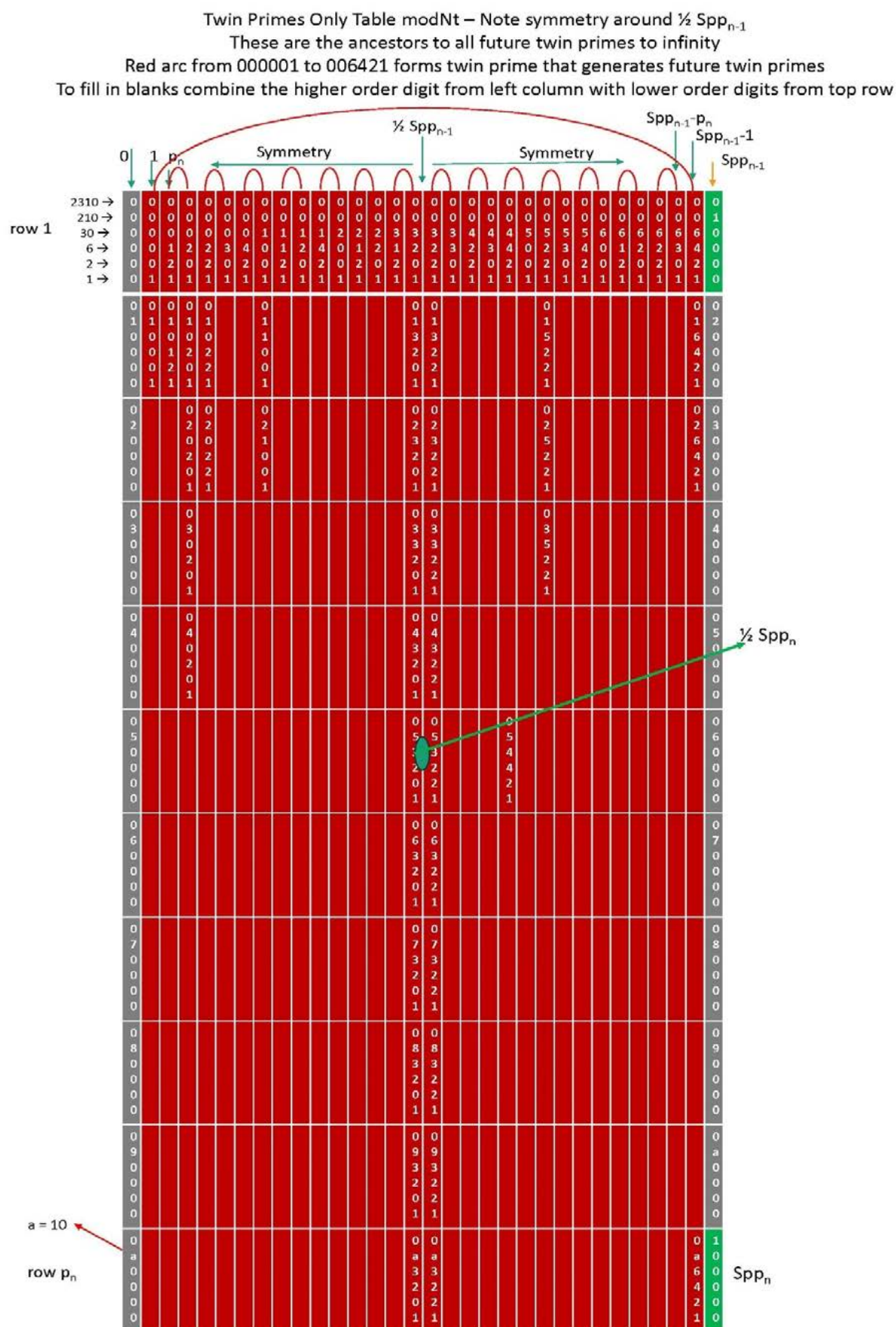


Figure 12)  $Spp_5$  twin prime only red cell only table

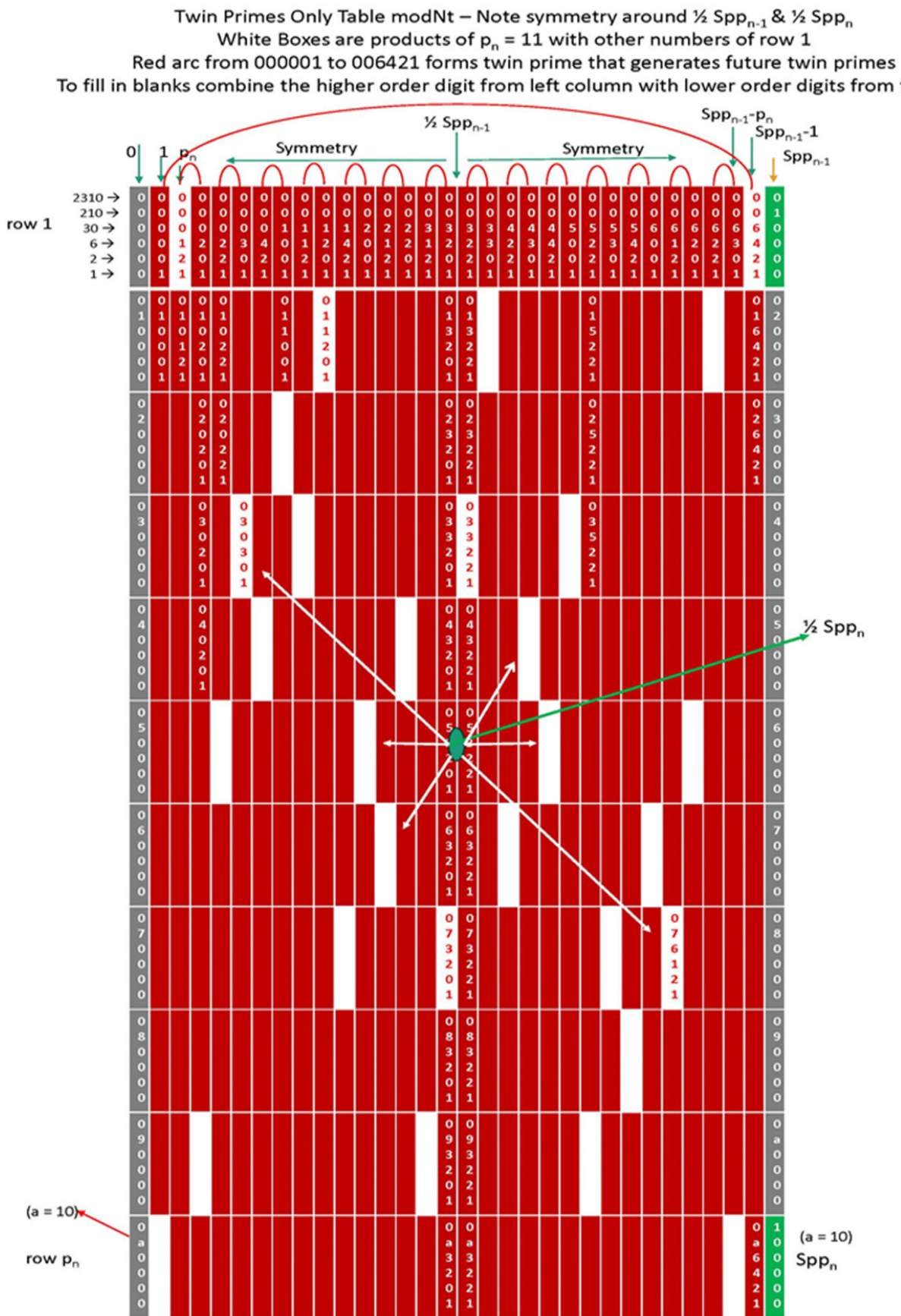


Figure 13)  $Spp_5$  twin prime table with white cells

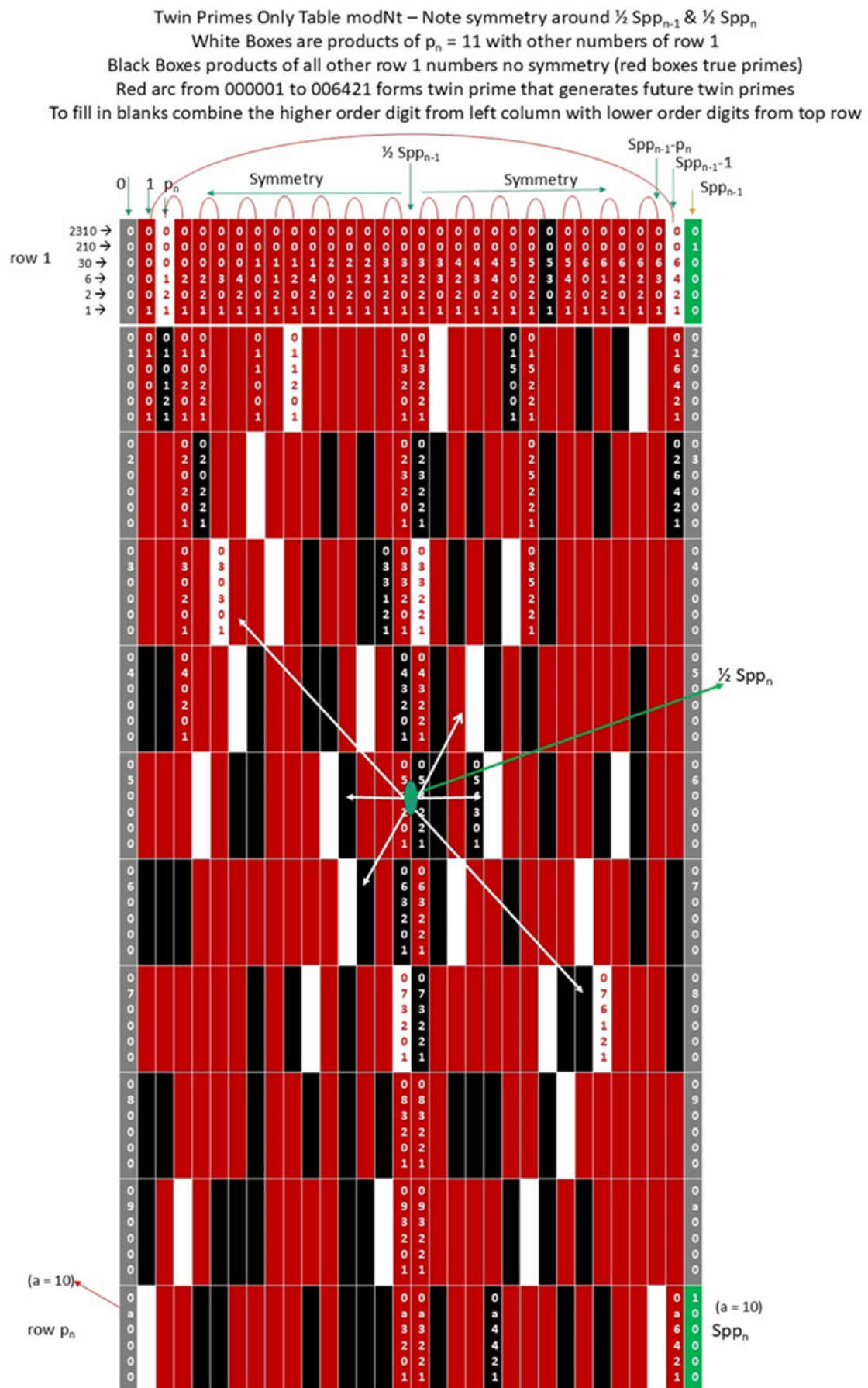


Figure 14)  $Spp_5$  final twin prime table - red cells are real twin primes

At this point we are going to transition to modNt number system to determine the real twin primes. The definition of modNt numbers up to 510,509 is given in Figure 8. In following the pair (167,169\*), the modNt equivalent values are (5221, 5301) found in row 1 of Figure 14.

The conversion of (167,169\*) to modNt values is as follows:

- $5221 \text{ modNt} = 5 \times 30 + 2 \times 6 + 2 \times 2 + 1 \times 1 = 167 \text{ mod10}$
- $5301 \text{ modNt} = 5 \times 30 + 3 \times 6 + 0 \times 2 + 1 \times 1 = 169 \text{ mod10}$

The red cell only twin prime table with white cells  $\text{Spp}_5$  (Figure 13) is the table used to generate the next twin prime only table  $\text{Spp}_6$  by stringing the rows 1 to  $p_5=11$  in sequence to build row 1 of the twin prime only table  $n=6$ . From this point forward the twin prime calculations are autonomous. They are a subset of the prime number red tables. Only the white cells (the cells not relatively prime to  $\text{Spp}_5=2310$ ) do not transfer to create row 1 of the next table. Notice the symmetry of both the red and white cells around the  $\frac{1}{2}$  point of row 1 and  $\frac{1}{2}$  point of the table (the green oval in the center of the table). For example, if you took the white cell pattern and rotated it 180 degrees it would show the exact same pattern. This table building process is the visualization of the McCanney Generator Function and its boundary condition. Figure 14 is the same table with all products (white and black cells) noted [5]. All the red cells that remain are true prime numbers. If both

members of a twin prime remain then it is a true twin prime. Twin primes are discovered in groups by direct calculations from prior groups of relative primes. This was the original intent of the book titled "Calculate Primes".

Now follow all newly generated pairs (including the black cells because they are relatively prime) to the next table  $n=6$  with  $p_6=13$ . Put into sequence rows 1 through  $p_5=11$  of table  $n=5$  in Figure 15 below (remove white cells before creating row 1 of table  $n=6$ ) where more relative twin primes are generated, all of which will move to row 1 of the  $n=7$  table to generate many more. In table  $n=7$  with  $p_7=17$ ,  $p_6=13$  will move into the Dead Zone  $\text{DZ}_7$  and although (167,169) it will not directly generate more twin primes, all of the relative twin prime pairs that it generated will continue to branch out and this process continues to infinity (if you doubt this continue the process as long as you like). Rather than carry out this Promethean or otherwise Sisyphean operation, this can be formalized into a proof by induction where it is shown that gap  $k=2$  will generate more relative prime pairs of gap  $k=2$  (twin primes) in each subsequent table continuing to infinity. The following is the formalization showing growth from one table  $\text{Spp}_n$  to the subsequent table  $\text{Spp}_{n+1}$  and proposed proof of the Twin Prime Conjecture that is both based on the new mathematical system and visualized in the  $\text{Spp}_n$  and  $\text{Rpp}_n$  tables.



Figure 15) Rows 1 through 11 table  $n=5$  ready to create row 1 of table  $n=6$

# Conservation of gaps – The key $(\sqrt{Spp_n})/Spp_{n-1}$ – $Rpp_n$ tables

**Solution the twin prime conjecture:** The topic of conservation of gaps was initiated in the prior papers of this series (especially Part III). The best way to present this is in the visualization of the  $Spp_n$  tables (the visual representation of the McCanney Generator Function) demonstrating the importance of  $(\sqrt{Spp_n})/Spp_{n-1}$  and  $Rpp_n$  tables as they generate groups of real primes, real twin primes and prime pairs of gap  $k$ . Although some of this may sound repetitive, this is the formal (text book) presentation.

First, we will formalize how a subsequent table  $Spp_{n+1}$  is created from a previous table  $Spp_n$  as developed in prior papers, but with a few more details that illustrate how the subsequent  $Spp_{n+1}$  table produces a greater than exponential increase in discovering real primes (and real twin primes or prime pairs of gap  $k$ ). Prior papers have constructed  $Spp_n$  tables up to  $n=6$  for  $p_6=13$  and  $Spp_6=30,030$ . This was to demonstrate the principles of table generation before formalizing with generalized equations using “red cell only tables”. The “red cell only” tables contain only real primes and composite numbers that are relatively prime to  $Spp_n$ . It is a self-contained system.

The process has properties of closure. All factors of the composite numbers are contained in row 1 of a given  $Spp_n$  table. The  $Spp_n$  tables are closed under the operation of addition in Abelian groups and this carries from one table to the next when generating subsequent tables. The process has completeness. All relative prime numbers (relative to  $Spp_n$ ) which includes all real primes are contained in the table from  $p_n$  up to the value of the sequential prime product  $Spp_n$ . The tables exhibit property of reciprocity because the process is reversible; with the modNt number system every digit represents the ancestry of the prime number or pair of prime numbers back to the prior table and eventually to the original ancestor pair. The system exhibits many symmetries. These include:

- Symmetry of red cells around the midpoint of row  $1=1/2 Spp_{n-1}$
- Row 1 symmetry carries to create symmetry for both red and white cells of the entire  $Spp_n$  table around the center point of the table  $=1/2 Spp_n$
- The  $comb_{n-1}$  equation of table  $Spp_n$ , which predicts all prime numbers to infinity, is symmetrical in that it predicts prime numbers when added to and subtracted from the sequential prime product  $Spp_{n-1}$  from the definition of the McCanney generator function.
- There are additional symmetries not discussed in this paper (e.g. around multiples of  $Spp_n$ )

Lastly, the system exhibits wave nature with the  $comb_{n-1}$  (all members of row 1 of table  $Spp_n$ ) added to multiples of the wavelength  $Spp_{n-1}=mSpp_{n-1} + comb_{n-1}$  ( $m=1, 2, 3, \dots \infty$ ). This explains the seeming patterns that emerge then fade in the prime number table that are interpreted as “random” or “pseudorandom”. It is the beating of these wave equations that creates this illusion. These patterns center around multiples of wavelength  $Spp_{n-1}$ . Pick any multiple of any  $Spp_{n-1}$  value and look to the right and left to see these patterns (taking the difference between prime numbers and these numbers). Also visible are regions like dead zones to the right and left of these numbers. Specific comb equations (subsets of the comb equations) predict for example all twin primes or all gaps of a given value  $k$ .

This is a result of a redefinition of prime numbers with a single boundary condition [6]. The new definition states that the only numbers that can be prime are those resulting from adding relative prime numbers to sequential prime products, with the single boundary condition that all numbers less than the square of  $p_n$  are prime. The entire point is to create new tools which allow proof by induction in addition to creating a new number system based solely on primes. The result is to define a new mathematical structure of primes for primes and by primes with the foundation in Peano’s Postulates. The system shows the importance of relatively prime composite numbers in this process.

The process of generating table  $Spp_{n+1}$  starts with table  $Spp_n$  which has  $p_n$  rows. The “red cell only”  $Spp_{n+1}$  table is constructed from the prior table  $Spp_n$  by the following:

1) Mark the “white cells” in table  $Spp_n$

- The “white cells” are the products  $p_n \times p_j$  (subscript “j” from the  $Rpp_n$  table) where  $p_n$  is the nth prime number associated with table  $Spp_n$  and

$p_j$  being the elements of  $comb_{n-1}$  which are all the red cells (real and relative primes of row 1 of table  $Spp_n$ ).

- There is one and only one white cell in each column of table  $Spp_n$ , therefore the total number of white cells is the same as the number of columns of table  $Spp_n$  uses the variable  $M_n$

2) Place rows 1 through  $p_n$  of red cell only table  $Spp_n$  in succession to form row 1 of table  $Spp_{n+1}$  with the first cell value=0 and the last cell value= $Spp_n$ . The rest of the cells of column 0 and  $Spp_n$  (rows 2 to  $p_n$ ) do not carry over, and the white cells do not carry over since they are not relatively prime to  $Spp_n$  and will no longer generate future relative primes. The resulting  $Spp_{n+1}$  row 1 values are:

- $[0, 1, p_{n+1}, p_{n+2}, \dots, (Spp_n - p_{n+2}), (Spp_n - p_{n+1}), (Spp_n - 1), Spp_n]$
- This is called  $comb_n$  of table  $Spp_{n+1}$  (note the subscripts) and has  $M_{n+1}$  values which are symmetrical around the midpoint  $1/2 Spp_n$ . This symmetry carries from one table to the next. When added to multiples of the wavelength  $Spp_n$ ,  $comb_n$  provides the equation that predicts all prime numbers to infinity. When compared to the prior  $comb_{n-1}$  (row 1 of the prior table),  $comb_n$  refines (reduces) the possible values and therefore becomes more accurate at predicting real prime numbers. These equations provide a monotonically decreasing prediction of all prime numbers (assuring there are no rogue primes or gaps) and predicts precisely where to find real primes, twin primes and prime patterns. The  $comb_n$  contains both real and all composite numbers that are relatively prime to  $Spp_n$  giving the predictive equation for all prime numbers as follows:
- $mSpp_n + [0, 1, p_{n+1}, p_{n+2}, \dots, (Spp_n - p_{n+2}), (Spp_n - p_{n+1}), (Spp_n - 1), Spp_n]$  where  $m=1, 2, 3, \dots \infty$ .
- This also provides a test or primality when using the modNt number system (all future primes must end in one of these sequences modNt ... if a future number does not end with one of these sequences, then it is not prime).
- Per the McCanney generator function on which these tables are based, 1 and  $(Spp_n-1)$  are always relatively prime to  $Spp_n$  and therefore will always carry to the next table row 1 (these are a primary source of twin prime pairs in every table and could be used to prove the Twin Prime Conjecture on its own).
- The prime number  $p_n$  related to table  $Spp_n$  is retired to the dead zone DZ <sub>$n+1$</sub>  of  $Spp_{n+1}$  as it will not generate any future primes (although all the primes and relative primes that it generated will continue to generate primes until they eventually enter the dead zone of their respective tables).

3) The numerical values of red cells of rows 2 to  $p_{n+1}$  in the columns of table  $Spp_{n+1}$  are calculated by adding multiples of  $Spp_n$  to the values in row 1 of table  $Spp_{n+1}$  proceeding up to row  $p_{n+1}$ .  $k_n$  is the gap between  $p_{n+1}$  and  $p_n$  ( $k_n=p_{n+1}-p_n$ ). There are various opinions of the definition of gaps but this one works well with this system. The total number of cells in row 1 of table  $Spp_{n+1}$  is equal to the total number of cells in row 1 of table  $Spp_n$  ( $M_n$ ) multiplied by  $(p_n-1)$ . The “-1” accounts for the white cells that do not carry from the prior table to form row 1. This is the basis for the generalized equation where the dimensions and other properties of a given table can be calculated without creating all the intermediate tables. Table  $Spp_n$  has  $p_n$  rows and  $M_n$  columns so there are  $M_n \times p_n$  red cells total (before removing white cells). Table  $Spp_{n+1}$  has  $p_{n+1}=p_n+k_n$  rows and  $M_{n+1}=M_n \times (p_n-1)$  columns so there are  $M_{n+1} \times p_{n+1}$  red cells. All patterns of prime numbers derived from all prior tables are maintained and visible in each table. That is why patterns seem to come and go in the prime number tables only to be replaced by other patterns. These patterns center around the values of  $Spp_n$  and its multiples where  $n=\alpha, 0, 1, 2, 3 \dots \infty$ , giving the illusion of “random” or “pseudo random” patterns. Instead they are like waves of different wave lengths on an infinite ocean beating against each other.

4) To generate the subsequent table  $Spp_{n+2}$  return to the top and begin again by selecting the white cells of table  $Spp_{n+1}$ .

5) The final step is to determine the number of real primes or real twin primes generated in the process. There are two methods used. Notation below uses the perspective of table  $Spp_n$  generating table  $Spp_{n+1}$  while determining the number of primes in table  $Spp_{n+1}$ . First understand that the table is closed

under multiplication. This means that if a number in this table has factors (is not prime), those factors are contained in row 1 [7,8]. Row 1 (comb<sub>n</sub>) contains all relative primes (composite and real primes) from the prior table.

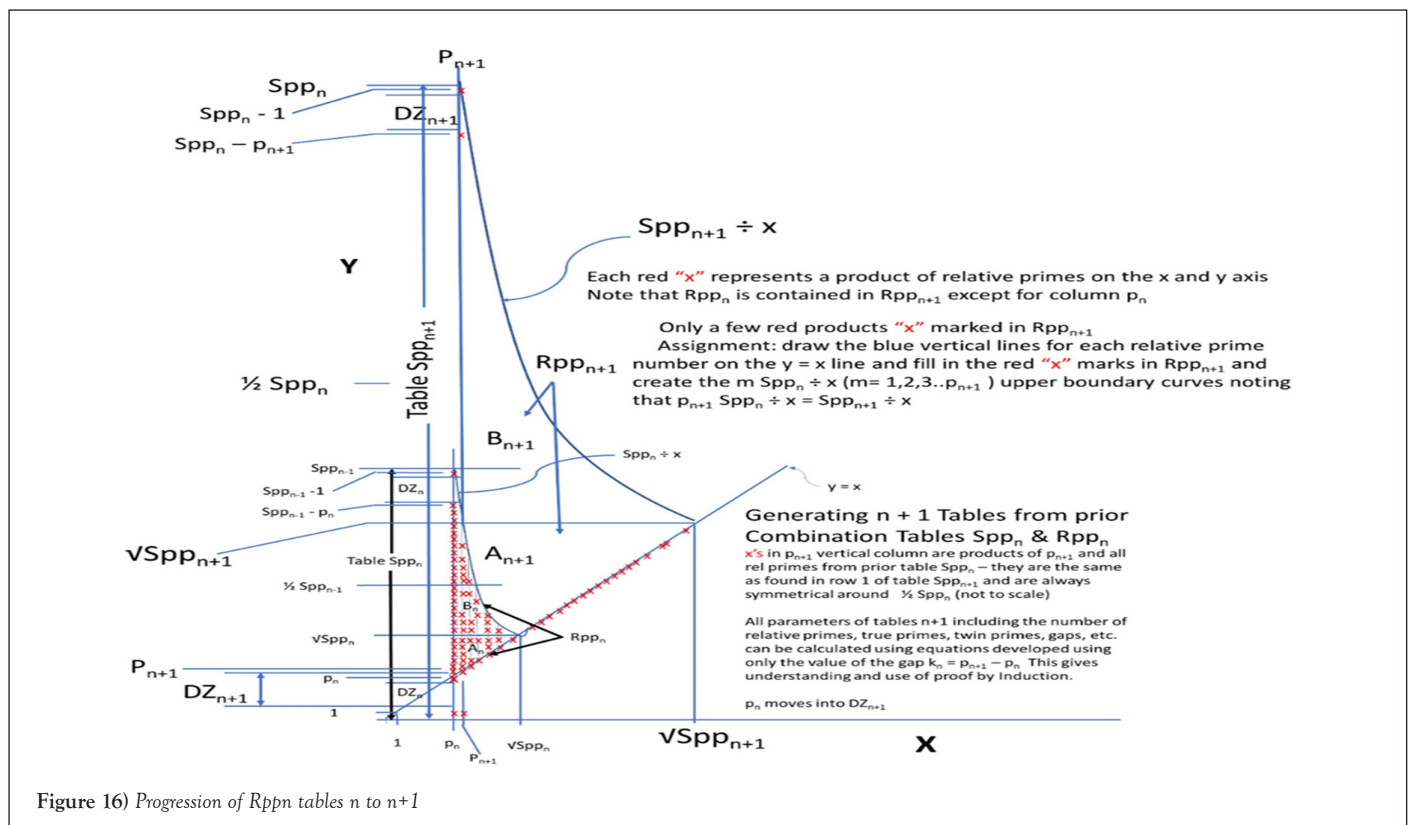
- The first method of selecting true prime numbers is the Boundary Condition of the McCanney Generator Function which is also the new definition of prime numbers. It states that all red cells of table  $Spp_{n+1}$  whose values are less than  $p_{n+1}^2$  are prime. This is because there can be no products in this region, therefore it is a region that is a “safe zone”. Since the prior table  $Spp_n$  discovered all real primes up to  $p_n^2$  the criteria are stated such that all comb members between  $p_n^2$  and  $p_{n+1}^2$  are newly discovered primes. This can be used to prove that there are an infinite number of individual prime numbers but is not useful in proving the Polignac or Twin Prime Conjectures.
- The second method of identifying prime numbers in table  $Spp_{n+1}$  is used in the proof of the Twin Prime Conjecture and can be seen in the tables below (derived from prior papers). The red only  $Spp_{n+1}$  table contains all relative primes between  $p_{n+1}$  and  $Spp_{n+1}$  (all composite numbers whose factors are relatively prime to  $Spp_{n+1}$  and real primes). The use of these tools is what has been missing from analysis of prime numbers, leading to the view of prime numbers in manageable pieces where one table generates the next allowing proof by induction. As noted above, the red cell only tables are closed under multiplication. This means that all the composite numbers in the table can be identified by multiplying elements of row 1 with other elements of row 1. All red cells not identified by this process will be prime. The  $Rpp_{n+1}$  table constitutes a mapping of all products  $p_i \times p_j$  of elements of row 1. The growth of the number of discovered true primes (and twin primes) with increasing counting integer “n” shows this progression. The solution is to generalize this progression of building a new  $Spp_{n+1}$  table from a prior table  $Spp_n$  to prove that each subsequent  $Spp_{n+1}$  table has exponentially more primes and twin primes than the prior table. Since this is the general case and not dependent of specific numbers, the proof will proceed by induction.
- Review of  $Rpp_{n+1}$  table generation (developed in prior papers of this series). Key values in the  $Rpp_{n+1}$  table are  $\sqrt{Spp_{n+1}}$  (located in row 1) and the ratio  $\sqrt{Spp_{n+1}}/\sqrt{Spp_n}$ .  $\sqrt{Spp_{n+1}}$  is the maximum value of a member of

row 1 of table  $Spp_{n+1}$  that can be multiplied by itself and fit inside the  $Spp_{n+1}$  table. The products  $p_i \times p_j$  from row 1 are  $p_i = p_{n+1}, p_{n+2}, p_{n+3}, \dots$ ,  $p_{\max} < \sqrt{Spp_{n+1}}$  and  $p_j$  are all elements of row 1 such that  $p_j = 1$  and  $p_i \leq p_j \leq (Spp_{n+1} \div p_i)$ . The first series of these products have a special purpose ( $p_{n+1} \times p_j$ ) and are labeled “white cells” since they are not relatively prime to  $Spp_{n+1}$ . The products are written in sequence as follows ( $p_i$  subscript “i” is from the x axis and  $p_j$  (subscript “j”) is from the y axis of table  $Rpp_{n+1}$  noting the order of subscripts to prevent duplication):

- $p_{n+1} \times [1, p_{n+1}, p_{n+2}, p_{n+3}, \dots, (Spp_n - p_{n+3}), (Spp_n - p_{n+2}), (Spp_n - p_{n+1}), (Spp_n - 1)]$
- Products of  $p_{n+1}$  with all elements of row 1 including  $p_{n+1} \times 1$  and  $p_{n+1} \times (Spp_n - 1)$
- Note the symmetry around midpoint of row  $1 = \frac{1}{2} Spp_n$
- This creates vertical line of red x's above  $p_{n+1}$  on  $Rpp_{n+1}$  graph (as an exercise draw in the red x's on the  $Rpp_{n+1}$  graph below)
- $p_{n+2} \times [p_{n+2}, p_{n+3}, p_{n+4}, \dots, p_{\max} < (Spp_{n+1}/p_{n+2})]$ 
  - Note the upper limit  $p_{\max}$  of the second term  $p_j$  is limited by  $Spp_{n+1}/p_i$
  - This creates second vertical line on  $Rpp_{n+1}$  graph above  $p_{n+2}$
- $p_{n+3} \times [p_{n+3}, p_{n+4}, p_{n+5}, \dots, p_{\max} < (Spp_{n+1}/p_{n+3})]$
- Continue until  $p_{\max} < \sqrt{Spp_{n+1}}$
- $p_{\max} \times p_{\max} = p_{\max}^2$

This creates the final red x on  $Rpp_{n+1}$  graph less than  $\sqrt{Spp_{n+1}}$

See the progression of tables below from  $Rpp_n$  to  $Rpp_{n+1}$  (Figure 16). The  $Rpp_{n+1}$  graph contains all the products (composite numbers) that can then be mapped back into the  $Spp_{n+1}$  table to identify composite numbers (leaving only true primes). If one can show that the progression from one  $Spp_n$  table to the subsequent table  $Spp_{n+1}$  produces not only more, but exponentially more true primes (and twin primes), then the original claim will be proven that each relative prime (and each relative twin prime) produces an infinite number of future real primes (and twin primes).



Tables 1 and 2 show the parameter results up to table  $n=11$  ( $p_{11}=31$ ) with the most notable found in Table 2 the fifth column and right hand column. This shows the growth of twin prime pairs discovered in  $Spp_n$  tables. The generalization of this growth using induction is the basis for the proof of the Twin Prime Conjecture. It is both mathematical and visual (thanks to the  $Spp_n$  tables). You do not have to generate all the  $Spp_n$  tables to infinity

since generalized equations are about to be demonstrated to show that each subsequent table generates more real twin primes than the prior table and by induction, gives the result that “every relative twin prime generates an infinite number of real twin primes”. The proof involves the understanding gained from the  $Rpp_n$  tables and the expression  $\sqrt{Spp_n}/Spp_{n-1}$ .

Table 1: Parameters for  $Spp_n$  tables to  $n=11$

| $n$      | $P_n \bmod 10$ (also number of rows in $Spp_n$ Table) | $P_n \bmod Nt$ | $Spp_n \bmod 10$ vs. $\bmod Nt$ $\bmod Nt=10^n$ (greater than exponential growth) | $\sqrt{Spp_n} \bmod 10$ upper limit relative prime $p_i$ of row 1 such that $p_i^2 < Spp_n$ | $(\sqrt{Spp_n})/Spp_{n-1}$ | Number columns red only table (also number of elements in $comb_{n-1}$ )=# red cells $Spp_{n-1}$ table less white cells in $n-1$ table | Total number red only cells $Spp_n$ Table=prior column $\times P_n$ | Prior column/ $Spp_n$ =fraction red cells vs. $Spp_n$ |
|----------|-------------------------------------------------------|----------------|-----------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|----------------------------|----------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------|-------------------------------------------------------|
| $\alpha$ | 0                                                     | 0              | N/A                                                                               | N/A                                                                                         | N/A                        | N/A                                                                                                                                    | N/A                                                                 | N/A                                                   |
| 0        | 1                                                     | 1              | $1/10^0$                                                                          | 1.0                                                                                         | N/A                        | N/A                                                                                                                                    | N/A                                                                 | N/A                                                   |
| 1        | 2                                                     | 10             | $2/10^1$                                                                          | 1.4                                                                                         | 1.414                      | 0                                                                                                                                      | 2                                                                   | 1.000                                                 |
| 2        | 3                                                     | 11             | $6/10^2$                                                                          | 2.4                                                                                         | 1.225                      | 1                                                                                                                                      | 3                                                                   | 0.500                                                 |
| 3        | 5                                                     | 21             | $30/10^3$                                                                         | 5.4                                                                                         | 0.913                      | 2                                                                                                                                      | 10                                                                  | 0.333                                                 |
| 4        | 7                                                     | 101            | $210/10^4$                                                                        | 14.4                                                                                        | 0.483                      | 8                                                                                                                                      | 56                                                                  | 0.267                                                 |
| 5        | 11                                                    | 121            | $2310/10^5$                                                                       | 48.0                                                                                        | 0.228                      | 48                                                                                                                                     | 528                                                                 | 0.229                                                 |
| 6        | 13                                                    | 201            | $30030/10^6$                                                                      | 173.2                                                                                       | 0.075                      | 480                                                                                                                                    | 6240                                                                | 0.208                                                 |
| 7        | 17                                                    | 221            | $510510/10^7$                                                                     | 714.5                                                                                       | 0.024                      | 5760                                                                                                                                   | 97920                                                               | 0.192                                                 |
| 8        | 19                                                    | 301            | $9699690/10^8$                                                                    | 3114.4                                                                                      | 0.0061                     | 92160                                                                                                                                  | 1751040                                                             | 0.181                                                 |
| 9        | 23                                                    | 321            | $223092870/10^9$                                                                  | 14936.2                                                                                     | 0.0015                     | 1658880                                                                                                                                | 38154240                                                            | 0.171                                                 |
| 10       | 29                                                    | 421            | $6469693230/10^{10}$                                                              | 80434.4                                                                                     | 0.00036                    | 36495360                                                                                                                               | 1058365440                                                          | 0.164                                                 |
| 11       | 31                                                    | 1001           | $200560490130/10^{11}$                                                            | 447839.8                                                                                    | 0.000069                   | 1021870080                                                                                                                             | 31677972480                                                         | 0.158                                                 |

Table 2: Parameters for  $Spp_n$  tables to  $n=11$

| $n$      | $P_n$ | Gap $n$ $k_{n-1}=P_n-P_{n-1}$ | Number of new primes discovered $p_{n-1}^2 < p_i < P_n^2$ (Generator function boundary condition) | Total number new primes discovered $Spp_n$ table using $Rpp_n$ table (rows 2 to $P_n$ ) | Number of column pairs - twin prime only table (also $2x$ this value=number of elements in twin prime $comb_{n-1}$ ) | Total number relative twin prime pairs in $Spp_n$ table (red cells only)= prior column $\times P_n$ | Number of new twin primes discovered $p_{n-1}^2 < p_i < P_n^2$ (Generator function boundary condition) | Total Number new twin primes discovered $Spp_n$ table using $Rpp_n$ table (rows 2 to $P_n$ ) |
|----------|-------|-------------------------------|---------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------|
| $\alpha$ | 0     | N/A                           | N/A                                                                                               | 1                                                                                       | 0                                                                                                                    | 0                                                                                                   | 0                                                                                                      | 0                                                                                            |
| 0        | 1     | 1                             | N/A                                                                                               | 1                                                                                       | 0                                                                                                                    | 0                                                                                                   | 0                                                                                                      | 0                                                                                            |
| 1        | 2     | 1                             | N/A                                                                                               | 1                                                                                       | 0                                                                                                                    | 0                                                                                                   | 1                                                                                                      | 0                                                                                            |
| 2        | 3     | 1                             | 2                                                                                                 | 2                                                                                       | 0                                                                                                                    | 0                                                                                                   | 1                                                                                                      | 2                                                                                            |
| 3        | 5     | 2                             | 5                                                                                                 | 7                                                                                       | 1                                                                                                                    | 5                                                                                                   | 2                                                                                                      | 3                                                                                            |
| 4        | 7     | 2                             | 6                                                                                                 | 36                                                                                      | 3                                                                                                                    | 21                                                                                                  | 2                                                                                                      | 10                                                                                           |
| 5        | 11    | 4                             | 15                                                                                                | 297                                                                                     | 15                                                                                                                   | 165                                                                                                 | 4                                                                                                      | 55                                                                                           |
| 6        | 13    | 2                             | 9                                                                                                 | 2903                                                                                    | 135                                                                                                                  | 1755                                                                                                | 2                                                                                                      | 398                                                                                          |
| 7        | 17    | 4                             | 22                                                                                                | 39083                                                                                   | 1485                                                                                                                 | 25245                                                                                               | 7                                                                                                      | 4168                                                                                         |
| 8        | 19    | 2                             | 11                                                                                                | 603698                                                                                  | 22275                                                                                                                | 423225                                                                                              | 2                                                                                                      | 52817                                                                                        |
| 9        | 23    | 4                             | 27                                                                                                | 11637502                                                                                | 378675                                                                                                               | 8709525                                                                                             | 4                                                                                                      | TBD                                                                                          |
| 10       | 29    | 6                             | 47                                                                                                | TBD                                                                                     | 7952175                                                                                                              | 230613075                                                                                           | 8                                                                                                      | TBD                                                                                          |
| 11       | 31    | 2                             | 16                                                                                                | TBD                                                                                     | 214708725                                                                                                            | 6655970475                                                                                          | 2                                                                                                      | TBD                                                                                          |

Take the ratio of primes discovered by  $Rpp_n$  (column 5 Table 2 above), noting that the number of prime numbers discovered are from rows 2 to  $p_n$  in their respective  $Spp_n$  tables. This is because the real primes in row 1 had already been reported on the prior table. The ratios (from  $n=1$  to  $n=9$ ) are as follows:  $2/1=2$ ,  $7/2=3.5$ ,  $36/7=5.14$ ,  $297/36=8.25$ ,  $2903/297=9.77$ ,  $39083/2903=13.86$ ,  $603698/39083=15.45$  and  $11637502/603698=19.28$  ... note the more than exponential increase. Take the difference between the increases and note that the gap 4 ratios have larger increases than gap 2 ratios. This is because the subsequent  $Spp_{n+1}$  table has more rows at the bottom of its table than the prior table (there are more red cells relative to the number of products that locate composite numbers). As tables grow with larger gaps, this is a factor that further increases the growth of discovered primes (general equations will be developed below).

Follow the same procedure finding the ratios of twin primes discovered by  $Rpp_n$ . They are as follows:  $3/2=1.5$ ,  $10/3=3.33$ ,  $55/10=5.5$ ,  $398/55=7.24$ ,  $4168/398=10.47$ ,  $52817/4168=12.67$  ... note the more than exponential increase. This progression continues to infinity. In the  $n=8$  table the process is discovering 603,609 real primes and 52,817 twin primes.

Next see the successive values of  $\sqrt{Spp_n}/Spp_{n-1}$  starting with  $n=1$  to  $n=11$ . The proportion (percentage) of products in each successive  $Rpp_n$  table is greatly reduced while the table size (number of red cells) is greatly increasing.

Each subsequent table has more than exponentially more primes and twin primes than the prior table. These numerical examples do not constitute proof but simply illustrate the mathematical relationship between tables as they grow. The proof relies on the generalization of this process showing that this growth continues to infinity.

The following is a detailed example (Figure 17) showing the importance of  $(\sqrt{Spp_n})/Spp_{n-1}$  and comparing it to this ratio for the subsequent  $Spp_{n+1}$  table  $(\sqrt{Spp_{n+1}})/Spp_n$ , noting that this number decreases rapidly with increasing values of the counting integer “ $n$ ”. The following is a visual representation of the generation of table  $Spp_6$  from table  $Spp_5$  also showing generalized formulae. This table is the key to understanding the growth from one table to the next such that there are exponentially more newly discovered primes in each table.

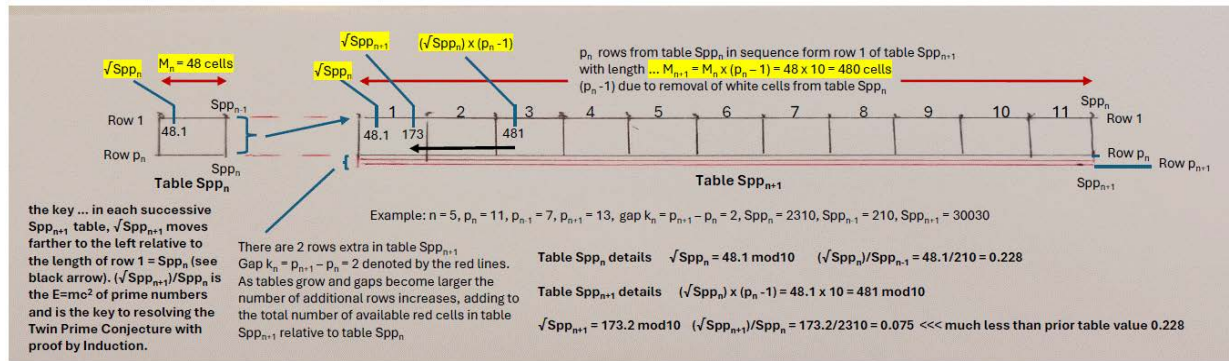


Figure 17) Building table  $Spp_{n+1}$  from  $Spp_n$  - general case

In Figure 17, the dimensions of table  $Spp_n$  are  $M_n=48$  cells (columns) and  $p_n=11$  rows. After removing white cells from table  $Spp_n$ , the rows from 1 to  $p_n$  are moved in sequence to form row 1 of table  $Spp_{n+1}$ . Note in the drawing this is being compared in table  $Spp_{n+1}$  to  $M_{n+1}=M_n \times (p_n-1)=48 \times 10=480$  cells (columns) but only  $p_n$  rows (see red dashed lines connecting the two tables). This is done on purpose to compare the number of products  $p_i \times p_j$  of each table with the same number of rows in both tables, leaving the 2 additional rows at the bottom of table  $Spp_{n+1}$  (noted by the two solid red lines) to be added in later. To get a proportional number of products as table  $Spp_n$  in the  $Rpp_{n+1}$  table (identifying composite numbers of table  $Spp_{n+1}$ ), one would use the cut off value of  $\sqrt{Spp_n} \times (p_n-1)=481$ . However, the table  $Spp_{n+1}$  cutoff value is  $\sqrt{Spp_{n+1}}=173.2$  with the ratio  $\sqrt{Spp_{n+1}}/Spp_n=173.2/210=0.75$  (compared to 0.228 for table  $Spp_n$ ). The generalized equation going from table  $Spp_n$  to  $Spp_{n+1}$  for the ratio  $R$  between  $\sqrt{Spp_{n+1}}$  and  $\sqrt{Spp_n} \times (p_n-1)$  is as follows:

$$\begin{aligned} (\sqrt{Spp_{n+1}})/Spp_n & R [\sqrt{Spp_n} \times (p_n-1)]/Spp_n \\ \sqrt{Spp_{n+1}} & R \sqrt{Spp_n} \times (p_n-1) \\ \sqrt{(Spp_n \times p_{n+1})} & R \sqrt{Spp_n} \times (p_n-1) \\ \sqrt{Spp_n} \times \sqrt{p_{n+1}} & R \sqrt{Spp_n} \times (p_n-1) \\ -\sqrt{Spp_n} \times \sqrt{p_{n+1}} & R \sqrt{Spp_n} \times (p_n-1) \\ \sqrt{p_{n+1}} & R (p_n-1) \\ \sqrt{(p_n+k_n)} & R (p_n-1) \end{aligned}$$

As  $n$  the counting integer increases and the gap  $k_n$  becomes small relative to the value of  $p_n$ , the right side increases as the square of the left side. This proves that for all cases of creating subsequent tables, the cutoff value  $\sqrt{Spp_{n+1}}$  is much smaller than the prior table. The same result occurs comparing the ratio between  $(\sqrt{Spp_{n+1}})/Spp_n$  and  $\sqrt{Spp_n}/Spp_{n-1}$  with the similar result  $\sqrt{p_{n+1}}/R p_n$ . This explains the increase in growth of primes, twin primes and prime

pairs of gap  $k$  in successive  $Spp_n$  tables.

Relative to the originator table  $Spp_n$  there are proportionately fewer products to create composite numbers in table  $Spp_{n+1}$ . This is why the ratio  $\sqrt{Spp_n}/Spp_{n-1}$  is called the  $E=mc^2$  of prime numbers. Understanding its effect on table generation holds the key to the growth of primes, twin primes and prime pairs of any gap  $k$ .

There are other effects that further cause a larger disparity between the number of products  $p_i \times p_j$  in the  $Rpp_{n+1}$  table from the prior table and therefore add additional real primes discovered as each new table is created. The first is the addition of the extra  $k_n$  rows at the bottom of table  $Spp_{n+1}$  due to the gap between  $p_n$  and  $p_{n+1}$ . The generalized equation is as follows with specific example  $n=5$  to  $n=6$ :

$$\text{General equation} = M_{n+1} \times k_n = M_n \times (p_n-1) \times k_n$$

Example Figure 17 table  $Spp_6=480 \times 2=960$  additional red cells.

As the gap sizes grow and the length of the tables increases, this adds a significant number of red cells and therefore real primes. These were not included in the original analysis which used the same number of rows= $p_n$  in both tables, so these extra rows are now added to the number of red cells (and therefore real primes) left after removing composite numbers generated in the  $Rpp_{n+1}$  table. This effect alone would be sufficient to prove the Twin Prime Conjecture.

The second effect is the fact that each subsequent  $Rpp_n$  table has more duplicate products than the prior table. This indirectly adds more red cells to the final count of discovered real primes because these products are not producing new unique composite numbers that would cancel potential real primes. Figure 18 above shows an example of table with duplicate products (color coded). The proof would proceed without this effect so is not stressed (it is a subject for future more esoteric studies).

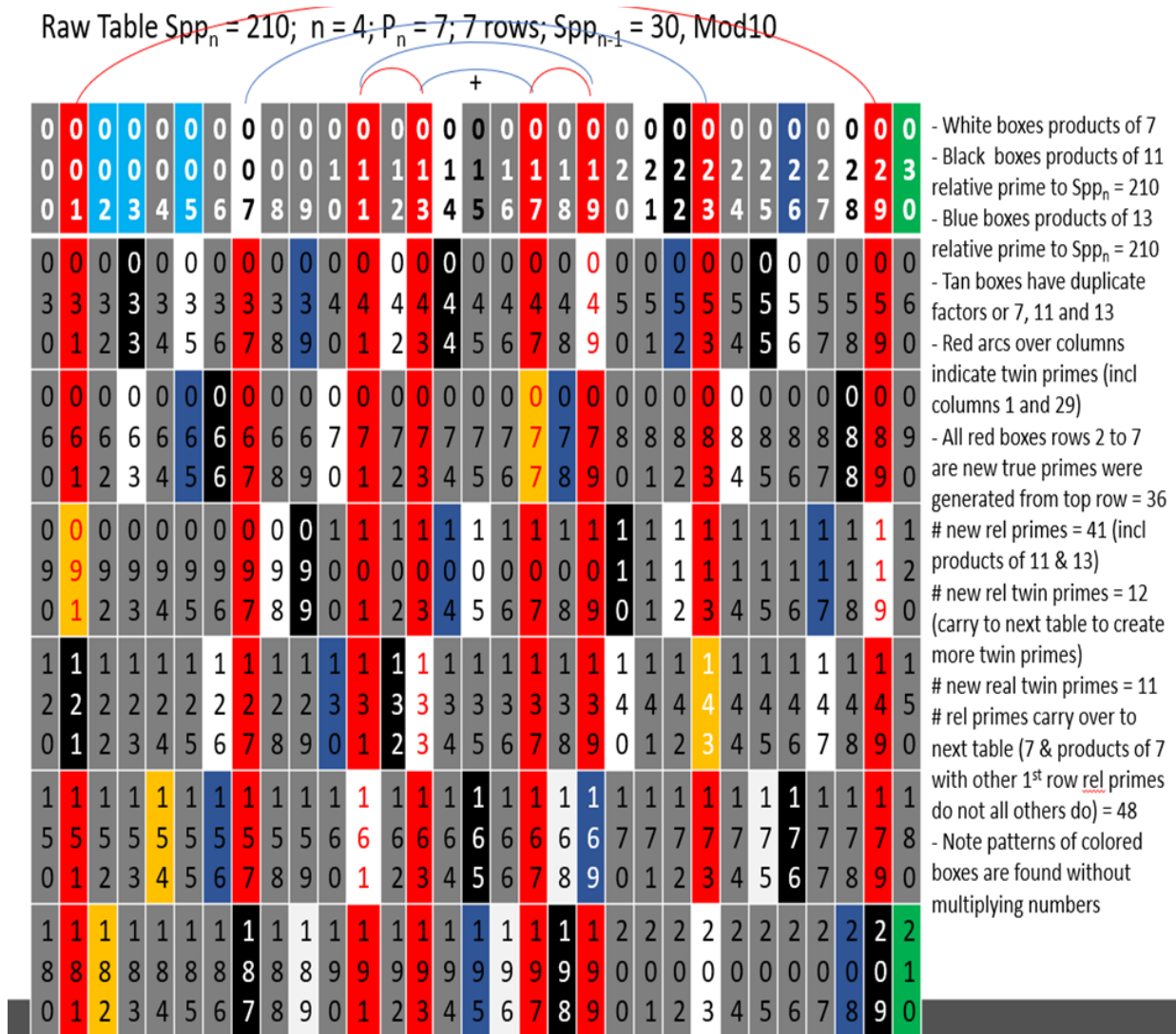


Figure 18) Color coded duplicate products

If at this point anyone has doubts about the process, please continue building tables and filling in the values until you are convinced that this will repeat to infinity. Then realize the generalized equations are sufficient to prove the result that:

- Every relative prime (relatively prime to  $Spp_n$ ) will generate an infinite number of real primes.
- Every relative twin prime pair (relatively prime to  $Spp_n$ ) will generate an infinite number of real twin prime pairs.

QED

**Goldbach conjecture -proofs by induction:** Presented here are concepts that help in understanding the basis for Goldbach Conjecture solutions. It has been pointed out in prior papers that the reason there are local maximums of the Goldbach pairs at values of  $Spp_n$  (or multiples of these values) is because of the symmetry of the  $Spp_n$  tables around the half point  $\frac{1}{2} Spp_n$ . The white cells eliminate relative primes symmetrically around the  $\frac{1}{2}$  point and therefore leaves all other red cells in the table. Every remaining red cell has a compliment cell that adds to  $Spp_n$ . The local GB maximums are a direct result of the symmetry of primes in the  $Spp_n$  table. As you move away from  $Spp_n$  the "alignments" become fewer and many times the adjacent alignment creates far fewer Goldbach pairs. To find the number of Goldbach pairs for the number  $Spp_n - 2$ , combine all the prior solutions for  $Spp_n$  and find only those that are the second member of a twin prime pair. Combine these with the other number and their sum will be  $Spp_n - 2$ . To complete the process, take any two elements with real primes that add to a given even number and

note their columns. In these columns there will be more prime pairs that add to that same number.

Looking at row 1 of the  $Spp_n$  table, find two primes that add to an even number less than the max value in row 1 which is  $Spp_{n-1}$ . Follow the two primes moving down in their respective columns and the numbers generated in these columns are Goldbach solutions to the row 1 plus a multiple of  $Spp_{n-1}$ . This creates families of solutions just as the twin primes generate other twin primes. These families are identifiable in the patterns of GB solutions as they fan out above the x axis. This is only introduced here and is a topic for one of the future papers in this series. The question is asked: "what would it take to break the Goldbach Conjecture"? This reverts back to the question of "conservation of gaps" with the caveat that for the GB Conjecture, the "gaps" may include real prime numbers between the gap end primes.

**The riemann hypothesis – insights – are there proofs by induction**

Work is proceeding in the background to map the Euler equation into a form that can lead to proof by induction. It was imperative to release the current information up to this point. It has been made clear by others that the solution to the Riemann hypothesis will not improve the ability to identify large primes. It is actually the contrary. If a large prime is discovered, it can map to a zero and not *vice versa*.

**Three methods of proof of twin prime conjecture**

Three methods have been discovered to prove the Twin Prime Conjecture

and are enumerated here. They all use the background developed in this set of papers.

- Using any method (prime number table, GIMPS, etc.) find any twin prime pair or relative twin prime pair  $(a,b)$  (where one or both of the pair are relatively prime to the  $Spp_n$  table they lie in). You can always find another twin prime pair by the following method. If  $(a,b)$  is in row 1 of the  $Spp_n$  table, then add multiples  $mSpp_{n-1}$  where  $m=1, 2, 3 \dots$  to both  $a$  and  $b$ . Eventually you will find at least one twin prime pair. If  $a$  and/or  $b$  lies in rows 2 to  $p_n$  then create the next table  $Spp_{n+1}$  and add multiples  $mSpp_n$   $m=1, 2, 3 \dots$  to both  $a$  and  $b$ . Eventually you will find at least one twin prime pair. In the second case ( $a$  and/or  $b$  in row 2 to  $p_n$ ), remember that twin primes and relative primes can be located in the columns 1 and  $(Spp_{n-1} - 1)$ , so  $a$  of the  $(a,b)$  pair would be in column  $(Spp_{n-1} - 1)$ , and  $b$  would be in row 2 of column 1. In this case you would create table  $Spp_{n+1}$  where the twin prime pair would appear completely in row 1. This method can be broadened to state that not only would you find at least one future real twin prime pair, but you would find an infinite number of them as they generate relative twin primes into future  $Spp_n$  tables. In most cases you would find at least one twin prime pair in the same pair of columns.
- Using any method as in #1 above, find any twin prime pair or relative twin prime pair  $(a,b)$ . In any modern search such as GIMPS, prime numbers are discovered with no knowledge of prior or future prime numbers. They have no predictive value as they are selected from lists such as Mersenne Primes and then verified by extensive computer calculations. Using the methods of this series of papers, locate the twin prime pair in a  $Spp_n$  table just less than and closest to the sample pair  $(a,b)$ . Convert the numbers to  $modNt$  number system and find the ancestry of the given pair by removing one by one the highest order digits. This will discover the entire lineage of relative twin primes (including real twin primes) back to the alpha prime  $p_\alpha=0$ . You will find many real twin prime pairs that were missed in the GIMPS search. This same method can be used for single primes, twin primes, prime pairs of any gap size (the gap size is relatively small compared to the value of  $Spp_{n-1}$ ). To show the method, select a twin prime pair from the  $n=6$  table.  $(a,b)=(18251 \bmod 10, 18253 \bmod 10)=(796121 \bmod Nt, 796201 \bmod Nt)$ . Both numbers are prime. Compare the ease of finding all the twin prime ancestors using the two number systems. In  $mod10$  you would have to first find all the values of  $Spp_n$  smaller than the table these were found in and then using division calculate individually all the ancestral twin prime pairs. Using the  $modNt$  numbers it is a simple matter of removing the higher order digits to find the ancestral sequence which is  $(796121, 796201), (96121, 96201), (6121, 6201), (121, 201), (21, 1)$ . Note the final pair translates to  $mod10$  as  $(5,1)$  which as explained in prior papers is the original twin prime pair. These numbers are easily translated to  $mod10$  by simply multiplying each digit by its  $Spp_n$  value as in the following example. Taking the first value of the pair  $a=796121=7 \times 2310+9 \times 210+6 \times 30+1 \times 6+2 \times 2+1 \times 1=18251$ . The  $modNt$  number system is made from and for prime numbers. It is the number system of prime numbers that has been missing for 2500 years. To complete the method, the  $mod10$  pairs in the ancestral path are  $(18251, 18253), (2081, 2083), (191, 193), (11, 13), (5, 1)$  with the \* denoting relative prime (non-prime) numbers in their respective tables. In this ancestry there are no relative primes they are all valid real twin prime pairs except for  $(5,1)$ . If there were relative primes, they would be as valid as real primes in their respective  $Spp_n$  tables but of course not counted in the final number of real twin primes. They would not prevent the sequence of discovery back to the original prime pair  $(5,1)$ . Note also the importance of having 1 and 0 as prime numbers. If they did not exist, we would have to invent them to complete the foundation of the mathematical system of  $modNt$  numbers beginning with Peano's Postulates. In the new definition of prime numbers, 0 and 1 are very much prime numbers. To prove another point, take any relative prime pair of gap  $k=4$  and find its ancestral path back to the original gap of 4 in table  $n=3$  with  $Spp_3=30$   $(1,5)$ . In this table the original pairs of gap 4 are  $(1,5)$  which generates  $(7,11), (13,17)$  and  $(19,23)$ . Noting that  $(25,29)$  is not because 25 is a multiple of  $p_5=5$ , is not relatively prime to  $Spp_3$  and therefore does not carry to create row 1 of table  $Spp_4$  to create more pairs of gap  $k=4$ . The other pairs mentioned all carry to generate all future pairs of gap  $k=4$ . If you find any prime or relatively prime pair of gap  $k=4$ , finding the ancestral path will ultimately arrive at these pairs.

The third method of proving the Twin Prime Conjecture is the one selected in this paper (using induction and generalized equations to build  $Spp_n$  tables) which states that "every relative twin prime pair will generate an infinite number of real twin primes".

The following are more topics for future papers:

**Example 2:** Analysis of larger gaps or gap patterns being created by white cells carrying to next table which then may continue to infinity in future tables

**Example 3:** Analysis of large local gap  $k=34$   $(1327, 1361)$ . Why it happens and why it does not carry to future tables. List all nearby prime gaps so show that locally there are small gaps to average out the large gap and how out of place it is/calculate average local gap and compare ... seems like a contradiction but is not. Explain why it does not affect future prime gaps. It is related to the fact that only white cells that are evenly distributed throughout the  $Spp_n$  table do not transfer to create row 1 of the subsequent table  $Spp_{n+1}$  so large "rogue" gaps do not affect future gaps.

**Example 4:** Discussion of max gaps vs average gaps becomes more stable into future ... post list of max primes and use as examples (conservation of gaps). In prior thinking of twin primes it was believed that they possibly would simply fade away as the prime numbers got farther apart. Reinforce the idea that the twin primes and all gaps  $k$  become more abundant when viewed using the  $Spp_n$  tables which grow more than exponentially. Since the primes (and twin primes) follow a logarithmic decline in density, and since the  $Spp_n$  tables grow faster than exponentially this is the cause of the increase in density of primes, twin primes and prime pairs of any gap  $k$ . Additionally, since the twin primes are calculated in groups based on prior prime numbers, this assures that they will continue to infinity and the process tells precisely where to find them.

**Example 5:** Reinforce concept of the wave nature of prime numbers using comb equations. List comb equations generalized are a wave representation. Reinforce the concepts that the  $Spp_n$  tables form abstract algebraic groups that exhibit the properties of closure, reciprocity, symmetry, completeness in addition to the wave nature. Reinforce the idea that all primes belong to families with unique ancestral paths back to the alpha prime 0. Reinforce the idea that the same prime then continues to have offspring to infinity. Reinforce the idea that relative primes in a given  $Spp_n$  table are as valid as real prime numbers and are essential in completing the process of finding all prime numbers (completeness).

**Example 5a:** Reinforce the idea that of combs for twin primes (subsets of the comb for the related complete  $Spp_n$  table) and that all future twin primes are calculated using just the twin prime only table. Express all comb and twin prime comb equations up to  $n=7$  showing that relative primes are included. Reinforce the idea that combs are defined as row 1 of a given  $Spp_n$  table and is designated as  $comb_{n-1}$  since it is actually derived from the prior table  $Spp_{n-1}$ .

**Example 6:** Show how future combs modify the prior combs so they are monotonically decreasing (no rogue primes or gaps) which was not previously available in prime number density or counting functions. The number of predicted primes diminishes but the wavelength increases so there are many more primes yet a subset of the prediction of the prior comb. This seems like a contradiction but is not because wavelength gets longer faster they become more accurate as you go to larger and larger  $Spp_n$  tables.

**Example 7:** Discuss the importance of the dead zone gaps around  $Spp_n$  numbers and multiples of  $Spp_n$ . Explain how the columns under the dead zone retired prime numbers carries into the columns below these numbers in row 1 (the rows 2 to  $p_n$  the gap persists) and are visible in the prime number tables if you know what to look for. For example, for  $Spp_3=2310$  note that there are no primes to the right or left in the gap between  $2310 \pm 1$  and  $2310 \pm 11$  or between any multiple of  $2310 \dots (m \times 2310) \pm 1$  and  $(m \times 2310) \pm 13$  where  $m=2, 3 \dots 13$ . This is a direct result of the McCanney Generator Function and its boundary condition. These are examples of the dead zones. As an exercise, in a list of prime numbers, locate the values of  $m \times 2310 = 4620, 6930, 9240, 11550, 13860, 16170, 18480, 20790, 23100, 25410, 27720$  and  $30030$ . These are the numbers from the right hand column of table  $Spp_6=30030$  where  $p_6=13$ . Verify that there are no primes between these numbers  $\pm 1$  and these numbers  $\pm 13$ . Also look for the prime patterns from these numbers to the right and left. Take the difference between these numbers and the primes to the right and to the left and these will be the

values of  $\text{comb}_5$ .

**Example 8:** Show how the comb values and its wave nature produce the prime patterns around  $\text{Spp}_n$  values and multiples of  $\text{Spp}_n$  values in both the + and - directions creating one of the symmetries of prime numbers. Other symmetries in an  $\text{Spp}_n$  table are found around: 1) The center point of row 1 (value  $\frac{1}{2} \text{Spp}_{n+1}$ ) and 2) The center point of the table  $\frac{1}{2} \text{Spp}_n$ .

**Example 9:**  $\text{Rpp}_n$  tables in 3 dimensions. Graph the product values of the  $\text{Rpp}_n$  table in the Z direction for a better visual understanding of the products that cancel red cells in the  $\text{Spp}_n$  tables. Show how one  $\text{Rpp}_n$  table is used to generate the next  $\text{Rpp}_{n+1}$  table.

**Example 10:** In the twin prime only tables show how to calculate the white cells in the twin prime table without products or factorization. Each cell has 2 equations that can be used to calculate the value. The first is the row 1 top cell value of the column using formula plus a known multiple of  $\text{Spp}_{n+1}$ . The other is the product of 2 elements of row 1. Solving these two equations provides the row in which the white cell occurs (necessary for the twin prime only table). Extend the tables showing factors between values of total number of twin prime pairs for  $n=2, 3, 4, 5, 6, 7$  showing the ratio of growth from one table the subsequent table as  $10/3, 55/10, 398/55$  etc.

### CONCLUSION

This paper is Part IV in the ongoing series of papers developing new mathematical tools that allow for proofs by Induction. The tools developed in Parts I, II and III are used to show that there is a greater than exponential growth in the occurrence of primes, twin primes and in general prime pairs

of gap  $k=2, 4, 6 \dots$  in the  $\text{Spp}_n$  Tables. This leads to the proof of the Twin Prime Conjecture in the form of the statement "every relatively twin prime pair (relative to  $\text{Spp}_n$ ) will generate an infinite number of real twin prime pairs".

### REFERENCES

1. McCanney JM. Principles of prime numbers - Part I - New definition of prime numbers with modNt number system & induction. J Pure Appl Math. 2024;8(3):01-59.
2. McCanney JM. Principles of prime numbers - Part II - Anatomy of  $\text{Spp}_n$  &  $\text{Rpp}_n$  tables with conversion from mod10 to modNt number system. J Pure Appl Math. 2024;8(4):01-12.
3. McCanney JM. Principles of prime numbers - Part III -  $\text{Spp}_n$  &  $\text{Rpp}_n$  table parameters with advanced topics and twin prime conjecture. J Pure Appl Math. 2024;8(5):01-15.
4. McCanney JM. Calculate primes with DVD lecture. JM McCanney Sci Press. Nashville (TN); 2007.
5. McCanney JM. Principles of prime numbers. Vol I. JM McCanney Sci Press. Nashville (TN); 2010.
6. McCanney JM. Breaking RSA codes. JM McCanney Sci Press. Nashville (TN); 2014.
7. McCanney JM. New definition of prime numbers with  $\text{Spp}_n$  tables and proofs by induction. 2024.
8. Cantor G. Contributions to the founding of the theory of transfinite numbers. Dover Publications; 1915.