Principles of prime numbers - Part II - Anatomy of Spp_n & Rpp_n tables with conversion from mod10 to modNt number system

James M McCanney

McCanney J.M. Principles of prime numbers - Part II - Anatomy of Spp_n & Rpp_n tables with conversion from mod10 to modNt number system. J Pure Appl Math. 2024; 8(4):01-12.

ABSTRACT

The prior paper "Principles of Prime Numbers - Part I - New Definition of Prime Numbers with ModNt Number System & Induction" must be read to put this paper into perspective. That paper is a summary of three books "Calculate Primes" (2007), "Principles of Prime Numbers – Volume I" (2010) and "Breaking RSA Codes" (2014). Another peer reviewed paper can be included in the list of prior papers "New Definition of Prime Numbers with Spp_n Tables and Proofs by Induction". In the prior books and papers, the concept of direct calculation of prime numbers was presented. It showed that the prime numbers constitute a complete number system modNt with a new visual representation in the form of "Spp_n and Rpp_n Tables".

The understanding of Prime Numbers as a complete system of mathematics generated from Peano's Postulates continues with visualizations of the results of using the "McCanney Generator Function" which directly calculates prime numbers in groups. The base N modulo number systems (e.g. the most common of which is mod10 or base 10) are shown to be inadequate to understand the true nature of Prime Numbers, which are the building blocks of all mathematics. It was shown that the mod10 or other base N number systems have been the hinderance to true understanding. The new modNt number system gives complete logical meaning to the prime numbers since every digit is related to the ancestry of the specific prime number showing them to have a unique "heritage" or "ancestry" back to the alpha prime 0 (zero). Many unexpected results arise on the order and properties of the prime numbers.

Key words: *Prime numbers; Number systems; Generator function; Nature's number system*

INTRODUCTION

This is part II of a multiple series set of papers building a complete mathematical structure for the prime numbers, showing they form a number system without the inclusion of other non-prime numbers. This and future papers will concentrate on the many details discovered by analyzing prime numbers as members of groups (they form a progression of groups with one generating the next) which gives rise to proofs by induction. This paper defines the structure of the Spp_n & Rpp_n Tables. It will also introduce the parameters that generate the next table from the prior table showing the nature of prime numbers from a completely different point of view [1].

Anatomy of Spp_n tables

Spp_n means "the nth Sequential Prime Product". It represents both the nth sequential prime product (product of all primes up to the nth prime) and is used to create the Spp_n Tables (with n incrementing from n = α, 0, 1, 2, 3 … ∞). The series of tables built below represent the visualization of the McCanney Generator Function (see prior papers referenced in the abstract). The sequence of tables is followed below by a generic Spp_n Table with some of the main features found in all tables. There are numerous forms of each table. The "raw" table has all numbers including non-primes with the "red columns" noted that eventually can be selected to create the "red only columns" tables (these uniquely contain all real and relative prime numbers). These can further be selected for "twin prime only tables" where only twin primes are managed in the given table. The "twin prime only tables" show that twin primes and only twin primes generate future twin primes as the tables are constructed from one to the next [2]. They can further be selected for other gap sizes or gap patterns. Each version has advantages depending on the problem one is trying to visualize. The "raw" all number table is useful in visualizing solutions to the Goldbach Conjecture, whereas the twin prime red column only table is essential in understanding the Twin Prime Conjecture. The full red column only table is best for understanding the symmetries of prime numbers, counting functions of prime numbers or sequences of higher order gaps (4, 6, 8 …) or gap patterns and the wave nature of prime numbers. Induction proofs are much easier to understand when visually enhanced. The tables below are first built with mod10 numbers and then followed by modNt numbers. The advantages of modNt numbers will become obvious whereas the limitations of mod 10 numbers will also become apparent. Tables n = α, 0, 1, 2, 3 & 4 follow (Figure 1).

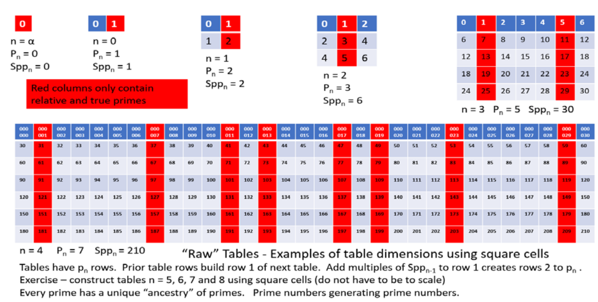


Figure 1) *Spp_n Tables n = α, 0, 1, 2, 3 and 4 mod10*

Encryption – Jmccanneyscience, LLC, United States

Correspondence: James M McCanney, Encryption – Jmccanneyscience, LLC, United States, e-mail: jmccanney@usinternet.com

Received: 4 May, 2024, Manuscript No. puljpam-24-7133, Editor Assigned: 5 May, 2024, PreQC No. puljpam-24-7133 (PQ), Reviewed: 16 May, 2024, QC No. puljpam-24-7133 (Q), Revised: 30 May, 2024, Manuscript No. puljpam-24-7133 (R), Published: 30 July, 2024, DOI:-10.37532/2752-8081.24.8(4).01-12

In above table Spp_n (n = 3) the twin primes occur in columns 1 and 5 (the origin and ancestors of all twin primes). They are (5, 7), (11, 13), (17, 19) and (29, 1). (23, 25) is not a pair that generates future twin primes because 25 (as shown in tables below) is not relatively prime to 30 = 2 x 3 x 5 since 25 has 5 as a factor. All future pairs of primes with gaps of 4 (using the definition of gap $k_n = p_j - p_n$) also originate in this table with n = 3. The table below is the same table but with red arcs over the twin prime pair columns including (29, 1). See part I paper for the explanation of this using group theory for closure. These tables form closed Groups and can further be reduced to just red column tables which contain only relative and real primes. They can further be reduced to having only twin prime red columns (where twin primes only are managed). Remember that relative primes are as much a prime as "real" primes in each table. Each table transforms in the progression of steps below where relative primes are identified in the table which will then generate the next table [3-6]. This may seem like a tedious effort, but it establishes the simple rules for creating future tables which become very large very quickly. It is the basis for solutions to unsolved problems using induction. The point is not to create large tables of prime numbers (although that is accomplished), but to create the mathematical structure to produce the induction proofs. Note that primes 2, 3 and 5 are colored with a light blue color showing that they are in the "Dead Zone" and will no longer generate future prime numbers (Figure 2).

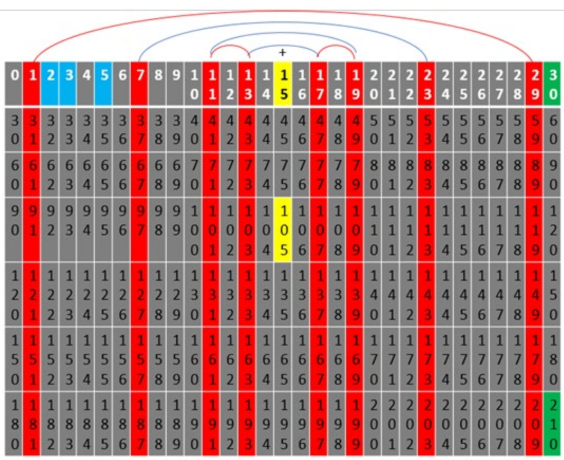


Figure 2) *Dead zone*

Below is the same table with white cells marked as products of $p_n = p_4 = 7$. They are symmetrical around the center point of the table $\frac{1}{2}Spp_4 = 105$. This is a fundamental property of all Spp_n tables. The white boxes are not relative prime to $Spp_4 = 210 = 2 \times 3 \times 5 \times 7$ because they have factors of 7, therefore they do not carry to build row 1 of the next table but all other cells including the black cells (products of 11 and 13) do carry over because they are relatively prime to 210. Since the white cells are symmetrical around the ½ point 105, this symmetry carries to the next table (symmetry is one of the basic properties of the Spp_n Tables and prime numbers in general). There is one and only one white box per column. The white cells are products of the red cells of row 1 with other red cells of row 1. These form a closed system so that when one eliminates the non-red cells, the red columns constitute a closed system containing only prime and relative prime numbers. In the future, the tables can be reduced to red column relative primes only (red columns) and further reduced to twin prime only tables (that generate only twin primes into all future tables). At this point one of the formulas for creating a new table from a prior table can be written. Since there is one white cell in each column, this constitutes the equivalent of a single row lost that does not carry to the create row 1 of the next table. So, the next table n = 5, $Spp_n = 2310$ for $p_n = 11$ row 1 will have 8 columns x ($p_n - 1$) rows = 8 x (7 – 1) = 48 cells symmetrical around 105 and 11 rows. The next table n = 6 will have 48 columns x (11 – 1) rows = 480 cells in row 1 symmetrical around 1155. This is the first of many equations that are developed to determine parameters of future tables without actually creating the tables, and thus forming the basis for proofs by induction. If you rotate the table with white cells only by 180 degrees, you see the same pattern of white cells showing symmetry (Figure 3). The black cells will exhibit symmetry when one arrives at the Spp_n tables for which they are p_n.

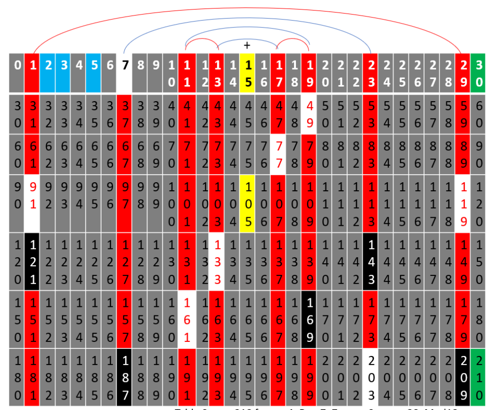


Figure 3) *The black cells will exhibit symmetry when one arrives at the Spp_n tables for which they are p_n*

The following table is the same table with all products colored boxes. Read the notes to the side of the table. As the tables grow (faster than exponentially) it will become apparent that there will be a higher percentage of true primes, true twin primes and prime pairs of other gap sizes in each successive table. Try to look at the patterns of colored cells (first white, then black only, etc.) to see the patterns (noting that the tan boxes have multiple factors so are common to more than one pattern). This is to show that the products that eliminate cells from being primes can be generated without multiplying numbers. When only the red columns are isolated, the entire pattern of primes *vs.* relative primes is contained in the table (without a single calculation). When the modNt system of numbers is used farther below, the entire process will gain another level of simplicity since only the top row and left column cells need to be filled in.

When the rows are placed in sequence to form row 1 of the following n = 5 table with only the white cells not transferring, and since there is just one white cell in every row (making each row unique but symmetrical around the ½ point 105) these are the patterns that will shape the prime number patterns in the next table. These patterns are observed in the prime numbers and what have given rise to the belief that the primes are "pseudo random" when it is actually these patterns that are emerging. Part I paper defined "combs" and the wave nature of prime numbers. The $comb_n$ is the series of relative primes of row 1 of the Spp_n table that repeats with wavelength Spp_{n-1} to infinity with all primes predicted by this wave (in the form of an equation). Each successive table becomes more accurate at predicting primes. This gives rise to the rapid methods of breaking RSA codes and the accurate prediction of large primes (Figures 4 and 5).

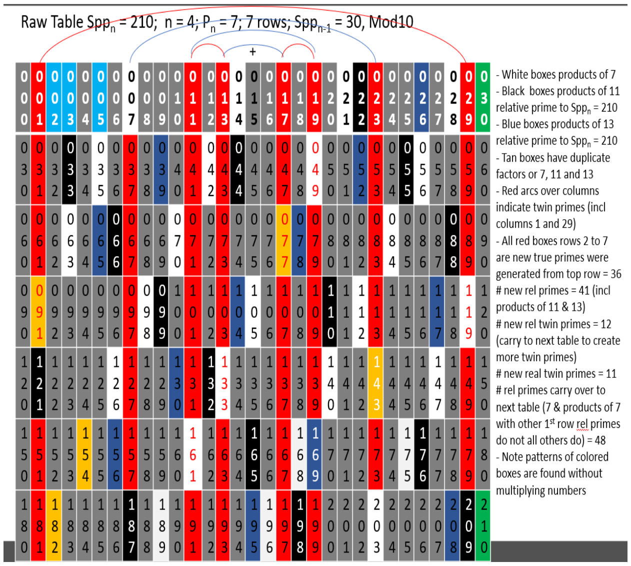


Figure 4) *This gives rise to the rapid methods of breaking RSA codes and the accurate prediction of large primes*

The following is the same table as those above but red columns only. This is done with each table creating the relative prime only tables. The tedious part of this process is over except for one more table farther below that isolates the twin pimes only red column table. Read the comments on the sides of this table. Each variation of the table (raw, red columns only or twin prime only) can generate the next table of the same type (raw, red columns only or twin prime only). Other variations exist such as gap = 4 or gap patterns but are not dealt with in this paper. In the side notes at the left of the table it notes that the black cells are symmetrical around the respective ½ points of the table when they are the current prime p_n of that table. This could alternatively be constructed using the n = 4 table below by simply extending the same table down from the current table 11 times (210 x 11 = 2310 with ½ point 1155 for p_n = 11). Only the white cells are symmetrical around the ½ point of the n = 4 table and since these are the only cells that do not carry to create the next table, the symmetry carries to the next table. The p_n prime p_4 = 7 where $Spp_n - p_n = Spp_4 - p_4 = 210 - 7 = 203$ are always white cells and therefore enter the Dead Zone DZ_{n+1} of the n+1 table. This is a natural process that eliminates these numbers on each successive table increasing the size of the DZ on each successive table. The next table will have "1" in row 1 the first column and the next column as $p_{n+1} = p_5 = 11$. On the right side of row 1 will have $Spp_n - 1$ which is always a relative prime number (and sometimes a real prime, however it will always be relatively prime to Spp_n and will always carry to the next table to be symmetrical with 1 the top left corner cell). The next cell from the right in row 1 of the n+1 table will always be $Spp_n - p_n$ to be symmetrical with p_n .

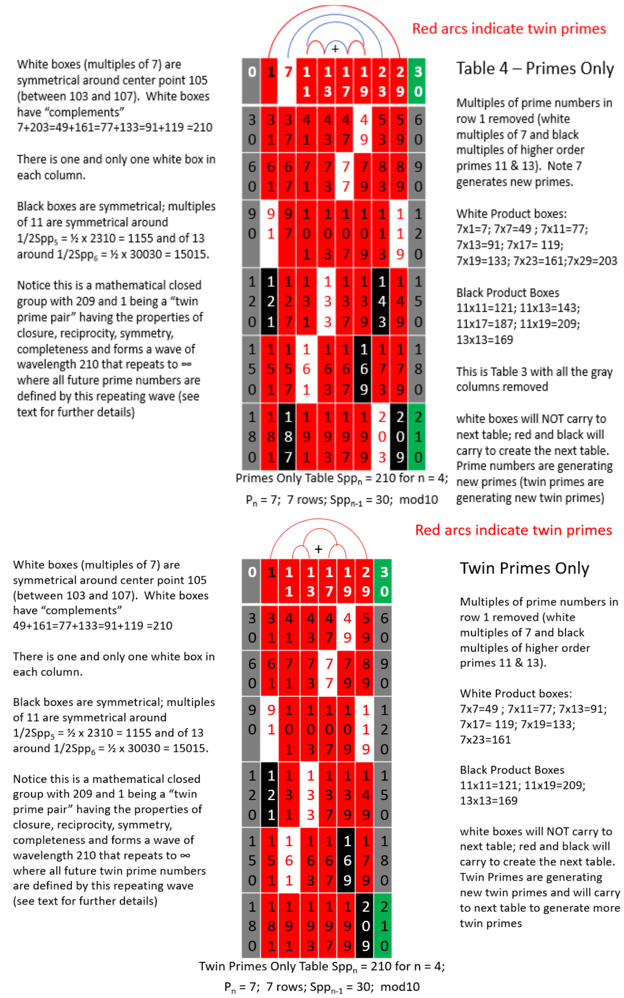


Figure 5) *The tedious part of this process is over except for one more table farther below that isolates the twin pimes only red column table*

As with any of the tables, when looking at the table with white boxes only, if you rotate the table by 180 degrees, it will have the same pattern of white boxes (symmetry) and this then comes out in the creation of the next table. The twin prime only table above contains all the twin primes to this point (besides those lost in the DZ) but these are the ancestors of all twin primes (there are no others). The twin primes similarly form a compete number system without the aid of any other numbers. All future twin primes have one of these twin prime pairs as ancestors which in turn have ancestors in prior tables back to table n = 3. Pick any prime number or twin prime pair and as an exercise, find its ancestry back to this table (see paper Part I for more information on this process).

The next set of tables are for n = 5 mod 10. All the same variations are created as above ending in the twin prime only tables. This will complete the mod10 set of tables since it will be a rudimentary exercise to continue creating larger tables. The next steps will be to create the formulae that generate the parameters of future tables to begin the process of proofs by induction.

To generate the next table which is the raw table for n = 5, go to raw n = 4 table to generate the following raw n = 5 table. The table numbers are too small to see here so enlargements of portions of the table are listed farther below (Figures 6-12).

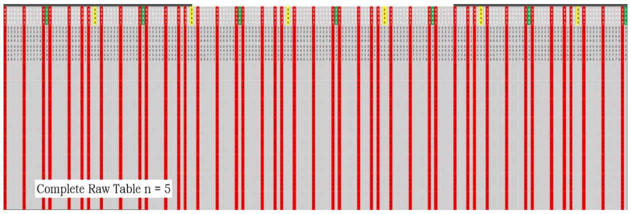


Figure 6) Raw n = 5 Spp_n Table - red columns contain all primes

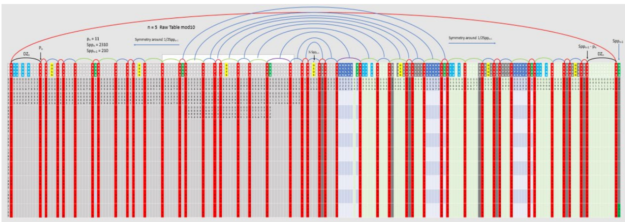


Figure 7) *Red arcs above adjacent red columns indicate twin primes and all other gaps are noted with arcs also. The end to end red arc is between column 1 on the left and column 209 on the right. The blue arcs represent symmetry of the patterns to the left and right of the center point 105 of row 1 (the blue arcs are not complete). See enlargements below for greater detail*

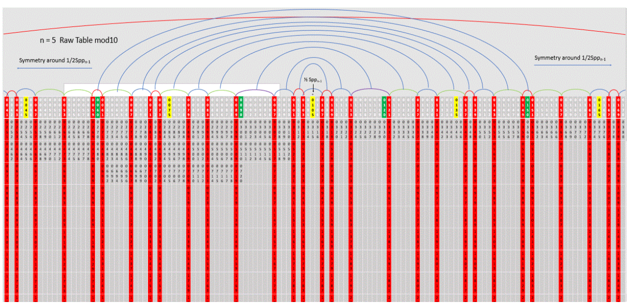


Figure 8) *Enlargement center region*

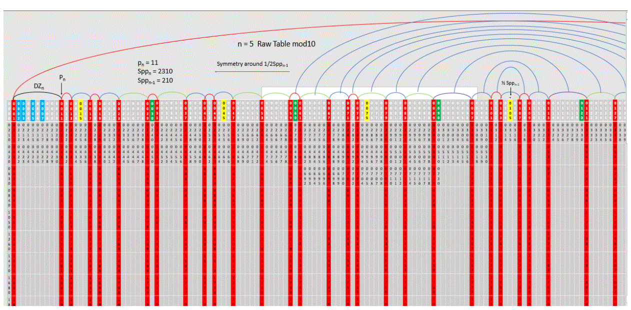


Figure 9) *Enlargement left region*

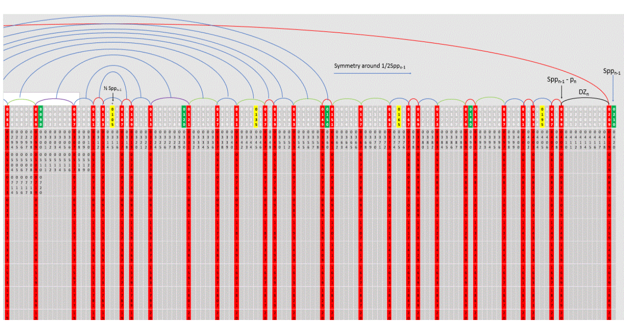


Figure 10) *Enlargement right region*

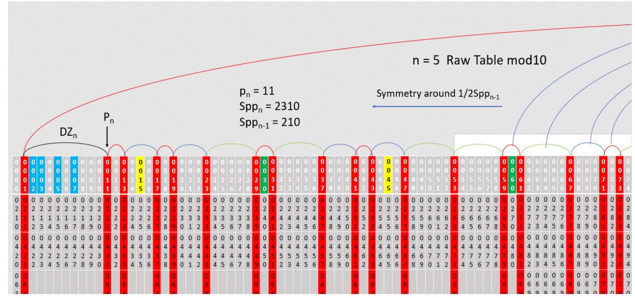


Figure 11) *Enlargement top left region*

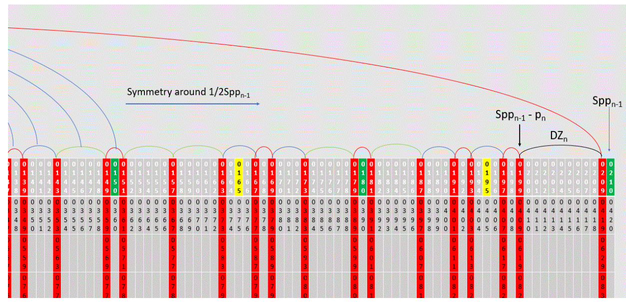


Figure 12) *Enlargement top right region*

Note in the above 2 diagrams that 7 and its complement 203 in row 1 have now moved into the Dead Zones (DZ_5) between 1 and p_5 = 11 on the left side of the table and between 210 – 1 = 209 and 210 – 11 = 199 on the right side of the table (column p_4 = 7 and the column Spp_4 – p_4 = 210 – 7 = 203). This is a subtle but natural process resulting from the Generator Function and its single boundary condition. The following is the red column only table derived from the above raw table with arcs above the columns indicating the gaps between the columns (Figure 13).

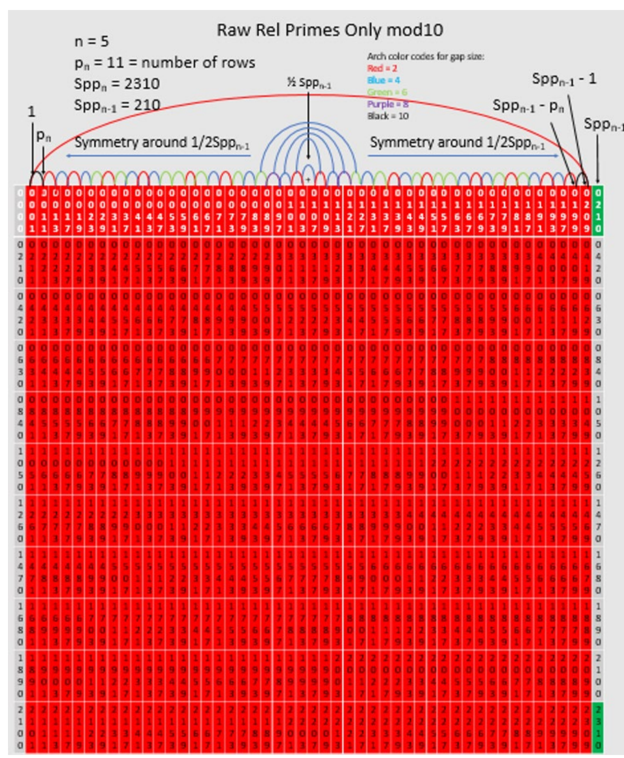


Figure 13) *Gaps between the columns*

The following is the red only table with arcs above the columns and white cells (noting products of p_5 = 11) with one white cell in each column. The black cells are products of 13. To create row 1 of the following table for n = 6, use the rows of this table less the white cells stinging them together (and of course not using the 0 or 210 columns except for the very first cell = 0 and last cell = 2310). As an exercise, complete this table and see how many relative primes, real primes, relative twin primes, real twin primes, relative primes of gap = 4 and real primes of gap = 4 (Figure 14).

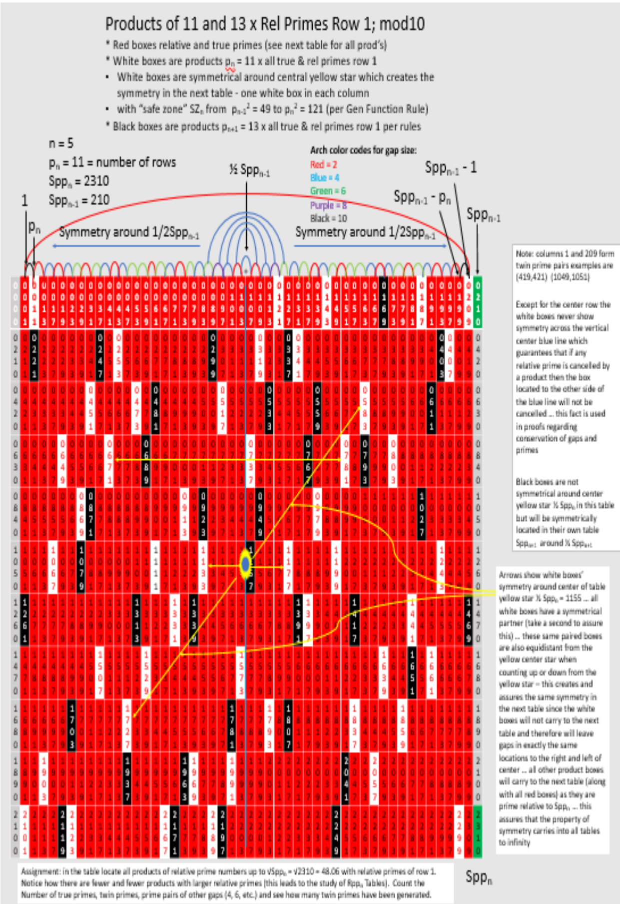


Figure 14) *See how many relative primes, real primes, relative twin primes, real twin primes, relative primes of gap = 4 and real primes of gap = 4*

The following is the red column twin prime only table mod10 for n = 5 (this table not completed left for exercise but will be completed below for modNt case farther below). It is not a closed system in that the products of row 1 (unlike the complete red column table). The products of row 1 may or may not land in the table. There is a simple method of finding the white and black cell values in each column without the full table. Since there are two equations that exist for the numerical value each cell, the solution of these quickly gives the values of the cells that will be white (multiples of p_n) and black (products of all other members of row 1 less than $\sqrt{Spp_n}$). The two equations for composite (non-prime) numbers are a) the product using numbers from row 1, and b) the top cell value plus a multiple of Spp_{n-1}. As twin prime only tables get larger, this is a simple method of determining real and relative primes (Figure 15).

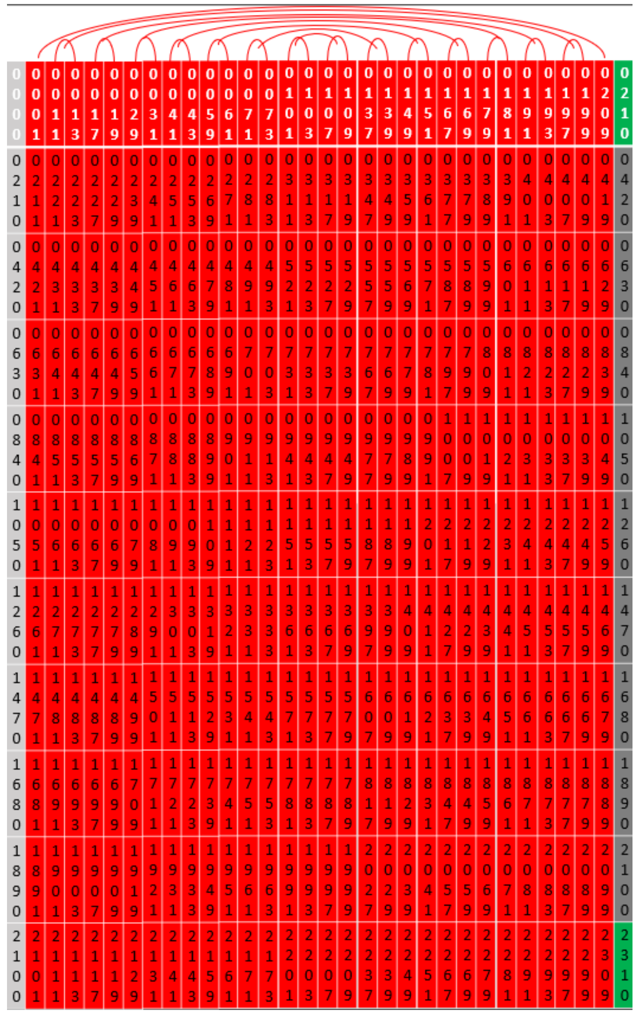


Figure 15) *The red column twin prime only table*

The following are the same tables but using the modNt number system. First there will be a quick review of the modNt number system before showing the modNt tables. For a more complete explanation review the paper Part I of this series. The following table gives the values of modNt *vs.* mod10 numbers up to 72 mod10. Note especially values of 1, 2, 6 and 30. These are the increments of digits in the modNt number system (as are 1, 10, 100 in the mod10 number system). Also note the values modNt for multiples of 30. Its usefulness will become apparent in the tables farther below. The limitations of the mod10 number system will also become apparent. Farther below is the formal definition of the modNt number system up to 510510 – 1 = 510509 (Figure 16).

000	000		200	012		400	024		1100	036		1300	048		2000	060
001	001		201	013		401	025		1101	037		1301	049		2001	061
010	002		210	014		410	026		1110	038		1310	050		2010	062
011	003		211	015		411	027		1111	039		1311	051		2011	063
020	004		220	016		420	028		1120	040		1320	052		2020	064
021	005		221	017		421	029		1121	041		1321	053		2021	065
100	006		300	018		1000	030		1200	042		1400	054		2100	066
101	007		301	019		1001	031		1201	043		1401	055		2101	067
110	008		310	020		1010	032		1210	044		1410	056		2110	068
111	009		311	021		1011	033		1211	045		1411	057		2111	069
120	010		320	022		1020	034		1220	046		1420	058		2120	070
121	011		321	023		1021	035		1221	047		1421	059		2121	071
200	012		400	024		1100	036		1300	048		2000	060		2200	072

Table - Nature's Number System modNt vs mod10 Number System to 72

Digit in column	7	6	5	4	3	2	1
n mod10	6	5	4	3	2	1	0
n modNt	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$p_n$mod10	13	11	7	5	3	2	1
$P_n$modNt	201	121	101	21	11	10	1
Spp_n mod10	30030	2310	210	30	6	2	1
Spp_n modNt $= 10^n$	1,000,000 $= 10^6$	100,000 $= 10^5$	10,000 $= 10^4$	1000 $= 10^3$	100 $= 10^2$	10 $= 10^1$	1 $= 10^0$
Max digit Value in mod10 $= p_{n+1} - 1$	16	12	10	6	4	2	1
Pattern each digit repeats the given pattern over and over (values given in mod10)	0 x 30030 1 x 30030 2 x 30030 3 x 30030 4 x 30030 5 x 30030 6 x 30030 7 x 30030 8 x 30030 9 x 30030 10x30030 11x30030 12x30030 13x30030 14x30030 15x30030 16x30030	0 x 2310 1 x 2310 2 x 2310 3 x 2310 4 x 2310 5 x 2310 6 x 2310 7 x 2310 8 x 2310 9 x 2310 10x2310 11x2310 12x2310	0 x 210 1 x 210 2 x 210 3 x 210 4 x 210 5 x 210 6 x 210 7 x 210 8 x 210 9 x 210 10 x 210	0 x 30 1 x 30 2 x 30 3 x 30 4 x 30 5 x 30 6 x 30	0 x 6 1 x 6 2 x 6 3 x 6 4 x 6	00=0x2 11=1x2 22=2x2	0=0x1 1=1x1
Max number in pattern mod10	510509	30029	2309	209	29	5	1
Max number pattern modNt	16.12.10.6.4.2.1	12.10.6.4.2.1	10.6.4.2.1	6.4.2.1	4.2.1	2.1	1

Figure 16) *modNt number system*

The following figures are the same tables as above but in modNt (see figure 17 below).

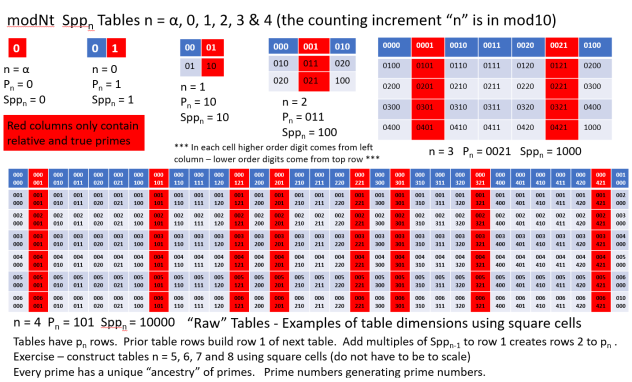


Figure 17) *Numbers in each cell in the modNt number system*

Determining the numbers in each cell in the modNt number system is easy (left column gives higher order digit and top row gives lower order digits (much simpler than mod10). To convert to mod10 simply multiply the digit by the value of the Spp_n for example the number from lower right-hand corner 006421 = 6 x 30 + 4 x 6 + 2 x 2 + 1 x 1 = 209 mod10. With modNt, you do not have to populate or write all the numbers since for any cell you can figure out the cell value by just combining the left column and top row. Note symmetry. With the same tables in modNt, the values of prime numbers can be easily determined without tedious long division nor multiplication and then easily converted to mod10.

See the patterns of the digits in the rows and columns with modNt numbers. Moving across in any given row the lower order digits follow the same pattern as row 1 numbers. Moving down in the columns only the higher order digits increment by 1. It is not necessary to write all the cell values because they are easily constructed with just the higher order digit from the left column and the lower order digits from the top row (Figure 18 below).

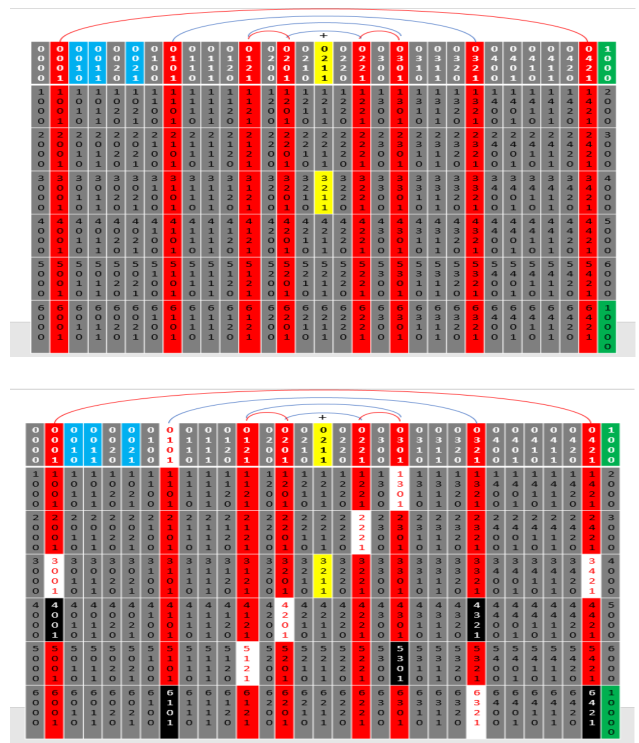


Figure 18) *Higher order digit left column - lower order digits top cell*

The following 2 tables are the red only table and the red only table with white cells (products of prime p_n = 101 modNt = 7 mod10. These are then used to create the n = 5 tables farther below. It is left as an exercise to create the twin prime only red column table (Figure 19 below).

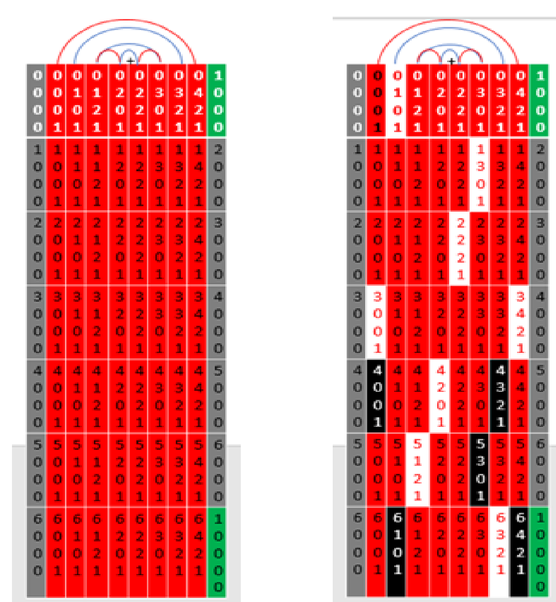
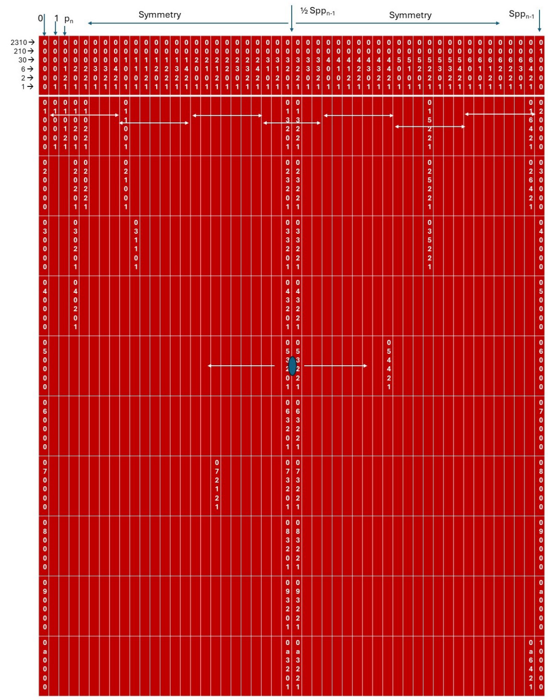
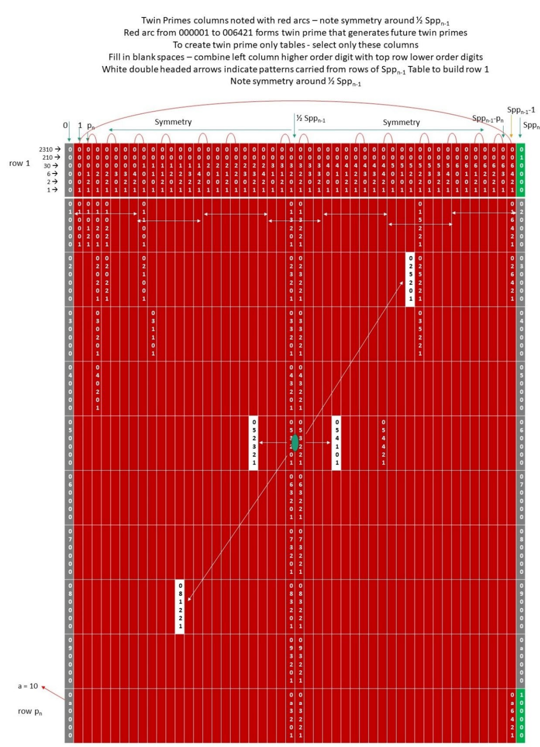


Figure 19) *Red only table and the red only table with white cells*

Below are the n = 5 tables in modNt number system. Note at the top left are the numbers with arrows 2310, 210, 30, 6, 2 and 1. These indicate the multipliers of the digits in the column for converting to mod10 and understanding the value of the digits in the numbers (3 tables build up to figure 20 farther below).





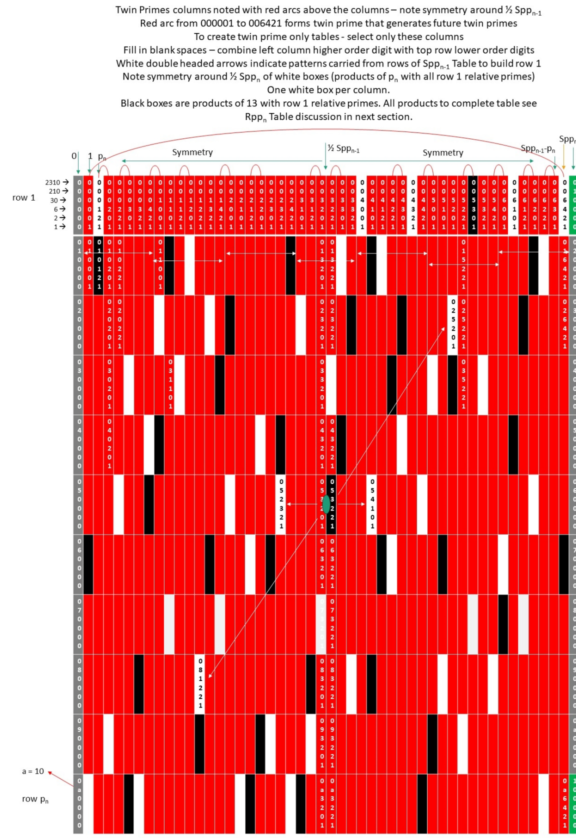
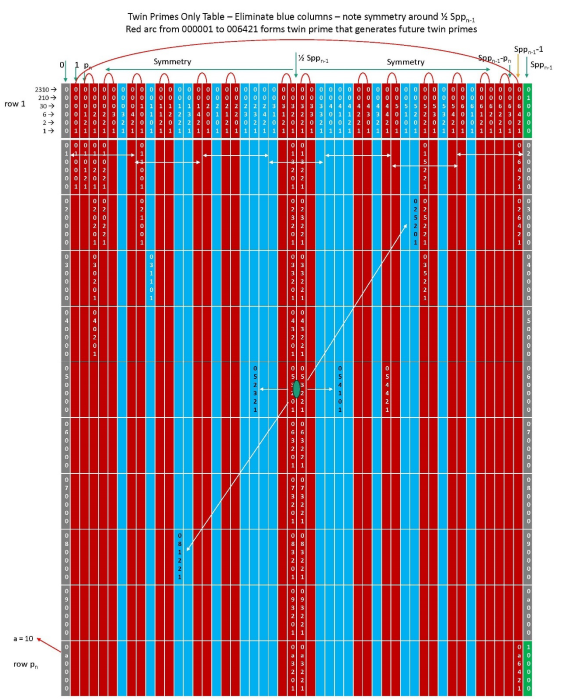


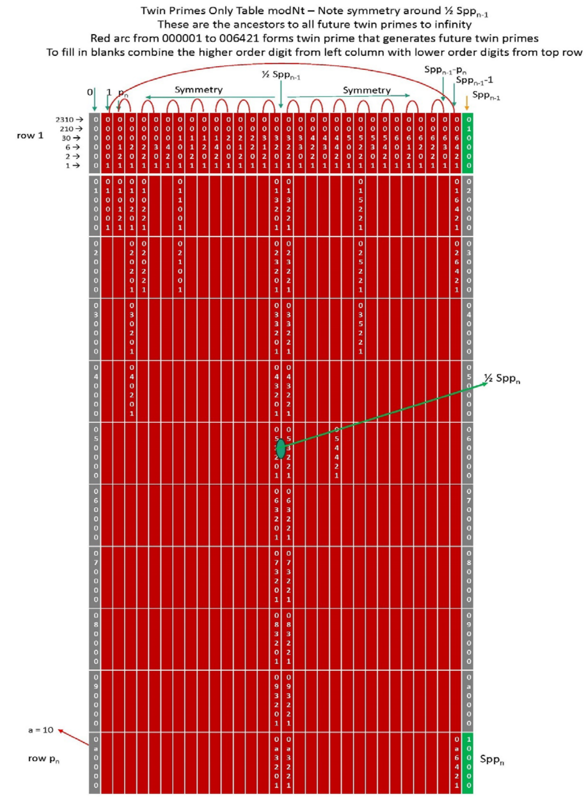
Figure 20) *Only white cells do not carry to build row 1 of next table*

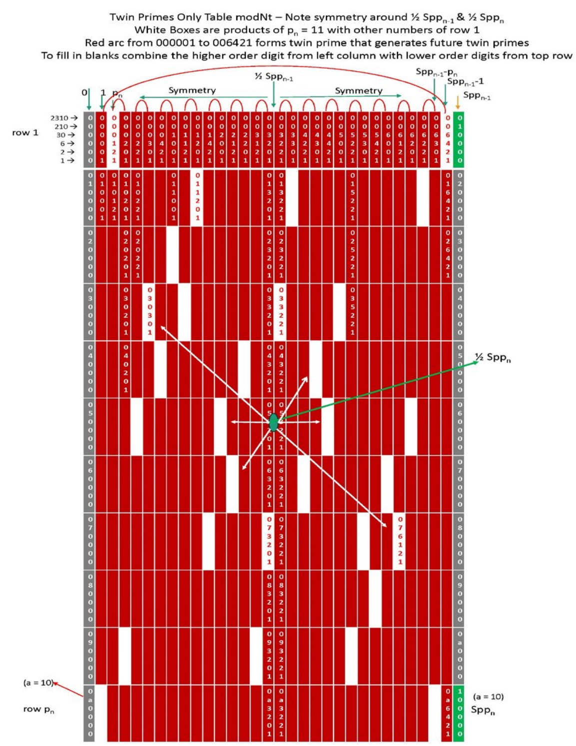
Using the modNt number system, create the next table remembering that the white cells do not carry over. Also complete the multiples of all relative primes from row 1 and complete the black boxes. This will illustrate the processes and show how easy it is to produce the Rpp_n products (developed in the next section of this paper) and noting the issue with duplicate products using the raw tables (which include the red columns). Since the multiples of any relative primes form patters you can populate the tables with products without ever multiplying numbers as shown previously with the mod10 tables. The row 1 patterns are spaced (eg multiples of 11 or 13 or any other relative prime and remember to include all the relative primes). They form patterns and when you get to row 2 they fall in an offset but the same pattern all the way to row p_n. You will find that the rule for preventing duplicate products is that if the row 1 cell pj has a factor less than the number pi then it will create a duplicate product. The goal is to have the number of cancelled cells subtracted from Spp_n = number of real primes, or in the case of products of p_n = the number of cells that carry to create the next table if you ignore the duplicate issue then you still will be left with all the real primes and without ever making a single multiplication.

Note the importance of $\sqrt{Spp_n}$ as the largest element of row 1 that will produce a product less than Spp_n and fit inside the table and therefore the quotient $(\sqrt{Spp_n})/Spp_{n-1}$ gives a value that becomes very small very fast. This gives a relative reading of the number of products of relative primes that will eliminate red cells from being real primes. There are more advanced modifications to this factor that will be more exact to show percentage more primes in each table, but just this factor is sufficient to show that the real primes, real twin primes, etc., will grow faster than exponentially in numbers with each successive table. It is the $e = mc^2$ equation of prime numbers. It is the key to understanding the large growth of prime numbers, twin primes, etc. using the modNt number system. Using the twin prime only tables, each successive table also has a "$comb_n$" equation that has a wavelength of Spp_{n-1} that repeats to infinity just like the full red column tables, which then predicts all twin primes. Also as with the full red column tables, the accuracy becomes much better with every successive table. This has always been the difficulty with attempts at creating formulae for the discovery of prime numbers. They were limited in scope and rapidly broke down, producing more and more false prime solutions. This is now overcome in the current system. In the current system, the opposite is true as it becomes much more accurate with larger numbers.

The following table shows the twin prime only columns in red and all other columns in blue. Removing the blue columns creates the twin prime only table for n = 5, noting that the symmetry is maintained around both ½ Spp_{n-1} = 105 (in row 1) and ½ Spp_n the center point of the table. As an exercise, pick an empty cell and determine its modNt value (using the left column and top row value) and then convert to mod10. The prime numbers are determined without division nor multiplication, but only using addition of the relative prime numbers of row 1 with the same multiples of Spp_{n-1}. In the third table below with red column twin prime only for n = 5 and white cells marked, you can generate the next twin prime only table for n = 6, however at this point the equations for generating the subsequent tables can be used without ever generating the tables (the following tables build up to figure 21 farther below).







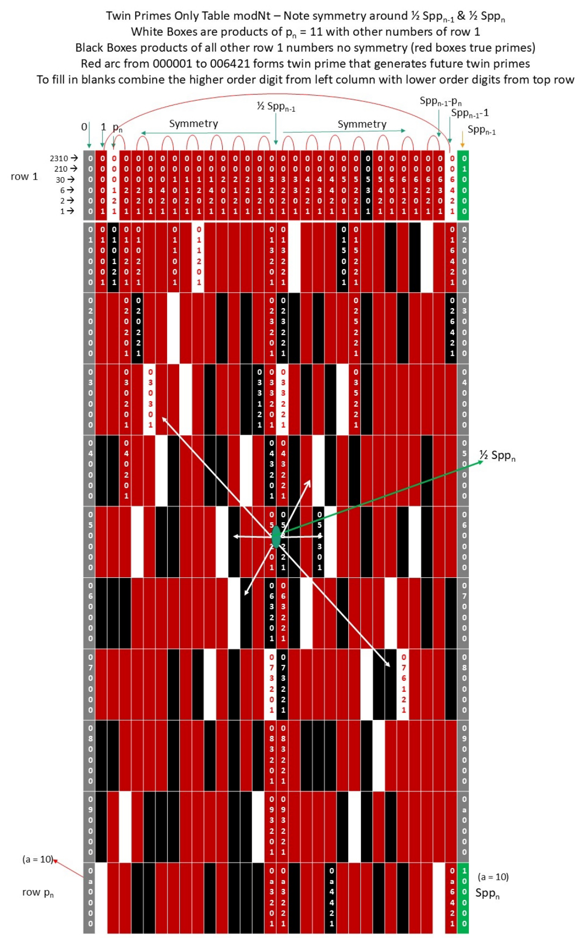


Figure 21) *The twin prime only n = 5 table*

In the tables above, prime numbers form a complete isolated number system with Group properties of - closure – reciprocity – symmetry – completeness - and wave nature, starting with Peano's Postulates and generating all prime numbers to infinity with each prime number having a unique ancestry back to the alpha prime 0.

Anatomy of Rpp_n tables

Rpp_n Table means "the nth Relative Prime Products Table". The Spp_n Tables described above (with n incrementing from n = α, 0, 1, 2, 3 … ∞) are generated starting with the n = α (alpha) prime 0 and Peano's Postulates. When each Spp_n table is completed and the products of p_n x p_i (the white cells) are removed where p_n is the nth prime number (associated with the nth Spp_n Table), and p_i = all relative primes of row 1 of the Spp_n Table, the remaining "cells" in the rows of the Spp_n Table are then used to generate row 1 of the next table Spp_{n+1}. There is one last step in working with the Spp_n Table before moving to the n+1 table. The step to complete table Spp_n is to determine the entire list of real prime numbers and gather parameters (for example: total number of real prime numbers, total number of twin primes, max gap values, value of $(\sqrt{Spp_n})/Spp_{n-1}$, etc.) and then to create tables and graphs showing the progress as the tables grow. An appendix in Part I of this series gives an extensive list of parameters.

The values of relative prime numbers in the Rpp_n Table come from row 1 of the Spp_n Table as noted below. Recall in the prior papers that it was established that the relative primes in an Spp_n Table are as valid a prime number as "real" primes and therefore must be used in calculating the products that cancel cells (in discovering real primes). This is a very subtle aspect of the nature of prime numbers and the direct calculation of prime numbers. It is what has been overlooked prior to the current work. This is the key that unlocks the true nature of prime numbers and their ancestry. As noted in the prior papers, "Every Prime number has a unique history and ancestry going back to the alpha prime 0". Some of the members of the ancestry may be relative prime numbers that as noted, are as real as "real" primes in a given Spp_n Table. These are composite numbers whose factors are prime numbers larger than p_n and therefore are relatively prime to the sequential prime product Spp_n which has as factors all prime numbers up to and including p_n. The unique ancestry of all prime numbers is seen in the modNt number that represents it. Additionally, each prime number has a family of related primes that were generated in the same "red column" and as such these primes are "siblings" and have "cousins" that were generated in adjacent columns in the same Spp_n Table. This also means that every twin prime pair has a unique ancestry as well as any other gap size or gap pattern. This conversely means that every twin prime (as well as relative twin prime pair) in each Spp_n Table will generate an infinite number of twin primes in future tables.

It is through the collection of parameters of the Spp_n & Rpp_n Tables (as they generate subsequent Tables) that begins the process of proofs by induction. Visual as well as mathematical understanding helps solidify the process just as geometric shapes help give visual confirmation of algebraic and trigonometric mathematics. The process of multiplying relative primes to locate real primes is a much more efficient process than using factorization division calculations to locate prime numbers since the prime numbers are generating future prime numbers. The goal is not to generate large tables of prime numbers (although this is a byproduct), but to create the parameters that will allow proofs by induction. The use of the modNt number system will facilitate the process since one does not have to fill in the numbers of every cell in the table, but only fill the top row and left hand column (these are used to generate the numbers in each cell making the entire process very fast). Any cell can easily be converted to mod10 numbers as will be shown. The tables become very large very quickly as noted in the prior papers. The Rpp_n Table shows the products that eliminate the relative prime numbers in the Spp_n Table, leaving only true prime numbers. There are simple rules to guide this process. Below the diagrams show a progression of Rpp_n Tables from n to n+1. A subtle issue is that there can be multiple products of relative primes from the Rpp_n Table that "cancel" a given "cell" so there are simple rules to prevent double counting the number of cancelled cells. Since the goal is to determine the number of real primes (the final count of cells that remain after all the relative prime products are calculated) this is not an issue.

One subtle feature of the products of any number from row 1 of a given red column only Spp_n table is that all the products of the given row 1 relative prime with any other number from row 1 results in a product that lies in the red column table. This means that each red column only table is closed (e.g. without the need for any other numbers). The prime number system is a closed system. This gives rise to the Rpp_n Tables (Figure 22).

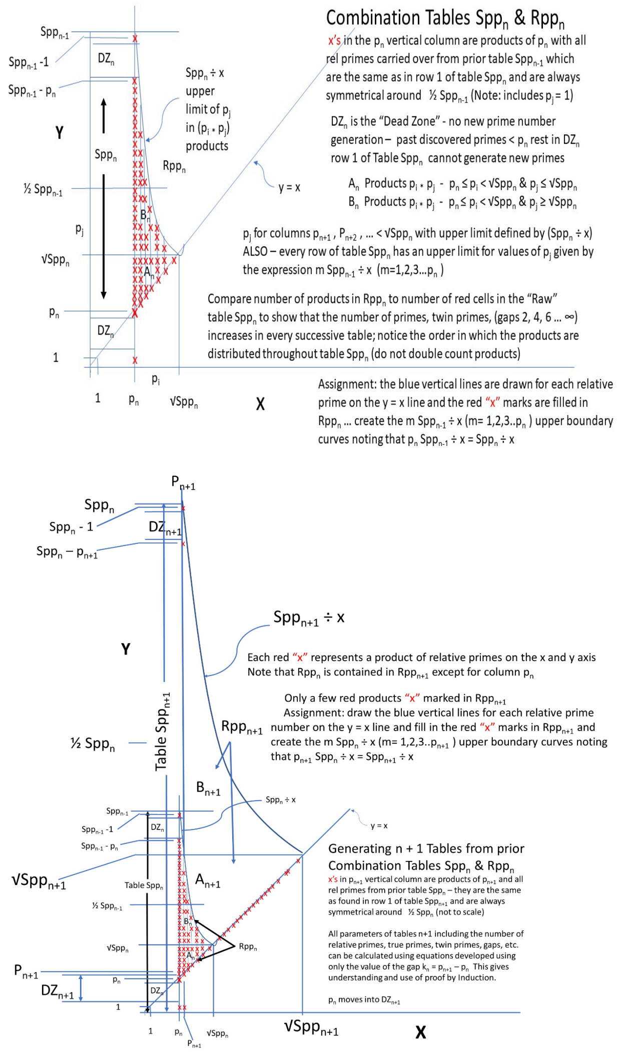


Figure 22) *Rpp_n Tables for n above and for both n and n+1 lower graph*

The newly developed methods break the problems into pieces that can be solved individually then combined to give a complete solution. Previously, primes were thought of as "random or pseudorandom numbers with no patterns". Trying to understand prime numbers as a linear progression has been partly at fault. Prime numbers are now organized and generate future prime numbers in groups and families with ancestors and descendants. This is key to creating the organization to solve more complex problems.

The "Calculate Primes" Generator Function system of directly calculating prime numbers is made more understandable in the current text because of the visualization using tables defined as "Spp_n and Rpp_n Tables". It is the same prime number solution, but visual. By dividing the direct calculation of primes into small manageable groups which have well defined parameters and mathematical properties, one can now solve problems with understanding that was not available before. Each group generates the next group with known parameters giving rise to proofs by Induction. This is the "tool" that everyone has been looking for over the past 2500 years. The mathematical system, as explained in "Calculate Primes", involves such mathematical properties as Closure, Symmetry, Reciprocity, Completeness and a wave pattern that allows equations to be generated with wave lengths that extend to infinity which become increasingly accurate with each successive table.

Finally, a system exists to predict the future of prime numbers and solves the issue of density of primes as one moves out on the number line. It is proven that the equations for prime number generation are simply bounded. The concept of finding "rogue primes" and "rogue gaps" is finally solved, and it is shown that in the Generator Function structure, prime numbers are a monotonically decreasing density function that is important in understanding prime patterns. Theorems are developed proving that all prime numbers generate an infinite number of future prime numbers and likewise all have ancestral patterns in the primes going back to the alpha prime "0". These are used to understand and offer solutions to the Twin Prime Conjecture and Goldbach Conjecture unsolved problems. The prime numbers are generated in groups using the Generator Function by only addition and subtraction of previously discovered prime numbers (discovered in the prior iteration of the Generator Function) (Figure 23).

Generic Spp_n tables

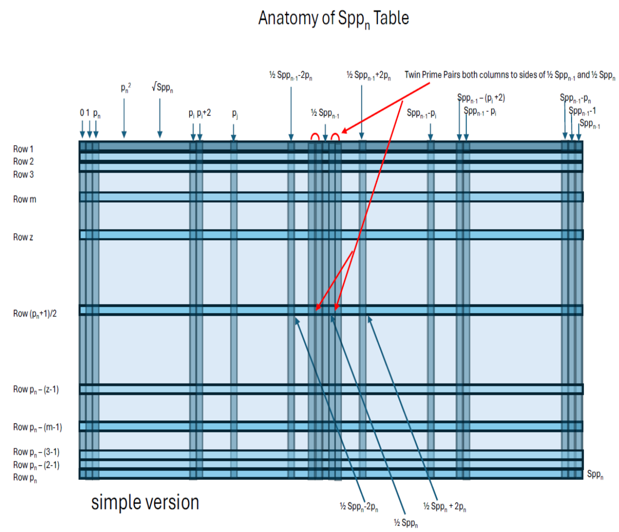


Figure 23) *Sample generalized Spp_n Table*

The example table above is a very simplified version of a generic Spp_n Table. This will be used as a basis for future papers to illustrate concepts. It may include symmetries for analysis of Goldbach pairs or it may be used to develop counting functions for twin primes. There are hundreds of variations of these tables showing different aspects of the primes with the list growing every day. Part of the usefulness is in understanding the subscript notation. There are many representations of prime numbers using the Spp_n and Rpp_n Tables and different ways of looking at the newly discovered sets of primes. The first is the rudimentary boundary condition where newly discovered real primes are all numbers p_i such that $p_n^2 < p_i < p_{n+1}^2$. These fall in the safe zone where there can be no composite numbers in the n+1 table. This is the natural process defined as the only boundary condition of the Generator Function. A second way of counting new real prime numbers is in rows 2 to p_n in each successive table using the Rpp_n Tables which find all real prime numbers in the table (without factorization).

There is another way of representing the numbers in Spp_n Tables. Since each number is locked in a grid of rows and columns and since each has a known set of factors (or is a true prime), these can be mapped onto a grid with mod10 integers and with a third number representing the lowest prime factor (p_{min}) of the cell (with 0 used to indicate a true prime). When discovering white cells of a given table to proceed with creating the next table (products of p_n with all other relative primes of row 1), and since there is symmetry around the ½

Spp_n value in the center of the table, one only must determine the white cells up to the center of the table since all the rest of the white cells are then found using symmetry. The 3 number representation (p_{min} , i_{row} , j_{col}) where i_{row} and j_{col} are the integer values of the row and column of the cell in the red column only table. With this information the entire Spp_n and Rpp_n Tables can be reconstructed but held with far less data than storing the entire set of numbers. The point of this Part II paper is to present the core ideas to understand the tables, but there remain many exercises for those interested in pursuing these leads. The point of this paper is to clarify the tables which lead to the proofs by induction.

Problem sets for Spp_n and Rpp_n tables

The following are examples of problems that can be solved using the information in Part I and Part II papers which give understanding to the order of the prime numbers as a complete autonomous number system. There are hundreds of similar problems that highlight the nature of prime numbers and eventually this series of papers will grow into a university level course complete with problem sets.

Calculate parameters of future Spp_n Tables and twin prime only tables.

- Determine the number of columns and rows (and therefore the total number of red cells) in the next 3 Spp_n Tables (for n = 6, 7 and 8). Do this without actually creating the tables. Note the number of cells in each row of the prior table that will carry to form row 1 of the subsequent table. Look for example in the n = 5 table with white cells and count the number of non-white cells. You will count 43 cells in row 1, 44 cells in row 2, 44 cells in row 3, 43 cells in row 4, 45 cells in row 5 and 42 cells row 6 containing ½ Spp_5. As you continue down the rows these numbers will reverse since the table is symmetrical. The patterns of white cells will be exactly reversed from the first rows (again due to symmetry of the table). Since there is only one white cell per column, all these patterns are different, and these sub patterns carry to the next and future tables. This constitutes an entire area of study.
- In doing so you will also be determining the number or "white cells" which are products of p_n with all other relative primes of row 1 of the Spp_n Table.
- Determine the number of twin prime columns and determine the number of relative twin primes and real twin primes in each table. These exercises will form the basis for the solution to the twin prime conjecture. Only the white boxes do not carry to the following table. Note how p_n and (Spp_n – p_n) do not carry to the next table and enter the Dead Zone of the next table naturally. They will no longer generate prime numbers, although all of the relative primes that they generated (and their offspring) will continue to generate future prime numbers.

Using the symmetry of the Spp_n Tables to understand the GoldBach Conjecture

- Show why the Goldbach Conjecture local maximum values are located at Spp_n and multiples of Spp_n. This is a starting point to understanding the solution to the Goldbach Conjecture. Each real prime number in the red only tables has a complement that is symmetrical around the center point of the table ½ Spp_n which when added will be equal to Spp_n . Some of these pairs will have a relative prime that does not count in the final solution, however it maximizes the alignment of numbers causing the local maximums of GB pairs to be at Spp_n .
- Show that for all the solutions to the Goldbach Conjecture found above (including if the numbers were relative primes), if you add or subtract 2 from either of the pairs of numbers to find a prime number (one of the GB pair is part of a twin prime), then this produces a GB pair for Spp_n ± 2. There are other ways of using the tables to analyze the GB Conjecture.

Deriving modNt numbers in Spp_n Tables and converting modNt numbers to mod10

- In any of the modNt tables above select an open cell or twin prime set of cells and determine the modNt number(s)
- Convert this number to mod10

Determine the ancestry and future generation of a relative primes using the modNt numbers

- Using the same numbers as the prior exercise, determine the ancestry of the modNt number(s) using the digits of the modNt number back to the alpha prime 0 (if a twin prime pair, also determine the original ancestor twin prime pair in table n = 3; also noting that some of the ancestors may be relative primes in that table).
- Use any twin prime pair of row 1 of Spp_5 Table (n = 5) and determine the number of relative twin prime pairs that carry to the next table and then follow all of these to the following table (noting that only the white cells do not carry to the next table).
- 169 mod10 = 5301 modNt is a relative prime in tables n = 4 and n = 5 since 13 is relatively prime in these tables. Determine the number of real prime numbers generated by 169 before it enters the Dead Zone of Table n = 6, remembering that it still will generate relative and real prime numbers in table n = 6 where p_6 = 13. Then determine the ancestry of 169 back to the alpha prime 0 (using the modNt number 5301) and convert to mod10 numbers. Note that all the prime and relative prime numbers generated by 169 (its offspring and their offspring) will continue to generate prime numbers to infinity. This leads to future discussion in future parts of this series dealing with the infinities of prime numbers (referring to concepts of Georg Cantor [6]).

Growth of primes, twin primes in future tables (counting functions)

- Since the size of the Spp_n Tables grows faster than exponentially, the counting functions grow faster than exponentially also. Look at the importance of the factor ($\sqrt{Spp_n}$)/Spp_{n-1} and what this means on a given table. The term $\sqrt{Spp_n}$ gives the upper limit of row 1 relative primes whose square fits within the Spp_n Table. As the tables become larger, this value rests farther to the left of row 1 whose maximum value is Spp_{n-1}. So, this ratio gives an estimate of the percentage of products in the Rpp_n Table compared to the total number of red cells in the table. Since this number gets small very fast, it is the key to understanding the growth of the counting function when viewed with Spp_n Tables. Since the tables grow faster than exponentially, and since the traditional counting functions are logarithmic, this gives a new method of understanding prime numbers. Rather than the prime numbers becoming fewer as you proceed out the lineal number line, they become more prolific.

- The $comb_n$ equations predict all future prime numbers (derived from the list of relative primes of row 1 of table Spp_n) with wavelength = Spp_{n-1} repeating to infinity to predict all future primes. Each successive $comb_n$ predicts with greater accuracy and each comb reduces the number of primes in the prediction (although the total number of primes increases because the wavelength grows). As a result, there are no "rogue primes" or clusters of primes. This brings up the topic of density of primes. If there is a large gap, then there will be an equalizing effect of many smaller gaps in the vicinity and visa versa. As the primes grow, the density of primes will become smoother (again the result of only white cells transferring to construct the next cell). This is a complex topic that requires analysis and is the topic of future parts of this paper.

Max Gap considerations (Max gaps do not carry to next tables only the evenly distributed p_n products with relative primes in each table carry forward).

- Understanding of Max Gaps comes from the subtle aspect of white only cells of the Spp_n Tables. White cells represent the products of p_n with all other elements of row 1 including 1. Since prime numbers generate future prime numbers, it might be considered that Max Gaps would also carry forward to affect future gaps. This is not the case. Since the white cells are the only cells that do not carry forward to form the next tables, and since the other relative primes (which cause the Max Gaps) do not carry forward, and since the white cells are evenly distributed throughout the Spp_n Table, the Max Gaps do not carry forward to affect gaps in the future lists of primes.
- This also gives rise to the assurance that solutions to the Goldbach conjecture will not be affected by large gaps that could cause it to fail. These are advanced topics that will be pursued in future parts of this series.

Gap 4 prime pairs (the definition of gap in this paper is the gap k_n = $p_j - p_i$ the difference between the two prime numbers)

- Since primes of gap 4 cannot be created from cancelling intermediate primes, they are like twin primes in that the only prime pairs that can have gap = 4 are descendants of original gap 4 primes (relative primes) in table Spp_3 with n = 3 (twin primes are separated by gaps of 4 so if one of the twin primes is cancelled then it results in a gap of 6). Select any gap 4 pair of primes and find their ancestry back to table n = 3.
- Use the same gap 4 pair of primes to determine all the future prime pairs of gap 4 (there will be an infinite number of them).

REFERENCES

1. Principles of Prime Numbers - Part I - New Definition of Prime Numbers with ModNt Number System & Induction
2. McCanney JM. Calculate Primes with DVD Lecture, jmccanneyscience Press, Nashville, TN, 2007.
3. McCanney JM. Principles of Prime Numbers – Volume I,jmccanneyscience Press, Nashville. TN. 2010.
4. McCanney JM. Breaking RSA Codes, jmccanneyscience Press, Nashville, TN. 2014.
5. New Definition of Prime Numbers with Sppn Tables and Proofs by Induction
6. Contributions to the Founding of the Theory of Transfinite Numbers (Dover Books on Mathematics) by Georg Cantor