

Small dimensionless interaction couplings in low-temperature amorphous solids and quantum electrodynamics

Mihail Turlakov*

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ABSTRACT

At low temperatures, phonons and localized Low-Energy Excitations (LEEs) in Quantum Strongly-Noncrystalline Solids (QSNS) and glasses are weakly coupled, long-lived quasi particles. This weak coupling, typically modeled as an interaction between phonons and two-level systems representing LEEs, is characterized by a small dimensionless coupling constant $\alpha_{\text{QSNS}} \approx 1/150$. In

the distinct field of Quantum Electrodynamics (QED), the weak coupling between photons and electrons is described by small fine structure constant $\alpha_{\text{QED}} \approx 1/137$.

Despite significant differences between QSNS and QED, I propose that the problem of calculating small dimensionless coupling constants in both fields from first principles belongs to a common universality class.

Key Words: Nuclear physics; Atomic nuclei; Nuclear reactions; Energy production; Quantum mechanics; Nuclear applications; Particle physics

INTRODUCTION

Quantum Electrodynamics (QED) and fine-structure constant are very familiar to physicists and to general public, while Quantum Strongly Non-Crystalline Solids (QSNS), which is simply another name for low-temperature amorphous solids and glasses, is not widely known area of the condensed matter physics. QED and QSNS are quite different areas of physics, but there are some constructive parallels. The problem of small dimensionless coupling constant of QSNS is only known to few experts, although the problem was clearly articulated long time ago. In the case of QED, the small coupling constant is the famous fine-structure constant. A striking observation is that both areas have small dimensionless coupling constants of similar magnitude. In this paper, I argue that there are more reasons to think that this striking observation is a consequence of universality rather than a coincidence.

QED is one of the best understood areas of physics, while QSNS is one of the least understood areas of physics in spite of multiple experimental and theoretical studies. Yet the comparative analysis of these two areas can possibly give valuable insights into both areas. In the rest of this introductory section, I introduce very briefly the problem of small dimensionless coupling in low-temperature QSNSs. In the main part of the paper, I compare the similarities and differences between excitations and their couplings in both areas of QSNS and QED.

LITERATURE REVIEW

Below several Kelvins, a large class of materials, amorphous solids or equivalently QSNSs, have nearly universal properties. Dimensionless sound attenuation is nearly independent of frequency and independent of material's type under certain conditions. Sound attenuation measures the coupling between phonons and Low-Energy Excitations (LEEs). The dimensionless interaction coupling constant between LEEs and phonons can be denoted as QSNS-coupling constant α_{QSNS} . Experimental measurements of internal friction, a measure of sound attenuation, give the value of $\alpha_{\text{QSNS}} \approx 1/150$.

The universality of internal friction, and therefore universal dimensionless coupling, was noticed over 14 orders of phonon frequencies. The Figure 1 of the internal friction taken from the reference shows for a large range of frequencies the universality of internal friction value in the relaxational

regime of LEEs [1,2]. For high-frequency phonons $\omega > k_B T$, approximately the same small value of internal friction is found in the resonant regime of LEEs.

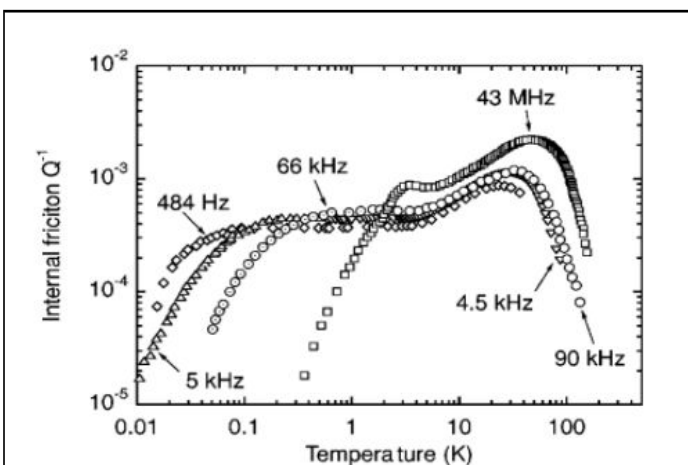


Figure 1) The dimensionless interaction coupling constant between LEEs and phonons can be denoted as QSNS-coupling constant α_{QSNS}

Quantum SNSs can be called quantum because LEEs become coherent and long-lived excitations below several Kelvins. The temperature $k_B T_Q \sim 1-3\text{K}$ defines the crossover regime, so that at temperatures lower than T_Q LEEs with energies $E < E_Q = k_B T_Q$ are quantum. The phenomena of echo and hole burning demonstrate quantum coherence properties of LEEs quite unambiguously. We can remark that phonons are also long-lived and display quantum Bose statistics at temperatures lower than the Debye temperature T_D .

Although the detailed microscopic nature of Low-Energy Excitations (LEEs) is still mysterious, there is no doubt that LEEs exhibit dominant and nearly linear-in-temperature contribution to the specific heat of QSNSs below T_Q . LEEs are localized in the sense that they do not contribute noticeably to the thermal conductivity at least below T_Q . The leading candidate for LEEs became Tunneling Two-Level States (TTLSs), proposed by Anderson-Halperin-Varma and Phillips (AHVP) [3]. One reason is why TTLS phenomenology became popular because this model provided a basic, even visual, picture of low-energy excitations (for example, see the discussion of

Departments of Physics, Oxford University, Oxford, United Kingdom

Correspondence: Mihail Turlakov, Departments of Physics, Oxford University, Oxford, United Kingdom; E-mail: mihail.turlakov@gmail.com

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the tunneling of ammonia molecule). TTLS phenomenology remains unsatisfactory and insufficient phenomenology for several reasons. The nature of LEEs - whether LEEs are always strictly two-level systems and are formed due to tunneling - is still the unresolved paradox, because there is no microscopic description of LEEs, which is sufficiently detailed and yet universal.

The similarities and the differences between QED and QSNS

There is appealing general analogy between QED and QSNS. Both fields of physics have two species of well-defined and weakly interacting particles (or excitations) of force and substance. QED has electrons and photons, while QSNS has LEEs and phonons. Both QED and QSNS have the most basic quantum field theory structure with one species of matter and one species of interaction. In what follows I discuss the similarities and the differences between QED and QSNS, while the quenched disorder of QSNS is an obvious and explicit difference, which plays no role in QED.

There is a very clear textbook similarity between photons of QED and phonons of QSNS. Photons and phonons are bosons. Photons and phonons have linear momentum-energy relation. Both particles are represented equivalently in the second quantization representation. Although one noticeable difference is that in the ground state (i.e., vacuum), phonons of QSNS are transverse and longitudinal, while photons of QED are only transverse.

There is no such a simple analogy between quasiparticles of matter in QSNS and QED. After all, even classical theories of elasticity and electrodynamics have different structure. LEEs of QSNS are analogous to Electron-Positron Pairs (EPPs) rather than directly to electrons of QED [4]. For instance, the elementary hollow spherical defect in any solid has dipolar radial stress. This example shows that Coulomb interaction $1/R$ is not possible and not sustainable in the elastic solid. There is no direct analog of singular charge (i.e., electron) in QSNS.

LEEs are gapless excitations and have finite constant density of states, while electron-positron pairs of QED are gapped $E > 2m_e c^2$. QED at energies $E \gg 2m_e c^2$ bears closer similarity to QSNS, where c is the light velocity. In this limit of QED, both, phonons of QSNS and photons of QED, have finite lifetimes due to the corresponding LEE-phonon and electron-photon couplings. A particularly simple result for phonons of QSNS is that the ratio of the phonon mean-free path l_{mfp} to the phonon wavelength λ is $l_{mfp}/\lambda = \alpha^{-1}_{QSNS} \gg 1$ is finite, frequency-independent, and is much greater than one. In what follows I continue comparing various aspects of QED and QSNS in a little bit more detail.

The lowest-order interaction vertex in the corresponding Hamiltonians describes the electron-photon coupling of QED and LEE-phonon coupling of QSNS correspondingly. Fine-structure constant and QSNS-coupling constant are the vertices squared in the loop calculation. Both couplings are small and of the order of 10^{-2} . It turns out that both α_{QED} and α_{QSNS} have logarithmic scaling as a function of energy in the renormalization theory. The general form of LEE-boson and EPP-boson coupling is broadly similar, where boson is phonon in QSNS or photon in QED correspondingly [5]. Some details of these couplings are yet different due to longitudinal and transverse parts. Both QED and QSNS have small dimensionless coupling constants, which describe the weak coupling of LEE-phonon and EPP-photon types [6]. LEEs interact between each other by dipolar-type interactions, and EPPs interact also naturally by conventional dipolar interactions.

Both theories have similar logarithmic corrections to the coupling constant with Landau pole at high energies. We can write down the explicit expressions for the logarithmic scaling of the couplings.

The fine structure constant is $\alpha_{QED} = e^2/(hc)$ defined by the corresponding constants of elementary electron charge e , Planck constant $\hbar$, and the speed of light c . The logarithmic screening in QED is

$$\alpha_{QED}(\epsilon) = \frac{\alpha_{QED}}{1 + \alpha_{QED} \frac{2}{3\pi} \ln(Am_e c^2/\epsilon)}, \quad (1)$$

where $A = \exp(5/6)$ and $m_e c^2/\epsilon \ll 1$. Virtual electron-positron pairs are responsible for the logarithmic screening. Landau pole energy is defined as the energy where the denominator becomes zero. The logarithmic screening in QSNS can be discussed in terms of screening of dipolar interaction α_{d-d} between two LEEs. The detailed angular dependence $f(\phi, \theta)$ for LEEs have been discussed, and this angular dependence is different from the textbook angular dipolar dependence of QED. Efros-Shklovsky screening effect in three spatial dimensions (3D) can be expressed as

$$\alpha_{d-d}(\epsilon) = \frac{\alpha_{d-d}}{1 + \alpha_{d-d} P_0 g \ln(E_Q/\epsilon)}, \quad (2)$$

as a function of the energy ϵ , where $E_Q/\epsilon \gg 1$ and $E_Q = k_B T_Q$. P_0 and g are the density of LEE states (per energy per volume) and purely numerical factor of order 1. The strength of dipolar interaction $\alpha_{d-d} = \Lambda^2/(\rho c^2 s)$, where Λ are local energy scale and local-mode coupling, and ρ and c_s are the mass density and the sound velocity of any particular QSNS substance.

DISCUSSION

In the first order of the approximation, both couplings are calculated as one-loop particle-antiparticle Feynman diagram. Both couplings of QED and QSNS have logarithmic screening due to the loop corrections. One difference between Eqns. (1) and (2) is also that they are applied when $\ln(Am_e c^2/\epsilon) < 0$ and $\ln(E_Q/\epsilon) > 0$ correspondingly. The important difference is that in QED the logarithmic correction appears from $R d^4p/p^4$ integral in the p -momentum space, while in QSNS the log-correction comes from $R d^3R/R^3$ integral in the real space. So the corresponding effective field theories are not identical, but they might belong to the same broad universality class. The analogy here is that although second order phase transitions have broadly similar critical behaviour around the phase transition, the detailed values of critical exponents can be different.

U(1) gauge symmetry of QED is considered to be fundamentally important. Although the gauge theory of elastic media with and without defects was developed, a gauge theory of interacting phonons and LEEs would be desirable to shed further light on the analogy between QED and QSNS.

It is insightful to consider the interpretations of dimensionless coupling constants of QED and QLTG as the ratios of two energies. Since both energies of the ratio scale equivalently as a function of distance, the ratio itself is constant and dimensionless. Fine-structure constant α_{QED} can be interpreted as the ratio of electron-positron energy and photon energy for the same scale R . The energy in the numerator is the attractive energy e^2/R of electron and positron separated by the distance R . The energy in the denominator is the photon energy hc/R with the wavelength R (up to a factor of 2π).

The QSNS-coupling constant α_{QSNS} can also be interpreted as the ratio of the dipole interaction between two QSNS sub-volumes separated by a distance of R (up to a factor of 2) and the mean-level spacing energy. The energy in the numerator is the dipole interaction energy α_{d-d}/R^3 between two sub-volumes. The energy in the denominator is the mean-level energy $1/(P_0 R^3)$, which is mean-level energy spacing of LEEs in the sub-volume of the radius R . Thus the standard expression for dimensionless coupling is $\alpha_{QSNS} = (\Lambda^2 P_0)/(\rho c^2 s)$.

The similarity between the two couplings, represented as energy ratios, is that in each case of QED and QSNS the interaction energy in the numerator is much smaller than some intra-volume energy. The difference is that the intra-volume energy in the denominator is the photon energy for QED and the mean-spacing energy for QSNS.

The obvious difference between QED and QSNS is that LEEs are localized while electron-positron pairs of QED are freely propagating in the vacuum.

As far as the screening of the coupling constant is concerned, both LEEs and EPPs tend to be local objects. Namely, the disorder might be irrelevant if disorder scale is larger than the size of electron-positron pairs or the spatial size of LEEs. The size of electron-positron pairs in QED is small $\sim \hbar/(m_e c)$ by energy-time uncertainty estimate, since EPPs have finite energy of order $2mc^2$, where m_e is the electron mass. Frozen disorder is definitely critical for the appearance of LEEs in the first place. Yet QSNS can be viewed homogeneous at the distances much larger than so-called mid-range order scale, so disorder is indeed not relevant for many phenomena at these large distances [7]. In spite of some suggestive arguments, it is not clear how the interplay between disorder and interactions works exactly in QSNS.

Another distinction between QSNS and QED is the role of the quantum statistics. LEEs, just like overlapping electron-positron pairs, do not have definite ensemble quantum statistics. In an earlier paper, I examined if QSNSs can be understood by assuming LEEs built out of fermions in the analogy with the Landau Fermi liquid for metals. Anyhow the role of quantum statistics in QSNS is also not clear at all at this stage. The further progress in understanding the interplay between the long thermalization times, the quantum statistics, and the localization of LEEs could come from the theory of many-body localization.

It is important to point out that Landau pole is only a signature of stable low-energy phases of QED and QSNS. Landau pole is not the evidence or the proof of the origin of the common universality class. The coupling constants in QSNS and QED change very slowly and logarithmically, this fact means that the phase can be observed over very large range of energies. The origin of the common universality class is likely due to the common high-energy theory and the mechanism of the phase transition (symmetry breaking in the terminology of the field theory) from the high-energy to low-energy phase.

In this paper, I mainly express the hypothesis of the common universality class of QED and QSNS. I hope to develop quantitative theoretical arguments and to propose possible experimental tests of the hypothesis in the future publication [8]. If one takes seriously this hypothesis, then QSNS can become a terrestrial laboratory for QED in testing of the origin of the small fine structure constant. In reverse, quantum field theory might be helpful in describing the generality of universal low-temperature properties of glasses and QSNS.

The common universality class of QSNS and QED is a hypothesis, while the full proof of this hypothesis looks very hard if not implausible. Such a proof would require the derivation of the fine structure constant α_{QED} and QSNS coupling α_{QSNS} . There was a lot of effort in both fields to derive in some way these dimensionless coupling constants, but none of these attempts became accepted in any of these fields. We can argue that any low-energy universality is a consequence of high-energy degrees of freedom. While in QSNS high-energy theory, which is the theory of disordered interacting ions and electrons, is clear, in QED such a high-energy theory is not known in spite of the Grand Unification Theory (GUT) hypothesis [9]. Interestingly,

the problem of the universality in QSNS is still much harder than many other problems of condensed matter physics because we do not know the reduced low-energy Hamiltonian of this general many-body problem in the presence of strong disorder.

CONCLUSION

QED and QSNS can be conventionally regarded as low-energy phases emerging from high energy phases. Each of these two areas of physics have only two species of particles identified with force and substance. Each of these two fields of physics are characterized by small dimensionless couplings of order 10^{-2} . The main quixotic proposition of this paper is that the low-energy phases of QSNS and QED belong to the same universality class determining the origin of the small dimensionless interaction coupling.

The qualitative differences between QSNS and QED are the role of disorder present only in QSNS (and not in QED) and the relativistic symmetry present only in QED (and not in QSNS). These differences may or may not be relevant in changing the universality class which defines these small dimensionless interaction couplings. The similar mechanism of logarithmic screening by electron-positron pairs in QED and by LEEs in QSNS is only indicative of the common universality class. The exciting possibility is that amorphous solids may turn out to be an experimental laboratory for studying some aspects of QED and particle physics.

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