

# The Collatz Conjecture – From 1 To Infinity, No Loops, No Gaps, No Instability, P Vs NP, Halting Problem

Barbara Clarke\*

Clarke B. The Collatz Conjecture – From 1 To Infinity, No Loops, No Gaps, No Instability, P Vs NP, Halting Problem. J Pur Appl Math. 2021; 5(3):38:42.

**ABSTRACT**

This paper describes the derivation of three functions that when used together generate all Collatz conjecture sequences starting from 1. Three functions are necessary as will become evident.

The tree structure that is defined by the functions is fractal in nature and not like any other tree structure I have seen presented (Utube). Irrespective of issues of proof the definition of data structure has the potential to

significantly impact current studies of the Collatz Conjecture. The work stands alone. There are no reference papers or associated organisations. The mathematics involved do not follow the general approach used by most mathematicians, in fact nothing more difficult than high school algebra is required.

Even so the paper addresses the issues of potential loops, possible gaps and the question of possible instability. The P vs NP question and halting issues for computer searches are touched on. Potentially the major impact of this work is the implication that hailstone data sets are the outcome of several functions rather than the product of a single function. Two of the functions referred to above are closely related. The third is not.

**INTRODUCTION**

My initial approach to the Collatz Conjecture was the same as most people, pick a number and generate a sequence heading to 1. After a few such numbers it becomes obvious that a tree structure is involved.

I decided to see if the problem could be approached from the opposite end. Accordingly I also decided that determination of the odd numbers was the key to the problem.

**Data Generation**

The tables below have been truncated to save space. There is enough information present to allow them to be extended indefinitely. The basic relationship of the Collatz Conjecture can be presented in two equations;

Equation 1:  $3*A + 1 = N$       A is odd, N is even.

Equation 2:  $N1 = N/2$       N is even, N1 may be odd or even.

If N is even and not a straight power of 2 then on division by 2 a sufficient number of times you will arrive at an odd number.

So N can be represented as follows:

Equation 3:  $N = B*2^n$       B is odd

Substituting for N in equation 1 we get

Equation 4:  $3*A + 1 = B*2^n$

Transposing terms we get

Equation 5:  $A = (B*2^n - 1)/3$     A, B odd,  $n \in 1, 2, 3, \dots$

So by selecting values for B and n we should be able to generate odd numbers for A which is shown in Table 1.

For a process supposed to generate odd integers this result looks a mess.

**Data Analysis**

By selectively removing elements of the table we can arrive at two reduced tables each containing only odd integers.

By inspection the following function can be arrived at:

**Function 1:**

$[A1_i] = ((3*p + 2)*2^n - 1)/3$      $p \in 1, 3, 5, \dots$      $n \in 1, 3, 5, \dots$

By inspection the following function can be arrived at:

**Function 2:**

$[A2_i] = ((3*p - 2)*2^n - 1)/3$      $p \in 1, 3, 5, \dots$      $n \in 2, 4, 6, \dots$

We now have two tables containing only integer values of odd numbers and the functions required to generate more odd numbers.

If  $[A1_i]$  or  $[A2_i] \equiv 1 \pmod{3}$  then  $[A1_i]$  or  $[A2_i]$  can be substituted for  $[B2_i]$

If  $[A1_i]$  or  $[A2_i] \equiv 2 \pmod{3}$  then  $[A1_i]$  or  $[A2_i]$  can be substituted for  $[B1_i]$

If  $[A1_i]$  or  $[A2_i] \equiv 0 \pmod{3}$  then  $[A1_i]$  or  $[A2_i]$  cannot be used in Function 1 or Function 2

The values of  $[A1_i]$ ,  $[A2_i]$  where  $[A1_i]$  or  $[A2_i] \equiv 0 \pmod{3}$  are not discarded. Essentially we have a third function

**Function 3:**

$[A3_i] = [A1_i]*2^n$      $[A1_i] \equiv 0 \pmod{3}$      $n \in 1, 2, 3, \dots$

$[A3_i] = [A2_i]*2^n$      $[A2_i] \equiv 0 \pmod{3}$      $n \in 1, 2, 3, \dots$

Bearing in mind the above statements it is possible by using Functions 1, 2 and 3 to construct the Collatz Conjecture tree starting from 1.

Function 1:  $[A1_i] = ((3*p + 2)*2^n - 1)/3$      $p \in 1, 3, 5, \dots$      $n \in 1, 3, 5, \dots$

Function 2:  $[A2_i] = ((3*p - 2)*2^n - 1)/3$      $p \in 1, 3, 5, \dots$      $n \in 2, 4, 6, \dots$

Function 3a:  $[A3_i] = [A1_i]*2^n$      $[A1_i] \equiv 0 \pmod{3}$      $n \in 1, 2, 3, \dots$

Function 3b:  $[A3_i] = [A2_i]*2^n$      $[A2_i] \equiv 0 \pmod{3}$      $n \in 1, 2, 3, \dots$

Generation of data to allow construction of the Collatz Conjecture tree is shown in below Tables 4 & 5

**Collatz Conjecture ‘tree’ starting from 1**

As shown in Figure 1 Every odd number is the root of a trunk of even numbers.

For trunks rooted on odd numbers such that  $[A1_i]$ ,  $[A2_i] \equiv 1 \pmod{3}$  or  $[A1_i]$ ,  $[A2_i] \equiv 2 \pmod{3}$  each even number shown in the diagram will give rise to a new odd number.

Adjunct Associate Professor, Education Peninsula, Australia

Correspondence: Barbara Clarke, Adjunct Associate Professor, Education Peninsula, Australia, E-mail: Barbara.Clarke@monash.edu

Received: April 11, 2021, Accepted: April 21, 2021, Published: May 02, 2021



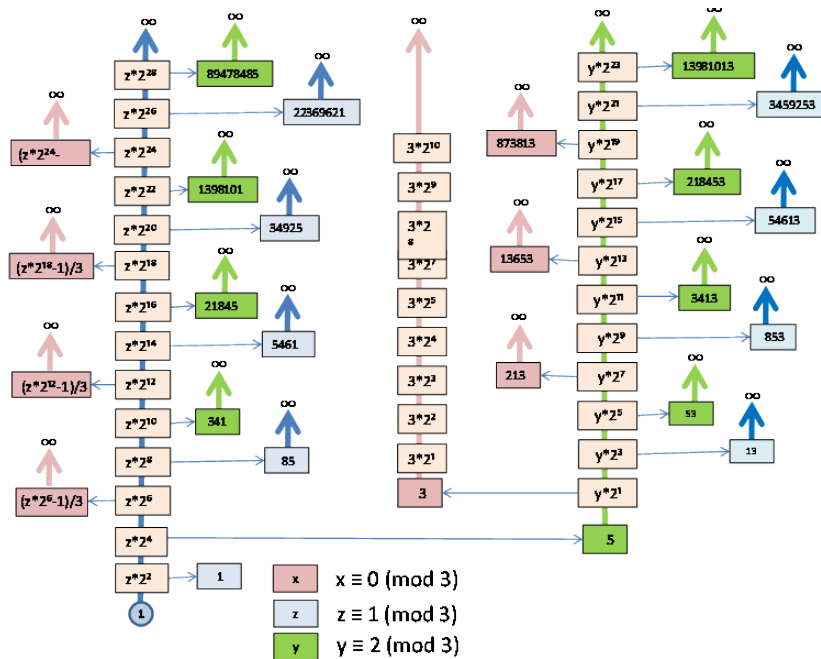
This open-access article is distributed under the terms of the Creative Commons Attribution Non-Commercial License (CC BY-NC) (<http://creativecommons.org/licenses/by-nc/4.0/>), which permits reuse, distribution and reproduction of the article, provided that the original work is properly cited and the reuse is restricted to noncommercial purposes. For commercial reuse, contact [reprints@pulsus.com](mailto:reprints@pulsus.com)

**Clarke B.**

**Table 1**

Calculated values of A.

n =	1	2	3	4	5	6
n^2 =	2	4	8	16	32	64
B	A	A	A	A	A	A
1	0.333	1	2.333	5	10.333	21
3	1.667	3.667	7.667	15.667	31.667	63.667
5	3	6.333	13	26.333	53	106.333
7	4.333	9	18.333	37	74.333	149
9	5.667	11.667	23.667	47.667	95.667	191.667
11	7	14.333	29	58.333	117	234.333
13	8.333	17	34.333	69	138.333	277
15	9.667	19.667	39.667	79.667	159.667	319.667
17	11	22.333	45	90.333	181	362.333



**Figure 1.** Collatz Conjecture 'tree' starting from 1.

For trunks rooted on odd numbers such that  $[A1_i], [A2_i] \equiv 0 \pmod{3}$  there are no odd numbers associated with such even numbers.

Every odd number is the root of an infinitely long trunk of even numbers.

While the diagram only shows the powers of 2 necessary to construct the diagram, each trunk contains all the powers of 2. This will become obvious if you start with a number such as 218453 and apply the original Collatz Conjecture process.

Note: There are infinitely more even numbers than odd numbers.

The above diagram is just the start of an infinitely repeating structure

**Can loops exist within the tree structure**

For a loop to exist then a value of A calculated along one path has to equal the value of A calculated along a different path, or the value of  $B \cdot 2^n$  calculated on one path has to equal the value of  $B \cdot 2^n$  calculated along a different path.

Function 1:  $[A1_i] = ((3 \cdot p + 2) \cdot 2^n - 1) / 3$   $p \in 1, 3, 5, \dots$   $n \in 1, 3, 5, \dots$

Function 2:  $[A2_i] = ((3 \cdot p - 2) \cdot 2^n - 1) / 3$   $p \in 1, 3, 5, \dots$   $n \in 2, 4, 6, \dots$

Reframing the two functions gives us

Function 1a:  $[A1_i] = ((3 \cdot p_1 + 2) \cdot 2^{(2n_1+1)} - 1) / 3$   $p_1 \in 1, 3, 5, \dots$   $n_1 \in 0, 1, 2, \dots$

Function 2a:  $[A2_i] = ((3 \cdot p_2 - 2) \cdot 2^{(2n_2)} - 1) / 3$   $p_2 \in 1, 3, 5, \dots$   $n_2 \in 1, 2, 3, \dots$

Addressing the question of loops within the data structure:

For odd numbers the answer needs to cover the following:

Case 1:  $[A1_i] = [A1_j]$

Case 2:  $[A2_i] = [A2_j]$

Case 3:  $[A1_i] = [A2_j]$

For even numbers where

$[B1_i] = (3 \cdot p_i + 2) \cdot 2^{(2n_i + 1)}$   $p_i \in 1, 3, 5, \dots$   $n_i \in 0, 1, 2, \dots$

$[B2_i] = ((3 \cdot p_i - 2) \cdot 2^{(2n_i)})$   $p_i \in 1, 3, 5, \dots$   $n_i \in 1, 2, 3, \dots$

$[B3_i] = [A1_i] \cdot 2^{n_2}$   $p \in 1, 3, 5, \dots$   $n_1 \in 1, 3, 5, \dots$   $n_2 \in 1, 2, 3, \dots$

$[B3_j] = [A2_j] \cdot 2^{n_2}$   $p \in 1, 3, 5, \dots$   $n_1 \in 2, 4, 6, \dots$   $n_2 \in 1, 2, 3, \dots$

We need to cover the following possibilities

Case 4:  $[B1_i] = [B2_i]$

Case 5:  $[B1_i] = [B1_j]$

Case 6:  $[B1_i] = [B3_i]$

Case 7:  $[B1_i] = [B3_j]$

Case 8:  $[B1_i] = [B3_i]$

Case 9:  $[B2_i] = [B2_j]$

Case 10:  $[B2_i] = [B3_j]$

Case 11:  $[B2_i] = [B3_j]$

Case 12:  $[B3_i] = [B3_j]$

**Case 1: Can loops exist between values of A1: i.e. can  $[A1_i] = [A1_j]$**

Function 1:  $[A1_i] = ((3^p + 2) * 2^{n1} - 1) / 3$   $p \in 1, 3, 5, \dots$   $n \in 1, 3, 5, \dots$

When you construct a table of values for  $[A1]$  with  $B1$  on one axis and  $2^{n1}$  on the other axis it becomes obvious that you cannot have two values of

$A1$  where  $B1_i = B1_j$  and simultaneously  $2^{n1} = 2^{n2}$

Let

$$((3^{p_i} + 2) * 2^{n1} - 1) / 3 = ((3^{p_j} + 2) * 2^{n2} - 1) / 3 \quad p_i, p_j \in 1, 3, 5, \dots \quad n1, n2 \in 1, 3, 5, \dots$$

This becomes

$$(3^{p_i} + 2) * 2^{n1} = (3^{p_j} + 2) * 2^{n2} \quad p_i, p_j \in 1, 3, 5, \dots \quad n1, n2 \in 1, 3, 5, \dots$$

$$(3^{p_i} + 2) / (3^{p_j} + 2) = 2^{n2} / 2^{n1} \quad p_i, p_j \in 1, 3, 5, \dots \quad n1, n2 \in 1, 3, 5, \dots$$

$$(3^{p_i} + 2) / (3^{p_j} + 2) = 2^{(n2 - n1)} \quad p_i, p_j \in 1, 3, 5, \dots \quad n1, n2 \in 1, 3, 5, \dots$$

If  $p_i = p_j$  then  $LHS = 1$ , because  $p_i = p_j$ ,  $n2 - n1$  cannot equal zero, therefore  $RHS$  is either even or a real number

I.e.  $LHS$  does not equal  $RHS$ .

If  $n2 = n1$  then  $(3^{p_i} + 2)$  cannot be equal to  $(3^{p_j} + 2)$

So while the  $RHS$  equals 1 the  $LHS$  is either odd (and not equal to 1) or a real number.

I.e.  $LHS$  does not equal  $RHS$ .

Therefore all values of  $A1$  are unique. Therefore no loops within  $A1$ .

**Case 2: Can loops exist between values of A2**

Function 2:  $[A2_i] = ((3^p - 2) * 2^{n1} - 1) / 3$   $p \in 1, 3, 5, \dots$   $n \in 2, 4, 6, \dots$

$[A2_i] = ((3^p - 2) * 2^{n1} - 1) / 3$   $p \in 1, 3, 5, \dots$   $n \in 2, 4, 6, \dots$

Let

$$((3^{p_i} - 2) * 2^{n1} - 1) / 3 = ((3^{p_j} - 2) * 2^{n2} - 1) / 3 \quad p_i, p_j \in 1, 3, 5, \dots \quad n1, n2 \in 2, 4, 6, \dots$$

$$(3^{p_i} - 2) * 2^{n1} = (3^{p_j} - 2) * 2^{n2} \quad p_i, p_j \in 1, 3, 5, \dots \quad n1, n2 \in 2, 4, 6, \dots$$

$$(3^{p_i} - 2) / (3^{p_j} - 2) = 2^{n2} / 2^{n1} \quad p_i, p_j \in 1, 3, 5, \dots \quad n1, n2 \in 2, 4, 6, \dots$$

$$(3^{p_i} - 2) / (3^{p_j} - 2) = 2^{(n2 - n1)} \quad p_i, p_j \in 1, 3, 5, \dots \quad n1, n2 \in 2, 4, 6, \dots$$

If  $(3^{p_i} - 2) = (3^{p_j} - 2)$  then  $LHS$  of equation = 1,  $RHS$  equals an even number or a real number.

If  $n2 = n1$  then  $2^{(n2 - n1)}$  equals 1 and  $(3^{p_i} - 2) / (3^{p_j} - 2)$  equals an odd number (not equal to 1) or a real number.

So all values of  $A2$  are unique. Therefore no loops within  $A2$ .

Case 3: Can loops exist between values of  $A1$  and  $A2$ :

Function 1:  $[A1_i] = ((3^p + 2) * 2^{n1} - 1) / 3$   $p \in 1, 3, 5, \dots$   $n \in 1, 3, 5, \dots$

Function 2:  $[A2_i] = ((3^p - 2) * 2^{n1} - 1) / 3$   $p \in 1, 3, 5, \dots$   $n \in 2, 4, 6, \dots$

Let

$$((3^{p_i} + 2) * 2^{n1} - 1) / 3 = ((3^{p_j} - 2) * 2^{n2} - 1) / 3 \quad p_i, p_j \in 1, 3, 5, \dots \quad n1 \in 1, 3, 5, \dots \quad n2 \in 2, 4, 6, \dots$$

$$(3^{p_i} + 2) * 2^{n1} = (3^{p_j} - 2) * 2^{n2} \quad p_i, p_j \in 1, 3, 5, \dots \quad n1 \in 1, 3, 5, \dots \quad n2 \in 2, 4, 6, \dots$$

$$(3^{p_i} + 2) / (3^{p_j} - 2) = 2^{n2} / 2^{n1} \quad p_i, p_j \in 1, 3, 5, \dots \quad n1 \in 1, 3, 5, \dots \quad n2 \in 2, 4, 6, \dots$$

$$(3^{p_i} + 2) / (3^{p_j} - 2) = 2^{(n2 - n1)} \quad p_i, p_j \in 1, 3, 5, \dots \quad n1 \in 1, 3, 5, \dots \quad n2 \in 2, 4, 6, \dots$$

Can  $3^{p_i} + 2 = 3^{p_j} - 2$

$$3^{p_i} + 2 = 3^{p_j} - 2$$

$$4 = 3(p_j - p_i)$$

$$(p_j - p_i) = 4/3$$

Now  $p_j$  and  $p_i$  are integers and the difference has to be an integer therefore  $(3^{p_i} + 2)$  cannot equal  $(3^{p_j} - 2)$

$(n2 - n1)$  can never be zero ( $n2$  is even,  $n1$  is odd),  $2^{(n2 - n1)}$  is either even or

$1/(\text{even number})$ .

$(3^{p_i} + 2) / (3^{p_j} - 2)$  is either an odd number or a real number. The equation can be arranged so the the  $RHS$  is an even integer while the  $LHS$  is odd or a real number.

Therefore there are no cases where a value of  $A1$  equals a value of  $A2$ .

A corollary of the above is that all even numbers used to generate values of  $A1$  and  $A2$ , are unique.

Given that all the powers of 2 are present in each trunk and the root of each trunk is a unique odd number then the even numbers in any trunk can be represented by the following:

$$N1_i = B1_i * 2^{n1}, \quad B1_i \equiv 1 \pmod{3}, \quad n1 \in 1, 2, 3, \dots$$

$$N2_i = B2_i * 2^{n1}, \quad B2_i \equiv 2 \pmod{3}, \quad n1 \in 1, 2, 3, \dots$$

$$N3_i = B3_i * 2^{n1}, \quad B3_i \equiv 0 \pmod{3}, \quad n1 \in 1, 2, 3, \dots$$

All the questions of loops between even numbers can be reduced to the general case where two even numbers are created by multiplying two different odd numbers by a power of 2. The two odd numbers are different because all odd numbers created by Functions 1 and 2 are unique.

General case:

Can  $A * 2^{n1} = B * n^{n2}$  where  $A \neq B$  and  $A, B \in 1, 3, 5, \dots$   $n1, n2 \in 1, 2, 3, \dots$

Let  $A * 2^{n1} = B * n^{n2}$

Then

$$A/B = 2^{(n2 - n1)}$$

$A/B$  is either an odd integer (not equal to 1) or a real number,  $2^{(n2 - n1)}$  is either 1 or an even number or the reciprocal of an even number.

If  $n1 > n2$  then the equation can be restated as  $B/A = 2^{(n1 - n2)}$  in which case  $B/A$  equals an odd integer or a real number and  $2^{(n1 - n2)}$  is an even number.

Therefore  $A * 2^{n1} \neq B * 2^{n2}$

From the above none of the even numbers in the Collatz Conjecture tree are duplicated.

Therefore there are no loops between even numbers.

Therefore there are no loops in the Collatz Conjecture tree.

The Gaps issue: Do the two functions below calculate all the odd numbers ?:

Function 1:  $[A1_i] = ((3^p + 2) * 2^{n1} - 1) / 3$   $p \in 1, 3, 5, \dots$   $n \in 1, 3, 5, \dots$

Function 2:  $[A2_i] = ((3^p - 2) * 2^{n1} - 1) / 3$   $p \in 1, 3, 5, \dots$   $n \in 2, 4, 6, \dots$

Consider

Function 1:  $[A1_i] = ((3^p + 2) * 2^{n1} - 1) / 3$   $p \in 1, 3, 5, \dots$   $n \in 1, 3, 5, \dots$

$$[A1_i] = ((3 * (2n1 + 1) + 2) * 2^{(2n2 + 1)} - 1) / 3 \quad n1 \in 0, 1, 2, \dots \quad n2 \in 0, 1, 2, \dots$$

$$[A1_i] = (3 * (2n1 + 1) * 2^{(2n2 + 1)} + 2 * 2^{(2n2 + 1)} - 1) / 3 \quad n1 \in 0, 1, 2, \dots \quad n2 \in 0, 1, 2, \dots$$

$$[A1_i] = (3 * (2n1)) * 2^{(2n2 + 1)} + 3 * 2^{(2n2 + 1)} + 2 * 2^{(2n2 + 1)} - 1) / 3 \quad n1 \in 0, 1, 2, \dots \quad n2 \in 0, 1, 2, \dots$$

$$[A1_i] = (2n1) * 2^{(2n2 + 1)} + (5 * 2^{(2n2 + 1)} - 1) / 3 \quad n1 \in 0, 1, 2, \dots \quad n2 \in 0, 1, 2, \dots$$

$$[A1_i] = n1 * 2^{(2n2 + 2)} + (5 * 2^{(2n2 + 1)} - 1) / 3 \quad n1 \in 0, 1, 2, \dots \quad n2 \in 0, 1, 2, \dots$$

For any given value of  $n2$  the above equation can be restated as

Function 1b:  $[A1_i] = k1 * n1 + k2$   $n1 \in 0, 1, 2, \dots$

Where  $k1 = 2^{(2n2 + 2)}$   $k2 = (5 * 2^{(2n2 + 1)} - 1) / 3$   $n2 \in 0, 1, 2, \dots$

Consider

Function 2:  $[A2_i] = ((3^p - 2) * 2^{n1} - 1) / 3$   $p \in 1, 3, 5, \dots$   $n \in 2, 4, 6, \dots$

$$[A2_i] = ((3 * p - 2) * 2^{n1} - 1) / 3 \quad p \in 1, 3, 5, \dots \quad n \in 2, 4, 6, \dots$$

$$[A2_i] = ((3 * (2n1 + 1) - 2) * 2^{(2n2)} - 1) / 3 \quad n1 \in 0, 1, 3, \dots \quad n2 \in 1, 2, 3, \dots$$

$$[A2_i] = ((3 * (2n1) * 2^{(2n2)} + (3 - 2) * 2^{(2n2)} - 1) / 3 \quad n1 \in 0, 1, 3, \dots \quad n2 \in 1, 2, 3, \dots$$

**Clarke B.**

$$[A2_i] = (2n_1) * 2^{(2n_2)} + (2^{(2n_2)} - 1) / 3 \quad n_1 \in 0, 1, 3, \dots \quad n_2 \in 1, 2, 3, \dots$$

$$[A2_i] = 2^{(2n_2+1)} * n_1 + (2^{(2n_2)} - 1) / 3 \quad n_1 \in 0, 1, 3, \dots \quad n_2 \in 1, 2, 3, \dots$$

Function 2b:  $[A2_i] = k_3 * n_1 + k_4 n_1 \in 0, 1, 3, \dots$

Where  $k_3 = 2^{(2n_2+1)}$   $k_4 = (2^{(2n_2)} - 1) / 3 \quad n_2 \in 1, 2, 3, \dots$

If you select an odd number N, then the number of odd numbers from 1 to N is given by  $(N + 1) / 2$ .

In order to use functions 1b and 2b to calculate the number of odd numbers in tables 2 and 3 it is necessary to redefine the work space.

Function 1a:  $[A1_i] = k_1 * n_1 + k_2 \quad n_1 \in 0, 1, 2, \dots$

Where  $k_1 = 2^{(2n_2+2)}$   $k_2 = (5 * 2^{(2n_2+1)} - 1) / 3 \quad n_2 \in 0, 1, 2, \dots$

Function 2a:  $[A2_i] = k_3 * n_1 + k_4 \quad n_1 \in 0, 1, 3, \dots$

Where  $k_3 = 2^{(2n_2+1)}$   $k_4 = (2^{(2n_2)} - 1) / 3 \quad n_2 \in 1, 2, 3, \dots$

For Table 2

For any given value of  $n_2$  the number of odd numbers present between 1 and N is given by

Let  $m_1' = (N - k_2) / k_1$

$m_1 = \text{Int}(m_1')$

The number of odd numbers for a given value of  $n_2$  is

$N(n_2) = m_1 + 1$

For Table 3

**Table 2**

Values of  $[A1_i]$ .

	n =	1	3	5
	n <sup>2</sup> =	2	8	32
p	$[B1_i] = 3p + 2$	$[A1_i]$	$[A1_i]$	$[A1_i]$
1	5	3	13	53
3	11	7	29	117
5	17	11	45	181

**Table 3**

Values of  $[A2_i]$ .

n =	n =	2	4	6
n <sup>2</sup> =	n <sup>2</sup> =	4	16	64
p	$[B2_i] = 3p - 2$	$[A2_i]$	$[A2_i]$	$[A2_i]$
1	1	1	5	21
3	7	9	37	149
5	13	17	69	277

Generation of data to allow construction of the Collatz Conjecture tree:

From Function 2:  $[A2_i] = ((3 * p - 2) * 2^n - 1) / 3 \quad p \in 1, 3, 5, \dots \quad n \in 2, 4, 6, \dots$

For  $p = 1$ ,

n	2 <sup>n</sup>	$[B2_i] = 3p - 2$	$N = [B2_i] * 2^n$	$[A2_i] = (N - 1) / 3$	$[A2_i] \equiv 0 \pmod{3}$	$[A2_i] \equiv 1 \pmod{3}$	$[A2_i] \equiv 2 \pmod{3}$
2	4	1	4	1	0.333	0	-0.333
4	16	1	16	5	1.667	1.333	1
6	64	1	64	21	7	6.667	6.333
8	256	1	256	85	28.333	28	28.667
10	1024	1	1024	341	113.667	113.333	113
12	4096	1	4096	1365	455	454.667	454.333
14	16384	1	16384	5461	1820.333	1820	1820.667

$m_2' = (N - k_4) / k_3$

$m_2 = \text{Int}(m_2')$

$N(n_2) = m_2 + 1$

The maximum value of  $n_2$  is exceeded when  $N(n_2)$  equals zero.

Summing all the values for  $N(n_2)$  returns a result equal to the number of odd numbers from 1 to N.

N can take any positive odd integer value.

This implies that there are no gaps in the odd numbers from 1 to N.

As I cannot prove this algebraically it will have to stand as Alf's conjecture.

**Stability**

The question of stability is usually raised in connection to applying the Collatz Conjecture process to a preselected number and seeing if it reduces to 1.

If you choose a number and then start following the path down the structure there is only one way to go. There are no loops or cross connections.

Accordingly there can be no instability in the process. Once you pass from one trunk to the next you never return to the previous trunk.

**The P vs NP question**

Using the conventional approach to creating a Collatz Conjecture sequence can result in the sequence reaching a value several hundred times the size of the starting number. The number of steps between starting the calculation and reaching this high point is unpredictable. The number of steps from the high point to the final outcome (reaching 1) is also unpredictable.

When starting from 1 you immediately have an infinite number of even numbers to choose from to create the next odd number. Once you select an even number and create the next odd number you again have an infinite number of even numbers to select from in order to create the next odd number from. And so it goes for every odd number you create. In trying to get to 41 you are faced with making this choice 39 times. So 39 times in a row you have to choose the right number out of an infinite number of possible numbers. The chance of correctly choosing the right sequence of numbers to reach 41 is  $(1/\infty)^{39}$ , ie your chance of correctly choosing the right path is infinitesimal.

However if you have the solution having worked it out the normal way then it is child's play to map the solution to the Collatz Conjecture structure defined by Functions 1, 2 and 3.

Thus checking a solution is very easy, creating a solution starting from 1 is impossible in most cases.

The Collatz Conjecture structure starting from 1 is an ever dividing tree without any cross connections. Every time you select an even number to reach the next odd number you have an infinite number of wrong choices available but only one right choice. Once you step off the correct path you can never regain it.

**The halting problem.**

When starting from 1 to find a selected number it is unknown as to what the maximum value of the sequence will be. It is also unknown as to how far beyond the maximum value you have to go to find the selected number. Thus there is no easily defined halting criteria for a search along any given path.

**Clarke B.**

From Function 1:  $[A1_i] = ((3^p + 2) * 2^n - 1) / 3$   $p \in 1,3,5,\dots$   $n \in 1,3,5,\dots$

For  $p = 1$ ,

n	$2^n$	$[B2_i] = 3p + 2$	$N = [B2_i] * 2^n$	$[A2_i] = (N-1)/3$	$[A2_i] \equiv 0 \pmod{3}$	$[A2_i] \equiv 1 \pmod{3}$	$[A2_i] \equiv 2 \pmod{3}$
1	2	5	10	3	1	0.667	0.333
3	8	5	40	13	4.333	4	3.667
5	32	5	160	53	17.667	17.333	17
7	128	5	640	213	71	70.667	70.333
9	512	5	2560	853	284.333	284	283.667
11	2048	5	10240	3413	1137.667	1137.333	1137
13	8192	5	40960	13653	4551	4550.667	4550.333

If you choose to search along every path for a fixed distance before moving to the next path the issue becomes the rapid multiplication of possible paths. In many cases it will be required to do several extensions of the distance covered by the search resulting in the number of possible paths expanding extremely rapidly.

It would not be difficult to select a number for which halting is unlikely to occur.

**CONCLUSION**

The functions described in this paper allow the calculation of the Collatz

Conjecture data set starting from 1. Issues of stability and whether or not any given sequence decays to one become irrelevant.

By changing the solution space from a rectangular table to the space between half an inverted parabola and the two axis, the problem of gaps in the data set disappear. As the value defining the shape of the parabola can take any value short of infinity then this approach truly defines all the possible Collatz Conjecture sequences.

While I have not read the paper produced by Sir Bryan Thwaites I believe this approach addresses the gaps issue raised in that paper.