OPINION

The K-equivalent, function equivalence in sense of Kamtchueng

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ABSTRACT

An open question relative to the definition domain of a polynomial function given by the following expression $f(x)=\frac{x^2+2x-3}{x+3}$ push us to define new relation between functions. By simple element decomposition, one could find $g(x) = x-1$ but even if intuitively the functions are equivalent, the first one is not defined for $x=-3$ in order to avoid the denominator to be null. In this short paper, we want to find the intuition back by defining a new relationship between two functions.

Key words: *Polynomial; Equivalence*

INTRODUCTION

This article is based on a reflexion from an exercice, I asked to my students if the two following function were equivalent.

$$f(x)=\frac{x^2+2x-3}{x+3}$$

$$g(x) = x-1$$

We consider two new types of functions: $g_a(x)$ equal to $x-1$ if $x \neq -3$ and a otherwise $g_a^n(x)$ equal to $x-1$ if $x \neq -3 \mid$ and $a+\frac{1}{n}$ otherwise

The point of this article is to introduce the Kamtchueng Equivalence of the two functions.

K-Equivalence

Theorem: f is K-equivalence to g under $D_f \cup \{x_0\}$ if and only if for all x_0 where f is not defined there is a compact centered in x_0 such as for all $x \neq x_0$ within the compact, limit of $f(x)$ when x tends $x^{\cdot}$ equal to $g(\dot{x})$.

$$\forall x_0 \in D_g \cap x_0 \notin D_f$$

$$\exists B_{x0}$$

$$\exists \dot{x} \in B_{x0} \setminus \{x_0\}$$

By definition of the K-Equivalence, one domain should be included strictly to the other, which is not the case for Dg.

$$\lim_{x \to x} f(x) = g(\dot{x})$$

With B_{x0} a compact centered in x_0 no empty and different of the singleton.

This definition can be extended to function domain with a countable number of not included points.

Theorem: Set $X = \cup_{1 \leq n}(x_n\}$, f is K-equivalent to g under $D_f^X = D_f \cup X$ with $D_f^X \subset D_g$ if and only if

$$\forall x_i \in X$$

$$\exists B_{xi}$$

$$\forall \dot{x} \in B_{xi} \setminus \{x_i\}$$

$$\lim_{x \to \dot{x}} f(x) = g(\dot{x})$$

The K-Equivalence is not commutative.

f K-Equivalent to g does not imply that g K-Equivalent to f under D

$$f \approx_D^{\mathbb{K}} g \quad \not\Rightarrow \quad g \approx_D^{\mathbb{K}} f$$

Demonstration: by considering the two functions f and g defined in Introduction, we have f Kequivalent to g but g is not K-Equivalent to f. Indeed the definition domain of g not included in the domain of f.

The K-Equivalence is transitive.

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f K-Equivalent to g under D_f^X and g K-Equivalent to h under D_g^Y imply that f K-Equivalent to hunder D_f^X

$$f \approx_{D_f^X}^{\mathbb{K}} g, g \approx_{D_g^Y}^{\mathbb{K}} h \quad \Rightarrow \quad f \approx_{D_f^X}^{\mathbb{K}} h$$

Demonstration:

Set $X = \bigcup_{1 \le n} \{x_n\}, f$ is K-equivalent to g under $D = D_f \cup X$ with $D \subset D_g$ if and only if

$$f \approx_D^{\mathbb{K}} g \Rightarrow$$

$\forall x_i \in X$

$\exists Bx_i$

$\forall \dot{x} \in B_{xi} \setminus \{x_i\}$

$\lim_{x \to \dot{x}} f(x) = g(\dot{x}) = h(\dot{x})$

Indeed the $g(\dot{x}) = \mathrm{h}(\dot{x})$ because of the $g \approx_D^K h$, therefore $f \approx_D^K h$

CONCLUSION

Lets focus on the function defined in Introduction f and g but also:

g_{-4}

$$g_{-4} = \begin{cases} x-1 & if \quad x \neq -3 \\ -4 & otherwise \end{cases}$$

$$g_{-4}^n = \begin{cases} x-1 & if \quad x \neq -3 \\ -4 + \frac{1}{n} & otherwise \end{cases}$$

$$g_3 = \begin{cases} x-1 & if \quad x \neq -3 \\ 3 & otherwise \end{cases}$$

In one hand, $g = g_{-4}$ but $f \neq g$ because of the apparent definition domain of f ; $D_f = \mathbb{R} \setminus \{-3\}$

In another hand $f \approx_{\mathbb{R}}^K g$ therefore $f \approx_{\mathbb{R}}^K g_{-4}$ It is interesting to note that $\lim_{n \to \infty} g_{-4}^n = g_{-4}$ but $\forall n, f \neq_{\mathbb{R}}^K g_{-4}^n g_{-4}^n$.

In addition, f, g, g_{-4}, g_{-4}^n and g_3 are equals in D_f

Firstly, what happen to the K-Equivalence when the non-defined set is not countable? Secondly is this definition really necessary? Are these functions really differents? what about all the $k^n(x) = \frac{f(x)}{\prod_{i=1}^n (x - x_i)} \prod_{i=1}^n (x - x_i)$?

Lets imagine two students look at a car, one say that the car is blue but the other one say that the car is green. The first one state that he believed it was green before but the builder of car state that the car is blue. If the builder say it but it is not so obvious, the second decide to create a color green blued which is very similar to green. At the end, maybe the car builder has done a mistake in fact one of the painting machine was leaking micro test of blue. Therefore even if it is neglieable the autochecker was telling the color blue instead of green… If it is the case, we create a new color especially for it! Would it have been better to just consider it as a mistake?