RESEARCH ARTICLE

The parameterization of all stabilizing minimum-phase controllers for minimum-phase strictly proper plants

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ABSTRACT

The problem of parameterization is to find all stabilizing controllers for the controlled system. At present, there are many methods to choose from for the controlled plants with the minimum-phase. However, most of the existing procedure do not introduce minimum-phase controller into the system. In this sense,Zhang et al. have a new view that a parameterization method for all minimum-phase controllers for minimum-phase biproper systems is given. This paper expands the research results of Zhang et al. for the minimum-phase strictly proper controlled systems and the parameterization of all minimum-phase stabilizing controllers is given. The internal stability and control performance of the closed-loop system are studied. At the same time, an algorithm which can be used to construct minimum-phase stability control is presented. Finally, an example is given to explain the characteristics of the algorithm proposed in this paper. Because this method can find all the minimum-phase stabilizing controllers well, it can be applied to many control problems.

Keywords: *Minimum-Phase System; Minimum-Phase Controllers; Strictly Proper Plants; Closed-Loop Characteristics .*

INTRODUCTION

For the minimum-phase strictly proper controlled system, the parameter expressions of minimum-phase stabilizing controllers are given in this paper. That is, this paper proposes the parameterization of the minimum-phase stabilizing controllers for the minimum-phase system. The problem of parameterization is to find all stablizing controllers for a plant, so as to make the system stable, and to obtain plants those can be stabilized [1–11]. This method can effectively solve all the parameterization of the minimum-phase stabilizing controllers for minimum-phase system, so it has been applied in many practical problems.

Compared with the non-minimum phase system, the minimum-phase system has fast response, small energy delay, stable inverse system and other advantages [12-13]. Comparing with the nonminimum-phase system constructed by configuring the right half-plane zero point or adding the time-delay, it is obtained that the minimum-phase system has the shortest response time [13].

At any time, the cumulative output energy of the minimum phase system is not less than that of the non-minimum phase system. It can be proved that the cumulative output energy of the minimum phase system is closer to time 0 and has the shortest energy delay [12]. And the inverse system of the minimum-phase system is stable, because the stable poles of the inverse system of the minimum phase system is the zeros of the original system which has no unstable zero. Benefiting from these advantages, the minimum-phase system is widely used in signal processing and other related fields, such as state system, design of causal stable digital filter, neural network and calculation and processing of cepstrum and inverse filtering [12-16].

Glaria and Goodwin provided a simple parameterization for the stability control of the minimum-phase [4]. However, there are two difficult problems. One of them is that the parameterization of stabilizing controllers proposed by Glaria and Goodwin usually contain improper controllers. In the specific process of use, a proper controller is needed. Second, the internally stability in the system is not parameterized. In order to solve the above

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problems, Yamada presents a parameterization for the class of all proper stabilizing controllers for linear minimum phase systems [5]. Parametric processing is carried out by using all the stabilizing controllers, such as all stabilization modification of the minimum-phase object, repetitive control, adaptive control, model feedback control, parallel compensation technology, PI control and PID control [5, 17-23]. For multiple-input/ multiple-output systems, a parameterized scheme of all stability control is given, and the research results of these schemes can be extended to multiple-input/multiple-output [17–24].

However, for the minimum-phase plants, there is still a problem whether its stability control can be used by the minimum-phase controller. If the stabilizing controller with non-minimum-phase is used, its unstable zeros will cause the transfer function of the closed-loop system to have zeros on its right half plane. This makes the closed-loop control system very sensitive to the disturbance of the external environment, thus affecting the control effect. In addition, if the feedback loop is truncated, that is, it is split into a feedforward, then the instability caused by it will lead to instability [7,8]. In this way, even though the controlled plant is of minimum-phase, the control system becomes an non-minimum system. If the minimum-phase control is adopted, the target of the minimum-phase will remain unchanged, and the magnitude of sensitivity of the whole system will become small. The lower the sensitivity curve, the greater the damping to external interference. If the minimum-phase controllers of the minimum-phase plants can be parameterized, a new control strategy for the minimum-phase system can be obtained. Therefore, for the strictly proper controlled plants with minimum-phase, the minimum-phase controllers must be parameterized.

In this paper, we propose the parameterization of all stabilizing minimum-phase controllers for minimum-phase strictly proper plants. That is, we consider the parameterization that the stabilizing controller makes minimum-phase plant stable, which the stabilizing controller is of minimum-phase. Analysis of the internal stability and control characteristics of closed-loop system are provided. We also present a design method of the minimum-phase stabilizing controllers that contributes to the construction of a minimum-phase closed-loop system. In addition, we show a numerical example to illustrate the characteristics of the proposed design approach.

PROBLEM DESCRIPTION

In this section, we introduce the problem considered in this paper. We consider a closed loop feedback control system as,

$$\begin{cases} y(s) = G(s)u(s)+d(s) \\ u(s) = C(s)\,(r(s)-y(s)) \end{cases}. \tag{1}$$

Here, $y(s) \in R(s), u(s) \in R(s), r(s) \in R(s)$ *and* $R(s)$ are the output, control input, reference input and disburtance of the control system respectively. $G(s) \in R(s)$ *and* $C(s) \in R(s)$ are the controller and the plant of the control system separately, and both of them are of minimum-phase, that means, all zeros of them are in the left half plane. In this paper, the controlled plant G(s) with minimum-phase is required to be strictly proper and it is possible to be stable or unstable. By using the parameterization of the minimum-phase stabilizing controller for the minimum-phase plant, the controller C(s) is required to be derived. In the specific process of use, a proper controller is needed and the internally stability and the robustness of the control system need to be considered. Here R(s) indicates the set of real rational functions for the set with s.

Before seeking the parametrization for the strictly proper plants, the preliminary result proposed by Zhang et al. is summarized [25]. For the minimum-phase biproper plants, the parameterization of all minimum-phase stabilizing controllers are given as follows.

Lemma 1

G(s) is assumed to be of minimum-phase and to be biproper [25]. Then the minimum-phase controller C(s) stabilizes the feedback control system in (1) if and only if C(s) is written by the form of

$$C(s) = \frac{Q(s)}{(1-Q(s))\,G(s)}, \tag{2}$$

where $Q(s) \in RH_\infty$ is any minimum-phase function to make $(1-Q(s))\,G(s) \in RH_\infty$. RH_∞ denotes the set of stable rational function with s [25].

The problem considered in this paper is to clarify the parameterization of all stabilizing minimum-phase controllers C(s) for minimum-phase strictly proper plants G(s).

THE PARAMETERIZATION OF ALL STABILIZING MINIMUM-PHASE CONTROLLERS FOR MINIMUM-PHASE STRICTLY PROPER PLANTS

In this section, we clarify the parameterization of all stabilizing minimum-phase controllers C(s) for minimum-phase strictly proper plants G(s).

This parameterization is summarized in the following theorem.

Theorem 1

Assume that G(s) is strictly proper and of minimum-phase. When K(s) exists in a system of equations, K(s) is a stable and biproper real rational function and make G(s) + K(s) be a biproper and minimum-phase real rational function. Utilizing the above K(s), for all proper minimum-phase stabilizing controllers C(s) of the plant G(s) with strictly proper and minimum-phase, the parameters are as follows

$$C(s) = \frac{\bar{C}(s)}{1+C(s)\,K(s)}. \tag{3}$$

Here, $\bar{C}(s)$ is expressed as

$$\bar{C}(s) = \frac{\bar{C}(s)}{1+Q(s)\,(G(s)+K(s))} \tag{4}$$

and $\bar{Q}(s)$ is the minimum-phase function belong to RH_∞ and to make $(1-\bar{Q}(s))\,G(s)+K(s)) \in RH_\infty$.

For proving the Theorem 1, the following theorems are needed.

Theorem 2

Assume that G(s) is strictly proper and minimum-phase. When K(s) exists in a system of equations, K(s) is a stable and biproper real rational function and make G(s) + K(s) be a biproper and minimum-phase real rational function.

Proof

At first, G(s) is factorized into the coprime factors with $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ on RH_∞ and G(s) is rewritten in the form of

$$G(s) = \frac{N(s)}{D(s)}. \quad (5)$$

Here, because G(s) is assumed to be of minimum-phase and strictly proper, $N(s) \in RH_\infty$ is of minimum-phase and strictly proper. In addition, G(s) + K(s) is donated as

$$G(s) + K(s) = \frac{N(s) + K(s)\,D(s)}{D(s)}, \quad (6)$$

Then, the exist condition of K(s) is that G(s) + K(s) is a biproper and minimum-phase, and consistent with that of $U(s) \in u$ and $K(s) \in RH_\infty$ satisfying

$$U(s) = N(s) + K(s)\,D(s) \in u. \quad (7)$$

Here, u is the set of unimodular functions on RH_∞, so $U(s) \in u$ implies $U(s) \in RH_\infty$ and $U^{-1}(s) \in RH_\infty$. The existence conditions of U(s) and K(s) is equivalent to the interpolation problem and are written as

$$\frac{d^j}{ds^j} U(s_i) = \frac{d^j}{ds^j} N(s_i) \quad (j = 0, \ldots m_i - 1; i = 1, \ldots l), \quad (8)$$

where $s_1, \ldots, s_l$ are different zeros of D(s) on the positive real axis, $m_1, \ldots, m_l$ are the corresponding multiplicities and l denotes the number of different zeros of D(s) on the positive real axis. Since G(s) is of minimum-phase, N(s) is also of minimum-phase. This implies that all of $N(s_i)$ are the same sign. From Theorem 2.3.1, there exists $U(s) \in u$ and $K(s) \in RH_\infty$ satisfying (8) [7]. This implies that there exists $U(s) \in u$ and $K(s) \in RH_\infty$ satisfying (7).

The remaining problem is whether or not, K(s) is biproper. Next, it is shown that if $U(s) \in u$ exists such that (7) holds true, then K(s) is biproper. From (7), K(s) is written by

$$K(s) = \frac{U(s) - N(s)}{D(s)}. \quad (9)$$

The assumption that U(s) holds (7) implies that K(s) written by (9) is stable. Since both U(s) and D(s) are biproper and N(s) is strictly proper, K(s) denoted by (9) is biproper.

We have thus proved the theorem.

Theorem 3

Assume that $\bar{G}(s) = G(s) + K(s)$ is the real rational function with strictly proper and minimum-phase. All minimum-phase stabilizing controllers $\bar{C}(s)$, its parameterization for the plant $\bar{G}(s)$ is denoted as

$$\bar{C}(s) = \frac{\bar{Q}(s)}{(1 - Q(s))\,G(s)}. \quad (10)$$

Here, $\bar{Q}(s) \in RH_\infty$ is any minimum-phase function and to make $(1 - \bar{Q}(s))\bar{G}(s) \in RH_\infty$.

Proof

Because $\bar{G}(s) = G(s) + K(s)$ is assumed to be the real rational function with strictly proper and minimum-phase, according to Lemma 1, the minimum-phase stabilizing controllers $\bar{C}(s)$, its parameterization for $\bar{G}(s)$ is denoted as (10).

We have thus proved the theorem.

Theorem 4

Assume that $K(s) \in RH_\infty$ is biproper and G(s) is strictly proper. If the minimum-phase controller C(s) stabilizes the plant G(s), then $\bar{C}(s)$ is written as

$$\bar{C}(s) = \frac{C(s)}{1 - C(s)K(s)} \quad (11)$$

stabilizes the plant $\bar{G}(s) = G(s) + K(s)$. Furthermore, the opposite is also true. That is, if the minimum-phase controller $\bar{C}(s)$ stabilizes the plant $\bar{G}(s) = G(s) + K(S)$, then the the minimum-phase controller C(s) is written as

$$C(s) = \frac{\bar{C}(s)}{1 + C(s)K(s)} \quad (12)$$

stabilizes the plant G(s).

Proof

First, we will prove that if the minimum-phase controller C(s) stabilizes G(s), then the minimum-phase controller $\bar{C}(s)$ written by (11) stabilizes $\bar{G}(s) = G(s) + K(s)$. $K(s)$ is assumed to be biproper and C(s) is assumed to be of minimum-phase. In (11) if the $(1 - C(s)K(s))^{-1}$ has unstable zeros, the unstable zeros are the unstable poles of C(s). Therefore, the $\bar{C}(s)$ has no unstable zeros, that is, $\bar{C}(s)$ is of minimum-phase. Then from (11) and simple manipulation, $1/(1+\bar{C}(s)\bar{G}(s))$, $\bar{C}(s)/(1+\bar{C}(s)G(s))$, $\bar{G}(s)/(1+\bar{C}(s)\bar{G}(s))$ and $\bar{C}(s)\bar{G}(s)/(1+\bar{C}(s)\bar{G}(s))$ are rewritten as

$$\frac{1}{1 + C(s)G(s)} = \frac{1 - C(s)K(s)}{1 + G(s)C(s)}, \quad (13)$$

$$\frac{\bar{C}(s)}{1 + C(s)G(s)} = \frac{C(s)}{1 + G(s)C(s)}, \quad (14)$$

$$\frac{\bar{G}(s)}{1 + C(s)G(s)} = \frac{(G(s) + K(s))\,(1 - C(s)K(s))}{1 + G(s)C(s)}, \quad (15)$$

and

$$\frac{\bar{C}(s)\bar{G}(s)}{1 + C(s)G(s)} = \frac{(G(s) + K(s))\,C(s)}{1 + G(s)C(s)}. \quad (16)$$

From the assumption that C(s) stabilizes $G(s)$, $1/(1 + C(s)G(s))$, $C(s)/(1 + C(s)G(s))$, $G(s)/(1 + C(s)G(s))$, and $C(s)G(s)/(1 + C(s)G(s))$ are all include in RH_∞. Therefore, all of transfer functions in (13), (14), (15) and (16) are include in RH_∞.

Next, we will show that if the minimum-phase controller $\bar{C}(s)$ stabilizes the plant $\bar{G}(s) = G(s) + K(s)$, then the minimum-phase controller C(s) written by (12) stabilizes G(s). In (12) if the

$(1+\bar{C}(s)K(s))^{-1}$ has unstable zeros, the unstable zeros are the unstable poles of C(s). Therefore, the C(s) has no unstable zeros, that is, C(s) is of minimum-phase. Then from (12) and simple manipulation, $1/(1+C(s)G(s))$, $C(s)/(1+C(s)G(s))$, $G(s)/(1+C(s)G(s))$ and $C(s)G(s)/(1+C(s)G(s))$ are rewritten as

$$\frac{1}{1+C(s)G(s)}=\frac{1+\bar{C}(s)K(s)}{1+G(s)\bar{C}(s)}, \tag{17}$$

$$\frac{C(s)}{1+C(s)G(s)}=\frac{\bar{C}(s)}{1+G(s)\bar{C}(s)}, \tag{18}$$

$$\frac{C(s)}{1+C(s)G(s)}=\frac{(\bar{G}(s)-K(s))(1+\bar{C}(s)K(s))}{1+G(s)\bar{C}(s)}, \tag{19}$$

and

$$\frac{C(s)G(s)}{1+C(s)G(s)}=\frac{(\bar{G}(s)-K(s))\bar{C}(s)}{1+G(s)\bar{C}(s)}. \tag{20}$$

From the assumption that $\bar{C}(s)$ stabilizes $\bar{G}(s)=G(s)+K(s)$, $1/(1+\bar{C}(s)\bar{G}(s))$, $\bar{C}(s)/(1+\bar{C}(s)\bar{G}(s))$, $\bar{G}(s)/(1+\bar{C}(s)\bar{G}(s))$ and $\bar{C}(s)\bar{G}(s)/(1+\bar{C}(s)\bar{G}(s))$ are all include in RH_∞. Therefore, all of transfer functions in (17), (18), (19) and (20) are include in RH_∞.
We have thus proved Theorem 4.
Theorem 1 is proved using the above-described theorems.

Proof

From Theorem 2, there exists biproper $K(s)\in RH_\infty$ to make $\bar{G}(s)=G(s)+K(s)$ of minimum phase. From Theorem 4, the parametrization of all internally stabilizing controllers C(s) for G(s) is same to that of all internally stabilizing controllers $\bar{C}(s)$ for $\bar{G}(s)=G(s)+K(s)$. The parametrization of all internally stabilizing controllers $\bar{G}(s)=G(s)+K(s)$ is given by (10), where $\bar{Q}(s)\in RH_\infty$ is any minimum-phase function to make $(1-\bar{Q}(s))\bar{G}(s)\in RH_\infty$. The equation (10) corresponds to (4). From Theorem 4, using $\bar{C}(s)$, C(s) is written in terms of (12). The equation (12) corresponds to (3). The proof of Theorem 1 is complete.

PROPERTIES OF THE CONTROL SYSTEM

We elucidate the properties of the closed-loop control system using the parameterization of all the stabilizing minimum-phase controllers given by (3) in this section.

First, we consider the reference tracking property. Here, using the parameterization of all minimum-phase stabilizing controllers for the minimum-phase plants in (3), the transfer function in (1) from the reference input $r(s)$ to the output $y(s)$ of the control system is given as

$$\frac{y(s)}{r(s)}=\frac{\bar{Q}(s)G(s)}{G(s)+K(s)}. \tag{21}$$

Therefore, in order to make the output $y(s)$ follow the step reference input $r(s)=1/s$ without steady-state error,

$$\frac{\bar{Q}(0)G(0)}{G(0)+K(0)}=1 \tag{22}$$

must be achieved. Thus, the output $y(s)$ follows the step reference input without steady-state error, if $\bar{Q}(s)$ satisfies the following condition

$$\bar{Q}(0)=1+\frac{K(0)}{G(0)}. \tag{23}$$

Next, the decay properties of the disturbance are described. The transfer function from the disturbance d(s) to the output $y(s)$ is given as

$$\frac{y(s)}{d(s)}=1-\frac{\bar{Q}(s)G(s)}{G(s)+K(s)}. \tag{24}$$

Therefore, in order to fully decay the step disturbance $d(s)=1/s$,

$$\frac{\bar{Q}(0)G(0)}{G(0)+K(0)}=1 \tag{25}$$

must be achieved. Thus, the step disturbance of $d(s)=1/s$, will effectively be rejected if $\bar{Q}(s)$ satisfies the following condition

$$\bar{Q}(0)=1+\frac{K(0)}{G(0)}. \tag{26}$$

DESIGN METHOD OF MINIMUM-PHASE STABILIZING CONTROLLERS

Next, we present a design method of the stabilizing minimum-phase controllers for the minimum-phase strictly proper plants. From Theorem 1, to design a minimum-phase stabilizing controller C(s), we need to obtain K(s) satisfying Theorem 2 and such that $\bar{G}(s)=G(s)+K(s)$ is of minimum-phase and biproper. Then, $\bar{Q}(s)\in RH_\infty$ is determined as a function of any minimum-phase such that $(1-\bar{Q}(s))(\bar{G}(s)+K(s))\in RH_\infty$. Furthermore, in order to make the output y(s) follow the reference input $r(s)=1/s$ without steady-state error, $\bar{Q}(s)$ needs to satisfy (23). A design method for $Q(s)\in RH_\infty$ and the minimum-phase stabilizing controller C(s) is concluded as follows.

1. Obtain K(s) that satisfies Theorem 2, and such that G(s) + K(s) is of minimum-phase and biproper.
2. Obtain $\bar{G}(s)=G(s)+K(s)$ that is of minimum-phase and biproper function.
3. Define a function $\hat{Q}(s)$ as

$$\hat{Q}(s)=-\frac{K(s)}{G(s)}. \tag{27}$$

4. Using $\hat{Q}(s)$ in (27), design $\bar{Q}(s)\in RH_\infty$ as

$$\bar{Q}(s)=1-\hat{Q}(s)\frac{k}{(\tau s+1)^{\alpha}}, \tag{28}$$

where $\tau\in R$, α is an arbitrary positive integer to make $\bar{Q}(s)$ proper and k is a constant.

5. Using $\bar{Q}(s)$ in (28), fix a minimum-phase stabilizing controller C(s) in (3).

NUMERICAL EXAMPLE

This section shows a numerical example to illustrate the features of the proposed design method.
Consider the problem to find a minimum-phase stabilizing controllers for the plant G(s) written by

$$G(s)=\frac{(s+3)}{(s-1)(s+2)}\,. \tag{29}$$

By Theorem 2, we obtain

$$K(s)=\frac{(s+1.657)(s+3.529)\,(s+5.814)}{(s+2)^2(s+9)}. \tag{30}$$

Then, $\bar{G}$ is written as

$$\bar{G}(s)\;=\;G(s)\;+\;K(s)\;=\;\frac{(s+5)(s+4)\,(s+1)^2}{(s-1)(s+2)^2\,(s+9)}\,. \tag{31}$$

From (27), $\hat{Q}(s)$ is written by

$$\hat{Q}(s)=-\;\frac{(s+1.657)(s+3.529)\,(s+5.814)(s-1)}{(s+9)(s+3)(s+2)}\,. \tag{32}$$

$\bar{Q}(s)$ is given by (28), where k, α, and τ are settled by

$$k\;=1, \tag{33}$$

$$\alpha=1 \tag{34}$$

$$\tau=1, \tag{35}$$

Then $\bar{Q}(s)$ is obtained as

$$\bar{Q}(s)=\frac{2\;(s+7.242)\,(s+3.272)\,(s+1.744)\,(s+0.242)}{(s+9)(s+3)(s+2)(s+1)}. \tag{36}$$

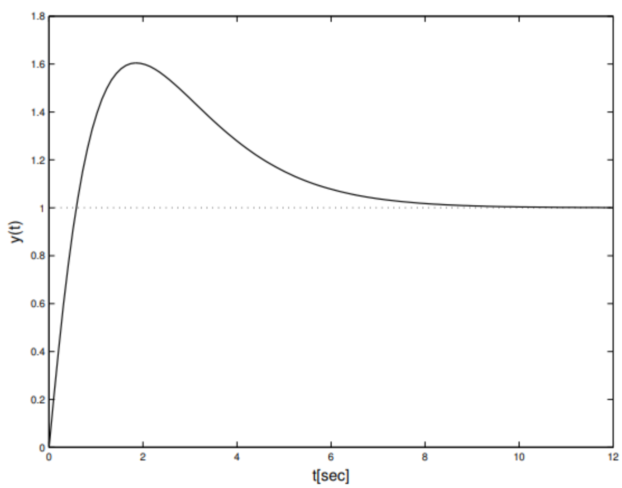

Figure 1) *The response of the output of y(t) of the controlled system corresponding to the step reference input r(t) = 1*

Then the minimum-phase stabilizing controller C(s) is obtained as

$$C(s)=\frac{2\;(s+7.242)\,(s+3.272)\;(s+2)(s+1.744)\,(s+0.242)}{s(s+5.814)(s+3.529)(s+3)(s+1.657)}. \tag{37}$$

For the step reference input r(t) = 1, the output of the closed-loop system y(t) reacts as follows 1 when using the minimum-phase stabilizing controller C(s) (37). In the curve of Figure 1, it is proved that the controlled system is stable under the equation (1), and its output y(t) is equal to the step reference input r(t) = 1, and there is no steady-state deviation.

On the contrary, if there is a step disturbance, the output response of the closed-loop is displayed in the Figure 2. The curve in Figure 2 confirms which the effect of this interference on d(t) = 1 is effective.

Next, to verify the robustness of the proposed method, we consider this case where we control a perturbed controlled plant with the controller that has been derived. The controlled plant is denoted as

$$G_1(s)=\frac{(s+10)}{(s-1)\,(s+2)}. \tag{38}$$

In this case, the output response corresponding to the step input r(t) = 1 is represented in Figure 3. The figure reflects that output y(t) follows the step reference input r(t) well with no steady state error. And the output response corresponding to the step disturbance d(t) = 1 is represented in Figure 4. The figure reflects the ability to effectively stop external disturbance.

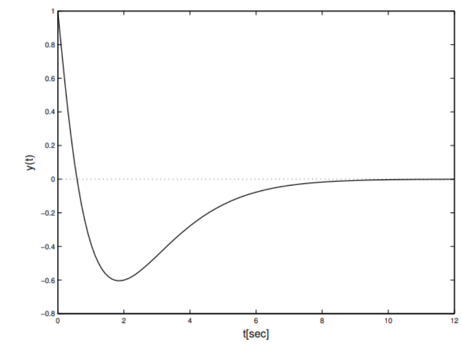

Figure 2) *The response of the output of y(t) of the controlled system corresponding to the step disturbance d(t)*

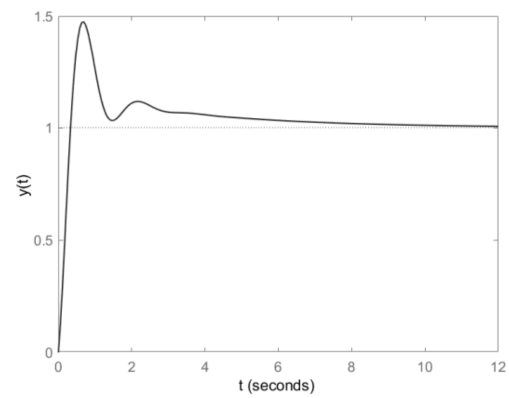

Figure 3) *The response of the output of y(t) of the controlled system with* G_1 *corresponding to the step reference input r(t) = 1*

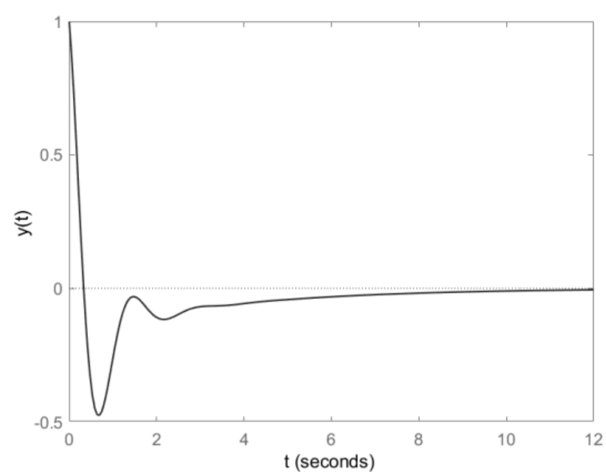

Figure 4) *The response of the output of y(t) of the controlled system with* G_1 *corresponding to the step disturbance d(t) =1*

CONCLUSION

In this paper, we clarified the parameterization of all stabilizing minimum-phase controllers for minimum-phase strictly proper plants. That is, we showed that if the stabilizing controller C(s) is written by the form of (3), the minimum-phase plant is stabilized. In addition, we showed a numerical example to illustrate that a stabilizing minimum-phase controller written by the form of (3) can stabilize the minimum-phase plant. In the future, we will present the parameterization of all stabilizing minimum-phase controllers for minimum-phase mutiple-input/mutiple-output plants.

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