The quantum motion of the nerve with a local defect

Miroslav Pardy

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ABSTRACT

We consider the nerve as the elastic string, the left end of which is fixed at the beginning of the coordinate system, the right end is fixed at point *l* and mass *m* is fixed between the ends of the string. We determine the classical and the quantum vibration of such system. The quantum motion is obtained by the so- called non-conventional oscillator quantization method by author. The proposed model can be also related in the modified form to the problem of the Mossbauer effect, being the recoilless nuclear resonance fluorescence, which is the resonant and recoil-free emission and absorption of gamma radiation by atomic nuclei bound in a solid. It is not excluded that our oscillator quantization of the string can be extended to generate the new way of the string theory of matter and physiology of nerves.

Key Words: *Quantum motion; Elastic string; Quantum vibration; Oscillator*

INTRODUCTION

A nerve is an enclosed, cable-like bundle of fibers (called axons) in the peripheral nervous system. A nerve transmits electrical and mechanical impulses. A nerve provides a common pathway for the electrochemical nerve impulses transmitted along each of the axons to peripheral organs or, in the case of sensory nerves, from the periphery back to the central nervous system.

The propagation of the nervous impulse is one of the oldest problems in biophysics. Luigi Galvani first described the contraction of a frog muscle after connecting two electrodes to the spine and the leg. He attributed the contraction to some kind of "animal electricity". In contrast, his contemporary Alessandro Volta was of the opinion that the pulse propagation is a purely electrical phenomenon [1].

In the middle of the 19th century Hermann Helmholtz was the first to measure the velocity of a nerve pulse quantitatively. In his dissertation Helmholtz found that the pulse velocity in frog sciatic nerves is about 30 m/s. This occurred practically simultaneously with his formulation of the first law of thermodynamics. Today, it is practically unthinkable to find careers that span the whole range from physiology to theoretical physics [1, 2].

The propagating electrical phenomena in nerves are called "action potentials". During a typical nerve pulse the voltage between the interior and the exterior of a cell changes locally by about 100 mV. The contemporary view of these phenomena originates from the model of Alan L. Hodgkin and Andrew F. Huxley from 1952, for which they received the Nobel Prize in Medicine in 1963.

Nerve signals can not only be triggered by voltage changes. The observed excitability caused by mechanical stimuli was interpreted by Wilke, that the nervous impulse cannot just be a purely electrical event. So, we consider here the mathematical and physical model of nerve which can reflect some new aspects of the nerve motion in the classical regime and in the quantum regime. We use so called the string model at the classical motion and at the quantum motion with the defect realized by the interstitial massive point [3].

Classical string motion

First, let us consider the string, the left end of which is fixed at the beginning of the coordinate system, the right end is fixed at point *l* and mass *m* is fixed interstitially between the ends of the string. The vibration motion of the string and the massive point with mass *m* is the problem of the mathematical physics in case that the tension is linearly dependent on elongation.

The differential equation of motion of string elements can be derived by the well-known way. We suppose that the string tension force acting on the element *dl* of the string is given by the law [4]:

$$T(x,t) = ES\left(\frac{\partial u}{\partial x}\right), \tag{1}$$

Department of Physical Electronics and Laboratory of Plasma physics Masaryk University Kotlarska 2, 611 37 Brno, Czech Republic

Correspondence: Miroslav Pardy, Department of Physical Electronics and Laboratory of Plasma physics Masaryk University Kotlarska 2, 611 37 Brno, Czech Republic, e-mail: nnnpppttt@gmail.com

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where E is the modulus of elasticity, S is the cross section of the string. We easily derive that

$$T(x+dx)-T(x)=ES\left(\frac{\partial u}{\partial x}\right)(x+dx)-ES\left(\frac{\partial u}{\partial x}\right)(x)=ESu_{xx}dx. \quad (2)$$

The mass dm of the element dl is ϱSdx, where ϱ is the mass density of the string matter and the dynamical equilibrium gives

$$\varrho Sdxu_{tt}=ESu_{xx}dx. \quad (3)$$

Or, after minimal modification we get

$$\frac{1}{c^2}u_{tt}-u_{xx}=0; \quad c=\left(\frac{E}{\varrho}\right)^{1/2}. \quad (4)$$

The last procedure was performed evidently in order to get the wave equation.

The string motion with the interstitial massive point

Now, let us consider the string with the point-like mass at coordinate s in the interval $(0, l)$. Then, the left part string motion of the string be $u_1(x, t)$ and the right side of the string motion is $u_2(x, t)$. The corresponding equation of motion of both part of the string are as follows [5]:

$$(u_1)_{tt}=c^2(u_1)_{xx}; \quad (0<x<s), \quad (5a)$$

$$(u_2)_{tt}=c^2(u_2)_{xx}; \quad (s<x<l). \quad (5b)$$

The boundary and interstitial conditions are S = 1:

$$u_1(x=0)=0, \quad u_2(x=l)=0. \quad (6)$$

$$u_1(x=s)=u_2(x=s). \quad (7)$$

The dynamical equation involving interstitial point is with $E=\varrho c^2$:

$$\varrho c^2(u_1)_x(s)-\varrho c^2(u_2)_x(s)=m(u_1)_{tt}(s)-m(u_2)_{tt}(s). \quad (8)$$

Let us look for the solution of the last equation in the form

Let us look for the solution of the last equation in the form

$$u_1(x,t)=C_1\sin\frac{\omega x}{c}\sin\omega t \quad (9)$$

$$\mu_2(x,t)=C_2\sin\frac{\omega(l-x)}{c}\sin\omega t. \quad (10)$$

$$u_1(x=0)=0, \quad u_2(x=l)=0. \quad (11)$$

After insertion of $u_1(x,t), u_2(x,t)$ from (9-10) into eqs. (7-8), we get the system of equations

$$C_1\sin\frac{\omega s}{c}=C_2\sin\frac{\omega(l-s)}{c} \quad (12)$$

and

$$C_1\varrho c\omega\cos\frac{\omega s}{c}=C_2\varrho c\omega\cos\frac{\omega(l-s)}{c}=$$

$$C_1m\omega^2\sin\frac{\omega s}{c}-C_2m\omega^2\sin\frac{\omega(l-s)}{c}. \quad (13)$$

In order to get the regular solution, the determinant of the system must be zero. Or,

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix}=0, \quad (14)$$

where

$$A=\sin\frac{\omega s}{c} \quad (15)$$

$$B=-\sin\frac{\omega(l-s)}{c} \quad (16)$$

$$C=\varrho c\omega\cos\frac{\omega s}{c}-m\omega^2 sin\frac{\omega s}{c} \quad (17)$$

$$D=-\varrho c\omega\cos\frac{\omega(l-s)}{c}+m\omega^2\sin\frac{\omega(l-s)}{c}. \quad (18)$$

It follows from eq. (14)

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix}=AD-BC=0. \ \sin\frac{\omega s}{c}\left[-\varrho c\omega+m\omega^2\tan\frac{\omega(l-s)}{c}\right]+ \quad (19)$$

$$\sin\frac{\omega(l-s)}{c}\left[\varrho c\omega - m\omega^2\tan\frac{\omega s}{c}\right] = 0, \quad (20)$$

So, we see, that the determination of the frequency ω involves the transcendent equation. The solution can be performed graphically, or by computer. Such problem is the integral part of the university mathematical methods [6].

Nevertheless, it is evident that the trivial solution is for $\omega = 0$ and for

$$\omega s = \pi nc, \quad n = 0, 1, 2, ...; \quad \omega(l-s) = \pi kc, k = 0, 1, 2, ..., \quad (21)$$

which implies that the correspondence between l and k is only for

$$\frac{l}{s} = \frac{(k+n)}{n}. \quad (22)$$

The determination of the ω from the transcendent equations

$$\left[-\varrho c\omega + m\omega^2\tan\frac{\omega(l-s)}{c}\right] = 0, \quad \left[\varrho c\omega - m\omega^2\tan\frac{\omega s}{c}\right] = 0 \quad (23)$$

is difficult and it can be solved by the appropriate mathematical methods [6].

The un-conventional quantization of the string motion by harmonic oscillators

The non-relativistic quantization of the equation for the energy of a free particle

$$\frac{p^2}{2m} = E \quad (24)$$

consists in replacing classical quantities by operators. We get the non-relativistic Schrödinger equation for a free particle. The operator replacings are $E \to i\hbar\partial/\partial t$, $\mathbf{p} \to -i\hbar\nabla$.

The Schrödinger equation suffers from not being relativistically covariant, meaning it does not take into account Einstein's special relativity.

It is natural to perform the special relativity generalization of the energy relation describing the energy:

$$E = \sqrt{p^2c^2 + m^2c^4}. \quad (25)$$

Then, just inserting the quantum mechanical operators for momentum and energy yields the equation

$$i\hbar\frac{\partial}{\partial t} = \sqrt{(-i\hbar\nabla)^2c^2 + m^2c^4}. \quad (26)$$

This, however, is a cumbersome expression to work with because the differential operator cannot be evaluated while under the square root sign.

Klein and Gordon instead began with the square of the above identity, i.e. $E^2 = p^2c^2 + m^2c^4$, which, when quantized, gives

$$\left(i\hbar\frac{\partial}{\partial t}\right)^2 = (-i\hbar\nabla)^2c^2 + m^2c^4. \quad (27)$$

So, we have seen that the quantization of classical mechanics is the simple replacing classical quantities by operators. We use here the novel quantization method where classical oscillators forming the classical systems are replaced simply by the quantum solution of quantum oscillators. The natural step is to apply the method to motion of the classical string.

It is well known that harmonic oscillator equation

$$\ddot{x} + \omega^2 x = 0; \quad \omega = \sqrt{k/m} \quad (28a)$$

has the solution

$$x(t) = A\cos(\omega t + \varphi). \quad (28b)$$

In case of the quantum mechanical oscillator motion, the solution for the stationary sates [7].

$$\psi_n = N_n H_n \exp\left(-\xi^2/2\right); \xi = x\sqrt{m\omega/\hbar}, \quad (29)$$

where N_n is the normalization constant

$$N_n = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\sqrt{\frac{1}{2^n!}} \quad (30)$$

and H_n are the Hermite polynomials defined by the following relation

$$H_n = (-1)^n e^{\xi^2}\frac{d^n}{d\xi^n}\exp\left(-\xi^2/2\right). \quad (31)$$

So, the wave function of the one string oscillator of the string with the periodic force at point c in the form:

$$\psi_i(x,t) = AC_1 \sin \frac{\omega(x_i - x)}{c} \; N_{n_i} H_{n_i}; \quad 0 < x_i < c \qquad (32a)$$

and

$$\psi_i\,(x,\,t) = AC_2 \sin \frac{\omega(x_i - x)}{c} \, N_{ni} H_{ni}; \quad c < x_i < l. \qquad (32b)$$

The total wave function of the string system of oscillators is then

$$\Psi_1(x,t) = \Pi_i^{\infty} \psi_i(x,t) = \qquad (33a)$$

$$\Pi_i^{\infty} AC_1 \sin \frac{\omega(x_i - x)}{c} N_{n_i} H_{n_i}; \quad 0 < x_i < c$$

and

$$\Psi_2(x,t) = \Pi_i^{\infty} \psi_i(x,t) = \qquad (33b)$$

$$\Pi_i^{\infty} AC_2 \sin \frac{\omega(x_i - x)}{c} N_{n_i} H_{n_i}; \quad c < x < l.$$

So, the quantization of string is possible only if we devide the string into elementary discrete points supposing that in every point of string $X \in (0, l)$, there is a quantum oscillator with the stationary states described by eq. (32) [8]. There is an analogue representation to eq. (33), which was applied by Feynman for determination of the quantum theory of the Mössbauer effect [9-15].

DISCUSSION & CONCLUSION

This elaborate is the integral part of modeling of signals in nerves. The general starting point is classical equations of mathematical physics deduced from basic equations describing electrical and mechanical dynamical processes and heat conduction. The equations used for describing physiological effects are based on experimentally observed phenomena. Such an approach means the interface of physics, continuum mechanics, thermodynamics and physiology. The mathematical model is a system of differential equations united into a whole by coupling forces. The general aim is to describe fundamental physiological effects as well as leaving the door open for quantum mechanical modifications.

The present elaborate was inspired by the author diploma work, in which the interaction of light with the crystal defect was calculated. At this theory the crystal was replaced by the Euler-Bernoulli linear chain with some defects.

The proposed model can be also related in the modified form to the problem of the Mössbauer effect, or, recoilless nuclear resonance fluorescence, which is the resonant and recoil-free emission and absorption of gamma radiation by atomic nuclei bound in a solid. In this effect, a narrow resonance for nuclear gamma emission and absorption results from the momentum of recoil transited to a surrounding crystal lattice and not to the emitting or absorbing nucleus alone. No gamma energy is lost. Emission and absorption occur at the same energy, resulting in strong, resonant absorption.

The generalization of our continual model can be performed in such a way that we replace one massive point m by the massive points $m_1, m_2, m_3, \ldots \; m_k$ at points $s_1, s_2, s_3, \ldots \; s_k$ and solve the adequate system of differential equations.

The string theory can be extended to the quark-quark interaction by the string potential, defined as the quark mass correction to the string potential, which was performed by Lambiase and Nesterenko. The calculation of the interquark potential generated by a string with massive ends was performed by Nesterenko and Pirozhenko and others. The propagation of a pulse in the real strings and rods which can be applied to the two-quark system as pion and so on, was calculated by author Pardy. So, it is not excluded that our oscillator quantization of the string can be extended to generate the new way of the string theory of matter and physiology of nerves.

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