RESEARCH ARTICLE

The role of pythagorean prime 137 in octonion dirac's equation in spacetime lattice and its links to the fine structure constant and the golden angle

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ABSTRACT

In this work, we present a theory to link the fine structure constant α to the Pythagorean prime 137. This model is based on quantized lattice spacetime and gauge symmetry of electromagnetism. Unlike the conventional Dirac equation which assumes an electron as a point-like particle without an internal structure. However, such a concept would lead to a divergence in self-energy. Instead, we treat an electron as a charged particle with an internal structure and a finite size due to the self's electrostatic interaction. We propose a modified equation using Cayley-Dickson's octonion operators of the Clifford geometric algebra formalism. We elucidate the number theory of the magic 137 as a Pythagorean prime triple with $1/\alpha = 137 = 4^2 + 11^2$, and a prime quintuple with $137 = 4^2 + 2^2 + 6^2 + 9^2$. This model leads to an internal structure for the electron with an effective radius of $R_e = 1.4395 \times 10^{-15}\ m$. We also obtain a simple relation for electrons' Coulomb and gravitational forces $F_C / F_G = 3 \times (137\pi)^{16}$. The formula's simplicity seems to imply intricate connections between 137 and other types of forces in nature. The volume of the fundamental hyper-cell of 432 formed by four axes with 2, 4, 6, and 9 in length has a golden angle ratio of $432/\pi \approx 137.50987083$ for a discrete 4D spacetime lattice. This value is extremely close to the conventional gold angle $360/\phi^2 \approx 137.50776405$ for the 3D continuous Euclidean space, where $\phi = (1+\sqrt{5})/2 \approx 1.61803399$ is the golden ratio. This dimensionless magic 137, as a geometric number in lattice spacetime, could join Euler's constant e and π as three important numbers in mathematical physics.

Keywords: *Fine Structure Constant; Spacetime Quantization; Dirac Equation; Pythagorean Prime; Octonion; Gauge Invariance; Golden Angle*

INTRODUCTION

One of the biggest mysteries in modern physics is the dimensionless fine structure constant, $\alpha = e^2/\hbar c$. which happens to be about 1/137 [1, 2]. This constant plays a crucial role in electromagnetism, and it strongly influences many physical processes in nature, including the evolution of stars and chemical reactions, yet no one knows why it takes such a value. The value of 137 has a substantial impact on life because should this value differ by more than 4%, stellar fusion would not produce carbon, and there will be no life on this planet [3].

In this report, we present a theory to explain the origin of this value by linking the number theory regarding Pythagorean primes to Einstein's mass-energy relation and the gauge theory in electromagnetism [4, 5]. In addressing this century-old mystery, we are compelled to postulate spacetime quantization (STQ), i.e., time and space are not divisible indefinitely, but possess a fundamental size. How STQ affects the foundations of physics would be like how Planck's energy quantization influenced the quantum revolution. In this work, we will show how the fundamental unit of length and time leads to an electron's quantized electric charge and mass. We will also show how the gravitational constant is intricately connected to the fine structure constant for electromagnetism, implying the possible existence of a common root that unifies these two forces, and the

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potential unification of the quantum theory for gravity with other forces. In Dirac's theory for electrons, the electric charge and mass are free parameters that must be determined experimentally [6]. Currently, no accepted theories explain the origin of the electron's charge and the value of the dimensionless fine structure constant α . In this work, we present an approach to link these values, along with the gravitational constant G, to more fundamental constants such as the speed of light and the Planck constant.

THEORY

In this work, we present a theory to link the magic prime 137 to the fine structure constant. This theoretical model is based on well-known physical principles and the prime number theory of 137. The constraint equations on 137 are a Pythagorean prime triple and a prime quintuple derived from STQ, gauge invariance (GI) of the Lorentz group, and Einstein's special relativity theory for the mass-energy relation [7].

STQ hypothesis

STQ is one of the three cornerstones of our model, in addition to the GI and Einstein's mass-energy relation. Discretization of spacetime has often been used in finite element analysis for computer simulations to solve differential equations involving continuous variables numerically. Such a technique is used for computer simulation and has nothing to do with the fundamental principles of STQ. At the same time, our STQ model posits that spacetime is fundamentally discrete, and the domain is also discrete in wave vector and frequency by Fourier series or z-function transformation. The most fundamental unit of the lattice structure is a cube. Assuming the fundamental unit of length is L and the time unit $T=L/c$, for a standing wave in such a cube, the side length must equal to an integer multiple of its half wavelength λ , i.e., $L = \lambda/2$, and the fundamental wave vector $K= 2\pi 1/\lambda=\pi/L$ and the fundamental angular frequency $\omega= 2\pi/T = \pi c/L$. The quantized energy for a single mode along each axial direction is given by $E=\hbar\omega= \pi\hbar c/L,$ and the total quantized energy for a state with one single excited method along each direction is $\pi\hbar c/L$. According to Einstein's mass-energy equivalence formula $m=E/c^2$, we obtain an effective quantized mass for such a fundamental-mode standing wave excitation, $m_u = \pi\hbar/cL$. Here, by defining a fundamental length unit, one obtains the fundamental time $T=L/c$, wave vector $K =1/2L$ frequency $1/T$, and mass-energy respectively.

Now, we discuss how STQ leads to the value of the fine structure constant being $\sim 1/137$ and an electron's charge, mass, and size in terms of these fundamental lattice units of space and time. According to Einstein's special relativity, one has $\left(E/c\right)^2 = m0c^4 + c^2 p^2$ and $p^2 = p_1^2 + p_2^2 + p_3^2$ for a relativistic particle with a rest mass m_0 , but for a massless gauge boson such as a photon, $m_0 = 0$. Such an equation implies that these values are continuous, i.e., spacetime can be divided indefinitely so that one can use derivatives of these variables in wave equations or particle dynamics. However, for a quantized spacetime with a lattice structure, energy and momentum have a natural unit dictated by the fundamental size of time and space. Consequently, in terms of fundamental units, one ha $E = n_0\hbar\Omega,$ $p_1 = n_1\hbar K_1$, $p_2 = n_2\hbar K_2$, $p_3 = n_3\hbar K_3$, and $p = n_4\hbar K_4$. According to the GI of electromagnetism, such symmetry leads to charge conservation. In continuous spacetime, the gauge transformation involves derivatives in time and space, but in discrete spacetime, one needs to quantize the gauge function. The derivation details will given in the Sec. 2.3., and we simply cite the results here,

GI in lattice spacetime as the origin of the fine structure constant

Based on the STQ, one needs to quantize the gauge function in a 4D spacetime lattice and the gauge function can be expressed as a sum of four anti-commutative orthonormal operators, as will be shown in section 2.3., we have derived the following equations involving only integers:

$$
\begin{aligned}
&n_0^2\, \hbar c/e^2 = n_1^2 + n_2^2 + n_3^2 + n_4^2 \\
&n_5^2 = n_1^2 + n_2^2 + n_3^2 \qquad (1) \\
&n_0^2\, \hbar c/e^2 = n_5^2 + n_4^2,
\end{aligned}
$$

where $\hbar c/e^2$ and n_4 must be a prime number for the solution to represent a fundamental mode. For the first set of integer solutions that satisfy Eq. (1) one must have $n_0 = 1$ and $\hbar c/e^2 = 137$ which is the lowest prime to satisfy the constraints $\left\{n_0, n_1, n_2, n_3, n_4, n_5,\right\} = \left\{1, 2, 6, 9, 4, 11\right\}$, The solution leads to $137 = 2^2 + 6^2 + 9^2 + 4^2$ 137 as a Pythagorean prime quintuple, 137 as a Pythagorean prime triple, and 11 as a Pythagorean quadruple. For greater than 1, the solution represents a higher harmonic mode. We have derived the relationship between the fine structure constant and 1/137, based solely on STQ, Einstein's relativity theory and the gauge symmetry in electromagnetism. Although the ideal value at 1/137 deviates slightly from the experimental value2, we think this slight deviation of about 0.026% is likely caused by weak interaction that breaks the symmetry of the Lorentz group or electron's interaction with its own field. This situation of 1/137 being the ideal fine structure constant is like Dirac's theory of a "bare" electron before the renormalization procedure in quantum electrodynamics [8]. We have found a formula for the empirical fit in power series of $\zeta \equiv 2\pi/137$. With two to five terms of series expansion, the error can be reduced to $1.8x10-5\%$ and $7.3x10^{-10}\%$, respectively. Such formulae provide a guideline for future development of the refinement theory involving the electroweak gauge theory. A slight increase in the effective fine structure constant observed at high energy over 90 GeV is not surprising, because at such a high energy regime hadronic contribution due to strong interaction would occur. In contrast, the ideal 137 value is derived based on the GI of electromagnetism. From Eq. (1) and the solution, one can see that the index n_4 of 11 is related to the magnitude of the momentum vector. In contrast, indices 2, 6, and 9 are associated with the individual momentum's component along three spatial axes. In addition, one can see that n_0 is linked to the number of quanta and n_5 is linked to internal energy due to EM interactions. The GI of electromagnetism and Einstein's mass-energy relations in the lattice spacetime lead to those equations involving six integers. For the solution to represent the fundamental mode, both must be a prime number. One may wonder what kind of a six-element set can satisfy such constraints.

Modified dirac's equation using octonion operators

In this section we present a modified Dirac's equation for an electron but in the Clifford geometric algebra (CGA) formalism with octonion operators [9-10]. In contrast to Dirac's original theory which involves four gamma matrix operators, in our modified equation we have eight operators. With this model an electron is considered not a point-like article without a volume but with an internal structure which can be described by three additional degrees of freedom involving anti-commutative operators. According to Dirac's theory, one has the following covariant derivative equation involving the gamma matrices [11].

$$\left(i\gamma^{\mu}\partial_{\mu}-m\right)\Psi=0$$
$$p_k=-i\partial_k,\ \ p_0=i\partial_0, \tag{2A}$$
$$\left\{\gamma^{\mu},\gamma^{\nu}\right\}=2\eta^{\mu\nu}\mathbf{I},$$

where $\eta^{\mu\nu}$ is the Makowski metric element $(+,-,-,-)$. Equivalently, one has

$$E=-\left(\gamma^0\gamma^1P_1+\gamma^0\gamma^2P_2+\gamma^0\gamma^3P_3-m_0\gamma^0\right)$$
$$\left(\gamma^0\right)^2=-\left(\gamma^k\right)^2=I,\ \ k=1,2,3. \tag{2B}$$

Because these idempotent operators with different indices satisfy the following anti-commutative relations $\left\{\gamma^{\mu},\gamma^{\nu}\right\}=2\eta^{\mu\nu}I$, one has obtained the well-known Einstein's mass-energy link in the natural unit $E^2=P_1^2+P_2^2+P_3^2+m_0^2$. Eq. (2) describes a structure-less point-like particle without a volume, and its mass in a free parameter which needs to be determined experimentally. In our modified DE, we consider an electron with an internal structure and the equation contains eight operators, representing three degrees of freedom for three spatial coordinates for the center of the mass as in Eq. (3), one degree of freedom for the total energy, and three degrees of freedom for the spatial axes to describe the internal dynamics concerning the center of the mass. With our modified model, the total energy can be expressed in terms of 8 octonions $\left\{I,G_1,G_2,G_3,G_4,G_5,G_6,G_7\right\}$ as

$$EI\left|\Psi\right\rangle=\left(G_5P_1+G_6P_2+G_7P_3+G_4Q_0+G_1Q_1+G_2Q_2+G_3Q_3\right)\left|\Psi\right\rangle$$
$$G_k=\begin{pmatrix}\sigma_k & I_2\\ I_2 & \sigma_k\end{pmatrix},\ G_{4+k}=\begin{pmatrix}I_2 & \sigma_k\\ -\sigma_k & I_2\end{pmatrix},\ G_4=\begin{pmatrix}0 & -iI_2\\ iI_2 & 0\end{pmatrix}$$
$$\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\ \sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\ \sigma_3=\begin{pmatrix}I_2&0\\0&-I_2\end{pmatrix},\ I_2=\begin{pmatrix}1&0\\0&1\end{pmatrix} \tag{3A}$$
$$\left\{G_m,G_n\right\}=0\ \ if\ m,\neq n$$
$$G_k{}^2=I,\ k=1,2,3,4.$$
$$G_n{}^2=-I,\ n=5,6,7.$$

In comparison, the four Dirac gamma matrices and the unit matrix of the set $\left\{\gamma^0,\gamma^1,\gamma^2,\gamma^3,I\right\}$ do not form a group because $\gamma^1\gamma^2,\gamma^2\gamma^3,\gamma^3\gamma^1$ do not belong to the same set. However, the eight octonion matrices form a group. In the reference of the center of mass for an electron, three spatial momentum components vanish, i.e., $P_1=P_2=P_3=0$, and Eq. (3) is reduced to a Hamiltonian involving four Q_k operators for the internal structural dynamics, i.e.,

$$E_0I\left|\Psi\right\rangle=\left(G_1Q_1+G_2Q_2+G_3Q_3+G_4Q_4\right)I\left|\Psi\right\rangle. \tag{3B}$$

The energy can be regarded as the rest mass energy for the electron due to its internal structural dynamics. The remaining three momentum components P_1,P_2,P_3 can be considered as the momentum for the electron of a rest mass m_0 concerning the laboratory reference frame, representing the conventional DE.

In the following, we present the derivation details of the magic 137 and its link to the fine structure constant, based on GI of the Lorentz group, STQ and Einstein's relation for a relativistic particle's total energy, rest mass energy and momentum. In continuous spacetime, a charge-conserving gauge symmetry exists for a charged particle's wave function in the presence of an electromagnetic field. The gauge transform will be invariant if $\lambda(t,r)$ satisfies

$$A\rightarrow A-\nabla\lambda(t,r),\quad \varphi\rightarrow\varphi+\frac{\partial}{c\partial t}\lambda(t,r)$$
$$\frac{\partial^2}{c^2\partial t^2}\lambda(t,r)-\nabla\cdot\nabla\lambda(t,r)=0, \tag{4A}$$

and in quantized spacetime with a discrete lattice coordinate $\left(t_n,x_i,y_j,z_k\right)$ one has

$$\lambda(t,r)\rightarrow\sqrt{\frac{c\hbar}{e^2}}\Lambda\left(t_n,x_i,y_j,z_k\right)$$
$$\Psi(t,r)\rightarrow\exp\left(i\sqrt{c\hbar/e^2}\Lambda\left(t_n,x_i,y_j,z_k\right)\right)\Psi\left(t_n,x_i,y_j,z_k\right), \tag{4B}$$

where both $\sqrt{c\hbar/e^2}$ and $\Lambda\left(t_n,x_i,y_j,z_k\right)$ are dimensionless $\Lambda\left(t_n,x_i,y_j,z_k\right)$ is related to electric potential ϕ and vector potential A. In quantized spacetime, momentum and $\Lambda(t,\mathbf{r})$ are quantized, and they are no longer scalar functions of time and space, and they need to be replaced by operators. The continuous space time needs to be quantized in terms a fundamental length unit L and a time unit $T=L/c$ Because the fundamental standing wave has a wavelength

λ equal to twice the lattice length, the fundamental units for the wave vector and frequency in the Fourier domain are given by $K=\pi/L$ and $\Omega = C\pi/L$, respectively.

Using our operator approach and discrete Fourier transform, the gauge transformation $\exp\left(i\sqrt{c\hbar/e^2}\Lambda(t_n, r_m)\right)$ becomes $\exp\left(i\sqrt{\hbar c/e^2}\mathbf{\Lambda}\right)$, and the operator $\mathbf{\Lambda}$ also becomes dimensionless in the 4D wave vector-frequency domain. Following Eq. (2) with quantized electric potential, vector potential, and the gauge function, by generalizing to discrete spacetime, one can express the quantized $\sqrt{c\hbar/e^2}\mathbf{\Lambda}$ in terms of four anti-commutative operators as

$$\sqrt{\hbar c/e^2}n_0|\Psi\rangle = \left(n_1G_1+n_2G_2+n_3G_3+n_4G_4\right)|\Psi\rangle. \tag{5}$$

Because $\{G_m, G_n\}=2\delta_{\mu\nu}$ I, $\mu,\nu=1,2,3,4,$ by squaring the above equation, one obtains

$$n_0^2\ \hbar c/e^2 = n_1^2+n_2^2+n_3^2+n_4^2 \tag{6A}$$

and because of the quantization of the total momentum and its three components, one also has

$$n_5^2 = n_1^2+n_2^2+n_3^2$$
$$n_0^2\ \hbar c/e^2 = n_4^2+n_5^2, \tag{6B}$$

where $\hbar c/e^2$ and n_5 must be prime numbers for the solution to represent a fundamental mode instead of high harmonic modes. After a search through all combinations of integers below 1000, we have found only one primary set of integers satisfying Eq. (6) for $n_0=1$, and $\hbar c/e^2=137$. This constant $\hbar c/e^2$ must be a prime number for the solution to representing the lowest fundamental set, instead of higher harmonic modes. With the value of $\hbar c/e^2=137$ settled, we found that 137 is a Pythagorean prime in a quintuple, and 137 is also a Pythagorean prime in a triple. One also shows showing 11 is a Pythagorean quadruple. From Eq. (6) one can notice that index 11 is related to the magnitude of the momentum vector. In contrast, the indices 2, 6, 9 are related to three individual momentum components along x, y, and z axes, and 4 is related to the internal energy due to the interactions. This set of integers is called the primary set, and all other solutions correspond to higher harmonic modes with an index n_0 greater than 1. Using the primary solution of six integers for Eq. (6), one can restore this equation to the original fundamental units for the wave vector $K=\pi/L$ and frequency $\Omega= c\pi/L$, and obtains

$$\alpha_0 = \frac{e^2}{\hbar c} = \frac{1}{137}, \tag{7A}$$

$$137\left(\frac{n_0\pi\hbar c}{L}\right)^2 = \left(\frac{n_0\pi\hbar c}{L/4}\right)^2 + \left(\frac{n_0\pi\hbar c}{L/2}\right)^2 + \left(\frac{n_0\pi\hbar c}{L/6}\right)^2 + \left(\frac{n_0\pi\hbar c}{L/9}\right)^2, \tag{7B}$$

and

$$\left(\frac{n_0\pi\hbar c}{L/11}\right)^2 = \left(\frac{n_0\pi\hbar c}{L/2}\right)^2 + \left(\frac{n_0\pi\hbar c}{L/6}\right)^2 + \left(\frac{n_0\pi\hbar c}{L/9}\right)^2. \tag{7C}$$

Eq. (7B) is analogous to Einstein's relation $E^2 = m_0^2c^4 + c^2P_1^2 + c^2P_2^2 + c^2P_3^2$ for a relativistic particle with a rest mass due to its internal structure, but now in discrete spacetime condition.

According to Eq. (7B) which represents a Dirac Equation (DE) but in a lattice spacetime. We consider such a cuboid solution to describe a building block. The term in Eq. (7B) is related to its relativistic mass-energy, and we can assign the electron's rest mass m_ec^2 to assume a side length of L_e for an electron with a radius of $L_e/2$. This scaling factor 137π equals approximately 432, which is the product of the hyper-cell side length and the hyper-cell volume. We obtain $R_e \equiv \pi\hbar c/\left(m_{es}c^2\times\sqrt{137}\times 36\right) = 1.4395\times 10^{-15}\ m$ with $m_ec^2 = 0.511\ MeV$.

The link of the fine structure constant to the number theory of Pythagorean primes

Using the primary solution of six integers for Eq. (6), one can restore this equation to the original fundamental units for the wave vector $K=\pi/L$ and frequency $\Omega= c\pi/L$ and obtains

$$\alpha_0 = \frac{e^2}{\hbar c} = \frac{1}{137}, \tag{8A}$$

$$137\left(\frac{n_0\pi\hbar c}{L}\right)^2 = \left(\frac{n_0\pi\hbar c}{L/4}\right)^2 + \left(\frac{n_0\pi\hbar c}{L/2}\right)^2 + \left(\frac{n_0\pi\hbar c}{L/6}\right)^2 + \left(\frac{n_0\pi\hbar c}{L/9}\right)^2, \tag{8B}$$

and

$$\left(\frac{n_0\pi\hbar c}{L/11}\right)^2 = \left(\frac{n_0\pi\hbar c}{L/2}\right)^2 + \left(\frac{n_0\pi\hbar c}{L/6}\right)^2 + \left(\frac{n_0\pi\hbar c}{L/9}\right)^2. \tag{8C}$$

For the primary mode $n_0=1$ Eq. (8B) is analogous to Einstein's relation $E^2 = m_0^2c^4 + c^2P_1^2 + c^2P_2^2 + c^2P_3^2$ for a relativistic particle with a rest mass due to its internal structure, but now in discrete spacetime condition.

According to Eq. (8) it represents a DE6 but with a quantized rest mass energy and kinetic energy along each axis in a lattice spacetime. This hyper-cell has 3 different spatial axial lengths due to GI in quantized spacetime. It can be regarded as one of the three degenerate eigenstates among all possible axial orientations with the same total energy. An electron can be considered in a coherent state from the superposition of these three degenerate eigenstates. The hyper-cell could be regarded as a building block, and we can use such hyper-cells to construct a more spherically symmetric electron. The unit length L in Eq. (4) is an unknown but scalable variable. One might link this length to the Planck length, but the corresponding energy is on the order of 1018 GeV, which is well beyond the energy scale of electromagnetism, weak and strong interactions [12]. We obtain a mass ratio concerning electron for a proton $m_p/m_e = (137\pi)\times 3\sqrt{2}\times 1.0055$. In addition to the constraints in Eq. (1), we also include a condition for the search of the prime value for $\hbar c/e^2$ with the ratio of the hyper-cell volume to the suitable prime being sufficiently close to an integer multiple of π. Because

$432/137 \approx 1.0037\pi, \sim\pi$ in our screening procedure for the prime value. We impose that the remainder of the quotient must be less than 0.5%. After using a screening algorithm to search all possible combinations of integers below 10000, we have only found one primary solution that meets the constraints. We choose electron mass-energy as a reference scale to determine a suitable unit length. We can assign the electron's mass-energy to $\pi\hbar c / \left(m_{es}c^2 \times \sqrt{137} \times 36 L_e\right)$, where factor 36 is the least common multiple of four axial lengths 4, 2, 6 and 9 for constructing a perfect 4D hyper-cube from hyper-cuboids. An electron can be regarded as in coherent superposition of these degenerate eigenstates, and because of the couplings of the 4D coordinates to those anti-commutative operators, an electron with a ½ spin can be regarded to possess a hyper-dimensional Möbius-type structure.

The term $137\hbar c / L$ on the left-hand side of Eq. (6B) represents the total mass-energy, and the first term $\hbar c / 4L$ on the right-hand side of the equation means the internal energy for an electron from the internal structure due to electromagnetic interactions. The last three terms $\hbar c / 2L, \hbar c / 6L, \hbar c / 9L$ are related to the electron's kinetic energy along three spatial axes. We obtain $L_e = 2.81867 \times 10^{-15}\, m$ with $m_e c^2 = 0.511\, MeV$. The corresponding electron's radius is, equals to $1.3103 \times 10^{-15}\, m$. By equating the mass-energy of an electron to the electrostatic energy of two point-like particles with the same electron's charge, the distance is found to correspond to the same previous radius of `$1.4090 \times 10^{-15}\, m$. In comparison, the theoretical classical radius of an electron is $2.818 \times 10^{-15}\, m$ the electron's Compton wavelength, the experimental proton's radius is about $0.842 \times 10^{-15}\, m$, and the radius of a quark is about $0.43 \times 10^{-18}\, m$. Without the scaling factor the corresponding length of $6.600 \times 10^{-12}\, m$ is on order as the electron's Compton wavelength $2.43 \times 10^{-12}\, m$.

We have found a simple relation for the ratio of Planck length to electron's radius R_e, $L_{Pkanck}/R_e = (27/2) \times (137\pi)^{-8} \times 1.003$, which has a power dependence on the scaling factor 137π. Other than the relation of 137 to the fine structure constant for electromagnetism, we have also found that the ratio of the Coulomb force and gravitation force for a pair of electrons can be described by a formula $F_C/F_G = 3 \times (137\pi)^{16} \times 1.0015$. The simplicity of these formulae and the power of 16 is unlikely to be coincident. First, it implies that Dirac's "large number" hypothesis of a gravitational constant decreasing along with the universe's age is questionable [13]. Secondly, these formulae hint that the prime 137 also plays an intricate role in gravity, and the power of 16 is likely related to sixteen pairwise tensor-product operators in 4D spacetime according to the geometry algebra formalism [14]. Our conjecture could provide a guideline for the theoretical development of quantum gravity.

DISCUSSION

In this work, we show that the prime 137 is linked to the inverse of the fine structure constant as a consequence of STQ, GI and Einstein's mass-energy relation. Out postulate of STQ means that the spacetime lattice has a finite fundamental unit size. Because time and space cannot be divided infinitely, therefore, the concepts of derivations in time or space can no longer be used. All derivative operators in the conventional wave equations such as the Schrodinger equation, Klein-Gordon equation, or DE need to be replaced by difference operators. The usual Fourier transform integrals in time or space needs to be replaced by *z*-transform or discrete Fourier series. Many divergence and singularity problems in physics can be avoided. Like Planck's introduction of quantized energy to solve the ultraviolet catastrophe in blackbody radiation, the proposal to quantize space and time avoids the vacuum catastrophe [15]. The present quantum field theory, based on continuous spacetime with time and space derivatives, predicts a vacuum energy 120 orders of magnitude greater than the experimental value. All elementary particles in the Standard Model commonly assumed to be point-like particles with an infinitely small size need to be replaced by a new concept of a finite size with their internal structures. This new paradigm is more logically consistent with why an electron can have a spin because it resembles a high-dimensional Mobius structure. The conventional continuous spacetime needs to be replaced by spacetime lattice, and one needs to treat time and space not just as a coordinate variable, but as operators [16-17].

CONCLUSIONS

In conclusion, we present a theory that links the fine structure constant to the inverse of the Pythagorean prime 137. Our model is based on quantized Makowski spacetime, gauge symmetry of electromagnetism and Einstein's special relativity. We propose a modified Dirac's equation for electrons using the octonion operator approach of the CGA formalism, and elucidate the role of the magic 137 as a Pythagorean prime triple with $1/a = 137 = 4^2 + 11^2$, and a prime quintuple with $137 = 4^2 + 2^2 + 6^2 + 9^2$. In our modified DE, it contains eight octonion operators. Among these operators, three momentum operators describe the internal structural dynamics with respect to the reference frame of the center of mass, and the other three operators describe the momentum components with the laboratory frame. One of the operators represents the effective rest mass energy due to the electron's internal dynamics. Unlike the traditional DE which assumes an electron is a point-like particle without an internal structure, our extended model describes an electron with a finite volume and an internal structure dictated by internal electromagnetic interactions. Our model avoids the divergence problem facing the conventional DE theory, and the vacuum catastrophe paradox for a continuum model.

Our analysis of the fine structure constant compels us to propose quantized spacetime which cannot be divided indefinitely, as also advocated by the loop quantum gravity theory. We demonstrate how $\hbar c/e^2 = 137$ is intricately tied to Einstein's mass-energy relation and quantized gauge function. These physical principles give rise to the constraints in Eq. (1): 1) a Pythagorean prime in a triple, i.e.,

$137 = 11^2 + 4^2$;2) a Pythagorean prime in a quintuple, i.e., $137 = 2^2 + 6^2 + 9^2 + 4^2$ while its decomposed element 11 needs to be a Pythagorean quadruple, i.e., $11^2 = 2^2 + 6^2 + 9^2$; 3) both 137 and 11 must be primes. After an exhaustive search for many possible combinations of six integers, 137 happens to be the only prime that satisfies the above three constraints. One may wonder how unique this value is. We propose here a conjecture that there exists no other prime for n_0 that satisfies constraints: 1) $n_0^2 \hbar c/e^2 = n_4^2 + n_5^2$; 2) $n_4^2 = n_1^2 + n_2^2 + n_3^2$; 3) both $n_0^2 \hbar c/e^2$ and n_4 must be a prime, otherwise, $n_0^2 \hbar c/e^2$ and n_4 cannot represent the most fundamental mode.

In this work, we unfold the mysterious role of prime 137 in the fine structure constant for electromagnetism. We also obtained a simple formula $F_C/F_G = 3 \times (137\pi)^{16} \times 1.0015$ for the ratio of Coulomb and gravitation forces between a pair of electrons. In addition, we also obtained a formula $L_{Pkanck}/R_e = (27/2) \times (137\pi)^{-8} \times 1.003$ for the ratio between the Planck length and electron's radius R_e . All these very simple relationships seem to imply that the magic 137 plays an important role not only in electromagnetism, but also has an intricate link to the other fundamental forces in nature. Our analysis compels us to propose the quantization of spacetime, which leads to no zero-point vacuum energy at absolute zero temperature, thus avoiding the vacuum catastrophe paradox. STQ also circumvents singularity divergence and renormalization procedures to remove infinities in quantum electrodynamics with continuous spacetime. In addition, it would lead to a different paradigm regarding our understanding of the universe. The product of four axial unit length, 4, 2, 6 and 9, in Eq. (6) of the hyper-cell is 432 which is equivalent to the hyper-cell's volume. The Pythagorean sum of 137 for the square of these numbers, which equals to the diagonal length of the hyper-cell and the diameter of the 4D hyper-sphere that encloses the hyper-cube. This ratio $432/\pi \approx 137.50987083$ which is extremely close to the gold angle defined as $360/\phi^2 \approx 137.50776405$ with a ratio discrepancy of 1.5×10^{-5} , where $\phi = (1+\sqrt{5})/2 \approx 1.61803399$ is the gold ratio. The golden ratio is known to play an important role in nature and fine arts, and this work also shows its interesting role in the fine structure constant and possibly other types of forces. As an end note, this magic prime137 might join the club of π , the ratio of circle's circumference to diameter, and the Euler constant e as three important universal dimensionless constants that play an important role in physics, mathematics, and the fundamental laws in the universe.

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