RESEARCH ARTICLE

The zero delusion

Julio Rives

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ABSTRACT

Zero signifies absence or an amount of no measure. This mathematical object purportedly exemplifies one of humanity's most splendid insights. Endorsement of the continuum consolidated zero as a cultural latecomer that, at present, everybody uses daily as an indispensable number. Zero and infinity represent symmetric and complementary concepts; why did algebra embrace the former as a number and dismiss the latter? Why is zero an unprecedented number in arithmetic? Is zero a cardinal number? Is it an ordinal number? Is zero a "real" point? Has it a geometrical meaning? To what extent is zero naturalistic?

A preliminary analysis indicates that zero is short of numerical competence, contrived, and unsolvable. We find it elusive when we dig into zero's role in physics, especially in thermodynamics, quantum field theory, cosmology, and metrology. A minimal fundamental extent is plausible but hard to accept due to zero's long shade. In information theory, the digit 0 is inefficient; we should replace standard positional notation with bijective notation. In communication theory, the transmission of no bits is impossible, and information propagation is never error-free. In statistical mechanics, the uniform distribution is inaccessible. In set theory, the empty set is ontologically paradoxical. Likewise, other mathematical zeroes are semantically vacuous (e.g., the empty sum, zero vector, zero function, unknot). Because division by zero is intractable, we advocate for the nonzero rational numbers, $\breve{\mathbb{Q}} \equiv \mathbb{Q} - \{0\}$, , to build a new physics that reflects nature's countable character. We provide a zero-free and unique $\breve{\mathbb{Q}}$ – based representation of the algebraic numbers punctured at the origin, $\breve{\mathbb{Q}} \equiv \mathbb{Q} - \{0\}$, the computable version of the complex numbers.

In a linear scale, we must handle zero as the limit of an asymptotically vanishing sequence of rationals or substitute it for the smallest possible nonzero rational. Zero, as such, is the predetermined power in- dicating the beginning of logarithmically encoded data via log (1). The exponential function decodes the logarithmic scale's beables back to the linear scale. The exponential map is crucial to understand advanced algebraic concepts such as the Lie algebra-group correspondence, the Laplace transform, and univariate rational functions in cross-ratio form. Specifically, linear fractional transformations over a ring lead to the critical notion of conformality, the property of a projection or mapping between spaces that preserves angles between intersecting conics. Ultimately, we define "coding space" as a doubly conformal transformation domain that allows for zero-fleeing hyperbolic (logarithmic) geometry while keeping relationships of structure and scale.

Keywords: *Zero; Infinity; Continuum; Special Relativity (SR); General Relativity (GR); Quantum Field Theory (QFT); Quantum Gravity (QG); Discreteness; Uncertainty Principle; Thermodynamics; Minimal Length; Information Theory (IT); Positional Notation (PN); Set Theory (ST); Rational number; Algebraic Number; Logarithmic Scale; Exponential Map; Lie Group; Laplace Transform; Möbius map; Linear Fractional Transformation (LFT); Cross-ratio; Conformality; Coding Space; Quantum Gravity; Beable; Logarithmic Scale; Exponential Map; Conformality*

INTRODUCTION

We present this essay and explain why zero is problematic.

Scope and rationale

This essay is a generalistic investigation of the number zero's role in diverse momentous fields of mathematics and physics. Because our society has internalized zero, one can hardly find academic articles about its usage or facets, except for some research on zero's genesis and a few analyses of the nothingness. We feel we are entering an untrodden area upon questioning zero in earnest.

Our motivation derives from a series of inquiries. Can humans naturally conceptualize zero? Why was number zero endorsed after so many centuries of ostracism? How did its historical and sociological context affect its endorsement in science? What differentiates zero from other numbers? Is it real? Are we applying and computing zero appropriately? Is this invention necessary or advantageous? Do we indeed

Dept. of Artificial Intelligence Univ. Nacional de Educación a Distancia (UNED), Spain

Correspondence: Juliot Rives, Dept. of Artificial Intelligence Univ. Nacional de Educación a Distancia (UNED), Spain *e-mail: jrives@dia.uned.es*

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understand what zero means? We aim to answer these questions and explain why zero is delusive to a degree. Our long-term goal is to raise awareness of the unfavorable consequences of overlooking foundational issues in mathematics critical to make headway in physics at the most fundamental level.

Zero's history is "full of intrigue, disguise, and mistaken identity", allegedly culminating in one of humanity's most splendid findings[1]. "The most useful symbol in the world, the naught", is only a recent cultural asset because humans since forever ignored or deemed it nonintuitive and unsettling before its adoption by the Europeans in the 16th century [2]. The late Sumer, Babylonian, Chinese, Maya, and Inca civilizations employed a place-value numeral system where a distinctive mark played the role of the current digit 0 [3]. However, it
was not used alone as the current number zero. Likewise, with minor exceptions, Ancient Egypt, Ancient Greece, and Rome did not recognize zero. Today, we utilize zero daily as an intellectual resource, but the extent to which it is universal and serves scientific purposes is unclear.

As a philosophical conception, zero symbolizes absence or void, whereas as a number or value, it "is an integer representing a quantity amounting to nothing" [4]. In this paper, however, we will not stress the dissimilarity between the abstract meaning and its representation for clarity and eloquence of our exposition. Moreso, some theorists introduce a further disparity between the numerical digit lone and 0 as a vacancy between other symbols of a numeral codeword (e.g., a string of digits) in positional notation (PN) [5]. Since the role of 0 has the same computational essence alone or accompanied, we will also dismiss such a difference. Consequently, we will adopt the widespread assumption that zero designates "a count of null balance" for analysis purposes.

Is zero authentic or fictitious? If the universe (or multiverse) is infinite, zero finds no "space" to exist; else, nothingness "is" out of our finite universe. In the latter case, can zero also live within our finite universe (or multiverse)? This prospect depends strongly on whether our universe is natural, which is unclear [6]. If it is not, zero might be a product of chance. Assuming that our finite universe (or multiverse) is natural due to a direct cause or the outcome of an evolutionary process, we can expect its stuff to be also natural and ask, where is zero?

On the one hand, zero is anything but naturalistic, unlike any other whole number, because it has no physical counterpart. Interactions and transactions always exchange nonzero stuff in physics, chemistry, biology, or sociology. On the other hand, zero is a potential source of human knowledge.

Primates, crows, and bees also catch some aspects of emptiness. However, is this zero-like sense a "perception of absence" or a "lack of perception" [7-9]?

No phenomenology offers evidence that zero is objectively true, suggesting that the animal world plausibly grasps zero as a posteriori knowledge (learned by experience), specifically as the logical complement to the presence of "something" instead of a simple null numbering of elements. We uphold this idea because zero is not innate in humans, as deduced from the history of zero and research projects proving that it is hard to integrate into our mental schemes [10]. Perhaps, we harness this tool supposedly unique to humans in the form of a somewhat different mathematical entity, say the asymptote of a vanishing sequence. For example, is a derivative "identically zero" or the limit of a tangent with no slope? Alternatively, we might take zero as a sheer tiny gap; mind the linear algebra's dual numbers $a+b\varepsilon$ to see how to formally extend a number a by adjoining a multiple b of the nonzero differentiation unit " with vanishing "n for a natural number $\eta>1$ [11].

Our inability to acknowledge that zero is unreachable has delayed advancement in physics. The fact that the "introduction of gravity into quantum field theory appears to spoil their renormalizability and leads to incurable divergences" has finally induced us to explore a fundamental limit to the resolution of spacetime [12]. A Minimal History chronologically examines the various approaches and tentative values, as well as several thought experiments related to Quantum Gravity (QG), concluding that the Planck length restrains the precision of distance measurements, regardless of the observable; to wit, lapse, position, radius, wavelength, string spread, connection distance, slit diameter, crosssection, deformation parameter, surface area, particle size, cell volume, sprinkling density, or lattice resolution [13]. Near the Planck length, we enter a realm of intractable natural indeterminacy, whence unpredictability. Still, a nonzero curvature radius at least makes finding a helpful metric possible.

Information theory (IT) also provides evidence that the essential attributes of a system cannot be nil. Assuming that "every physical quantity, every it, derives its ultimate significance from bits", our rationale is twofold [14].

First, the universe arguably utilizes a positional (logarithmic) system to encode data. A natural code must store numbers efficiently, implementing some form of PN to concentrate more or less information in every numeral's place depending on its position, the attribute, and context [15]. The omnipresence of Benford's law and the amazing Gauss–Kuzmin distribution confirm this presupposition [15-19. "It's a Logarithmic World", where the properties of a system are data spaces to record information on a logarithmic (or harmonic number) scale. Since reserving space for a null feature is wasteful, zero does not need a fundamental representation. In other words, if nature links extent with information univocally, zero is never an encoded number.

Second, "Computation is inevitably done with real physical degrees of freedom, obeying the laws of physics, and using parts available in our actual physical universe" [20]. These parts are open systems with minimal internal activity to support the logarithmically encoded data. Eventually, the system will decode its properties back to the linear world utilizing the exponential function. Because zero is the value of the unit encoded in any base, i.e., $\exp(\log(1)) \equiv 1$, zero is unnecessary in this case, too. Then, the system will share the decoded information via its communication channels with the surrounding systems, and these will react to the system seeking to minimize the information differential. Ultimately, to preserve the entropic gap, the system will encode the new environmental scenario to update the content of its logarithmic data structures. This cycle implies a relentless interplay between temperature and thermodynamical entropy on one side and the coding radix and the information entropy on the other. We will briefly discuss this parallelism between quantum thermodynamics and IT.

Science accepts offhand that zero is a number and infinity is not, but this mindset is somewhat incongruent. We find the symmetry between them in everyday life; depending on the context, we will recognize zero signifying "nothing", "false", "start", "never", "origin", "transmitter", "timelessness", or "annihilation", whereas infinity meaning "everything", "true", "end", "always", "destination", "receiver", "eternity", or "creation". Effectively, these pairs of terms transform into each other by reciprocity. The interdependence between zero and infinity requires us to accommodate or expel them in tandem.

No number set elucidates the zero-infinity polarity as $\mathbb{Q}$ does. A rational number embodies relativity and mutuality between a pair of magnitudes, suggesting that we can conceive it more appropriately as a two-dimensional relational object, and relationships are fundamental to comprehending the cosmos. It turns out that zero can be a numerator but not a denominator, i.e., one of the two dimensions has a point, namely 0, less than the other, so we cannot exchange the two axes, hampering the view of a rational number as a pair of integer components handled on an equal footing. Because zero, like infinity, is generally uncomputable, we propose the numerator never to be zero.

Thus, we will restrict ourselves to the nonzero rational numbers, $\breve{\mathbb{Q}} \equiv \mathbb{Q} - \{0\}$, a fully symmetric set graphically representable on a square grid of nonzero integers $\breve{\mathbb{Z}} \equiv \mathbb{Z} - \{0\}$. For $\breve{\mathbb{Z}}^2$ excludes the axes, we can calculate a reciprocal by swapping the coordinates without exception; for example, $\forall x, y \in \breve{\mathbb{Z}}\ (x, y) \to (y, x)$ or $\forall x, y \in \breve{\mathbb{Z}}\ {}^{x}\!/_{y} \to (\tan(\arctan 2(y, x))$. Moreover, we will explain how an ordered list of nonzero rationals can unambiguously represent a nonzero algebraic number. "Algebraic numbers, which are a generalization of rational numbers" [23], are the "algebraic closure" of $\mathbb{Q}$ and represent the constructive version of the set of complex numbers $\mathbb{C}$. Focussing on the algebraic numbers punctured at the origin, $\breve{A} \equiv \breve{A} - \{0\}$, we maximize computability without forfeiting the limiting values 0 and ∞.

The shade of the nonzero integer, rational and algebraic numbers is long. An irreducible double ratio $\frac{a/c}{b/d}$ is a rectangle in the $\breve{\mathbb{Z}}^2$ lattice. The rectangles of unit size $(ad - bc = 1)$ form the modular group, the group of linear fractional transformations (LFT) $(az+b)/(cz+d)$ acting on the upper half of the $\breve{A}$ plane $(z \in \breve{A}\ and\ \Im[z] > 0)$, , where $\{a, b, c, d\} \in \breve{\mathbb{Z}}$. Instead, if $\{a, b, c, d\} \in \breve{\mathbb{Z}}$, we obtain the Möbius group, the group of LFT that maps circles to circles preserving angles between crossing or touching circles throughout $\breve{A}$. Specifically, for any three points $\{a, b, c, d\} \in \breve{\mathbb{Z}}$, there is a Möbius map $f\angle$ that takes $z \in \breve{A}\ to\ f\angle(z) \in \breve{A}, A to - 1,\ and\ C\ to\ \infty$

Möbius transformations are the most straightforward examples of conformal transformations, mappings or diffeomorphisms (smooth deformations) that do not alter angles within a point's neighborhood but possibly distort extent or curvature. Although many conformal functions are not Möbius in two dimensions, it turns out that an exponential map is locally a Möbius map conformal at any point of A. Moreover, the exponential's inverse function is also a conformal map within the principal branch. The give-and-take between the $base-\alpha$ logarithm $\log_\alpha z$ and power α^z maps, with $\alpha \neq 0$, provides a universal method of information encoding-decoding. The Lie group-algebra correspondence and the Laplace direct-inverse transform undertake the same two-way procedure.

Conformality is principally a local property generalizable over rings; all conformal groups are local Lie groups represented by a class of LFT. Conformal maps preserve conic forms and angles between intersecting conics through the almighty cross-ratio, another double ratio of differences between four points or vectors. This rational construct is the paradigm of regulating zero and infinity in the same framework and a building block of conformal maps. Taking the crossratio's logarithm, a subset of the ring's domain becomes a coding space, a region of negative curvature characterized by a coupling factor between distance and distortion of angles. Within a coding space, a geodesic line is a conic, the calculation of distances uses hyperbolic geometry, and the zeroes and poles of an LFT are limiting values of (vanishing and diverging) sequences, all of which allows relaxing the notion of proximity, opening the door to quantum nonlocality. Physically, conformality is closely related to randomness in two dimensions and causality and scale invariance in all dimensions. Because of the fundamental curvature factor that blocks flatness, i.e., zero, conformally compactified spaces are at the heart of Quantum Field Theory (QFT) and many gravitational theories, representing one of the avenues to a robust theory of QG.

The following sections delve into zero from different stances. First, we introduce the main trouble with zero; it is inseparable from infinity because both comprise the same fundamental duality. Then, we explain how zero causes havoc on General Relativity (GR) and QFT, setting off an unsolved crisis that QG will someday overcome. Zero fictitiousness leads to the prospect of a universal minimal length. We analyze the role of zero in IT; physicality is computability, and zero is uncomputable. We posit that change is equivalent to information flow, both unstoppable. Because information coding must be efficient, the universe likely uses PN; unlike standard PN, bijective PN creates zero-free and unique numeral representations that boost productivity. We scrutinize Set Theory (ST) concerning the signification of emptiness, finding that the null class is a waste stone, an obstacle for the number sets $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$, et cetera. Next, we explain the paramount significance of a "natural" ratio, an irreducible fraction between nonzero magnitudes. The irregularities and inconsistencies that the number zero provokes A to the rationals resound through the $\mathbb{R}-\textit{based}$ number sets, so we bet that $\breve{\mathbb{Q}}$ is the primordial number system and instead of $\mathbb{C}$ is the number system upon which computability grows. Then, we point out where and how to apply the nonzero "polyrational" numbers to obtain versatile mathematical structures with a high impact on physics, e.g., LFT of the Möbius, Laguerre, and Minkowski planes. Finally, we review the cross-ratio, a prominent rational construct that gives rise to generalized conformal transformations and the idea of coding space. In the concluding remarks, we underline that zero's involvement in science is not innocuous and that we have already devoted too much effort.

This work has taught us that zero is a foreign construct whose utilization separated from infinity is deceptive. We have also discovered that zero and infinity are dual beables, so we can throw light on how to respond to the following existential question; "Lump together literally everything contained in ultimate reality. Now call it all by the simple name 'Something'. Why is there 'Something' rather than 'Nothing'?" In picturesque language, the sum of (an infinite amount of) "nothing" (absolute nonexistence, 0) and (a finite amount of) "everything" (complete existence, ∞) constitutes the primeval duality principle (valid for the universe, multiverse, or metaverse) pivoting on the identity (relative nonexistence or existence, 1) as a brute fact. In turn, multiples of this unit ("something") appear in two versions, the encoded or intensive arrangement, which nature keeps on a logarithmic scale, and the decoded or extensive one, which the cosmos exhibits on a linear scale. The infinite nothingness, i.e., zero, "lives" on the boundary of the encoded world as a potential unit, namely log (1), lingering where spacetime begins (see Fig. 1).

The troubles with zero

This subsection anticipates that the main troubles with zero are its unreality, irrationality, and inseparability from infinity.

Zero is a recent creation "destined to become the turning point in a development without which the progress of modern science, industry, or commerce is inconceivable." Theoretically, "the integer immediately preceding 1" is the additive identity of the mightiest algebraic structures. "Zero's absence would stunt the growth of mathematics". Nowadays, most people think zero is a number like others. Many mathematicians will contend that saying the contrary is a sad mistake, and some will even discredit those who impugn the verisimilitude of zero. Not so fast! Zero is peculiar and causes complications immediately. For instance, in Frege's view, "0 is the number which applies to the concept unequal-to-itself". Mathematics deals with it ad hoc; zero is an even number (the evenest number!), but the only function is odd and even. It produces disturbing dichotomies, too; zero is a positive and negative value, neither a prime number nor a composite number.

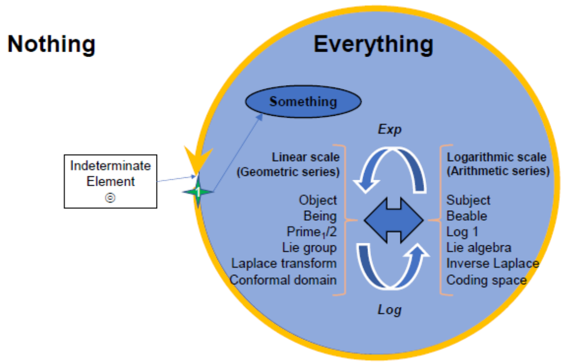

Figure 1) *Nothing, Everything, Unit, Something, the two natural scales, and the "natural projective line" in yellow with the indeterminate element* ⊙

Detractors of zero affirm that it is not cardinal or ordinal and cannot even be an object's property. For example, the class "apple" disappears if we have no apples and the attribute "age" is not in a birth certificate. Zero is even an unneeded number. Arithmetical operations with the number zero are spurious. For instance, consider the arithmetic mean calculation of the number of things in a group of bags; should we regard the empty ones? If affirmative, the answer is the total number of pieces in bags with content divided by the sum of empty bags and bags with content; zero is not involved in the calculation. We can write the equation $x-y-z=0$ as $x-y+z$. Mathematical definitions and theorems largely allude to nonzero numbers, entities, or solutions because the scope of applicability of zero is usually bland or negligible.

Zero uselessness appears not only in algebra (zero number, vector, matrix, tensor, ring, et cetera). Functional analysis, ST, and Category Theory consider the zero function, the empty set, and the empty categorical sum as additive constants, despite being trivial elements. In knot theory, a zero knot (unknot) is topologically equivalent to a circle, and any two closed curves in three-dimensional space with a linking number zero are unlinked. In computer science, the zero Turing degree is the equivalence class that contains all the algorithmically solvable sets. All these zeroes are mathematical sugar, flimsy stuff denoting the same idea of "nothing" addressed to a specific branch in mathematics; to wit, no number, no vector, no matrix, no tensor, no ring, no function, no set, no categorical sum, no knot, no link, and no uncomputability. We will only examine the empty set among these zero objects because of its impact on $\mathbb{R}$.

Furthermore, some facts sustain that zero is beyond our sensory and processing faculty, an imaginary and often false friend. According to Richardson's undecidability results, determining whether a simple expression that involves polynomials and the sine function vanishes is theoretically unsolvable, not to mention in practice. If "the coordinate system remains as the necessary residue of the ego-extinction", the geometry of the origin (zero) is mute, nay, impregnable subjectivity. The probability of encountering nothing in the cosmos is so infinitesimally small that it is de facto nil; "it is impossible for there to be nothing", nothing is physically outside our universe, and even debatable that nada existed with exclusiveness before the Big Bang. The following section will further reason that zero is unobservable and unbelievable.

We have already introduced the contention that zero is neither naturalistic nor motivated by dependable scientific criteria. Despite living in a quantum universe where information is physical, possesses a discrete character, and serves computational purposes, our central mathematical tools give off the continuum's aroma. The immaculate real numbers, "true monsters", govern the n-dimensional Euclidean space, a mythical realm of maximum density having no room for holes. However illogical as it can be, we need the continuum because its perfection provides us with protection, which is again not a scientific reason but a psychological one. Zero satisfies our longing for (mathematical) "connected compactness". Our fear of empty spaces and Newton, Cantor, and Poincaré's long shade have been barriers to unmasking zero.

Nothing points to nothingness in IT. A system's information (lack of entropy) correlates with its internal thermodynamical activity, which depends on the system's quantum mechanical degrees of freedom. These reify the system's properties to sign the mantra "quantum physics is an elementary theory of information". The system balances when its entropy reaches a maximum. However, given that a generic quantum system is "contextual", excepting possibly the universe, and never definitely hermetic, the environment causes decoherence interacting with the system's degrees of freedom that bear its intensional information. The fluctuation of the corresponding microstates permanently unchains processes that take the system out of equilibrium, permeating the universe with a renewed distribution of information. This emergent thermodynamics indicates that nature associates "transformation" with "information exchange". The present constantly varies through interactions, so we always perceive a series of material effects and a notable phenomenology linked with information currents. Since propagating null information is nonsensical, inaction is equivalent to a lack of information flow. Because inactivity is never absolute (e.g., like a time crystal reveals), information flow is incessant.

The notion of infinity as the limit of a diverging sequence sounds logical, despite considering its twin, zero, not a limit value but a number. This inconsonance has negative consequences in thought, algebra, and calculus. Perhaps, the most significant proof of the intimate fellowship between zero and infinity comes from logic. According to Gödel's theorems, we can find unprovable statements in a contradiction-free and enumerable logical system. Vice versa, if such a system is complete (the truth or falsity of any axiomatically-constructed piece of knowledge is provable within the system), we can find incongruencies. Thus, a system requires null or uncountably infinite cardinality to reach axiomatical completeness and consistency simultaneously; logical insufficiency and uncomputability are crucial properties that unite zero and infinity.

In mathematical physics, the Dirac delta "function" is a prime object that masters the zero-infinity duality. It serves as a distribution gateway to the continuum, and vice versa, from the reals to discontinuous math. Its output vanishes everywhere except when the domain value is zero, whereas its integral over $\mathbb{R}$ equals one. This function is preeminent in physics because it allows calculating a system's response to the input's total, i.e., as an impulse (e.g., an instantaneous collision) or potential (e.g., a point mass), escaping from details, the transition processes or layers at lower integrative levels.

Is the Dirac delta distribution physical? No, it is only the abstraction of an immeasurable perturbation extended over no period. We will justify that a being's existence needs to take up nonzero spacetime, and a change requires nonzero spacetime to perform. Even algebraically, we must reject the utter punctuality of zero and infinity; "with the extension of variable magnitudes to the infinitely small and infinitely large, mathematics, usually so strictly ethical, fell from grace". We understand the Dirac delta better as the limit rendered by a finite sequence of distributions $\delta_\alpha(x) = 2\big/\left(\sqrt{\pi\,\alpha^2}\,\exp((x/\alpha)^2)\right)$, with $\alpha \in \breve{\mathbb{Q}}$. As α dissolves, the bumps get sharper and sharper, concentrating on a prong or pin at the origin. Thus, we recover the mathematical and physical sense of this duality of beables operating as a logarithmic scale's communicating vessels; when x approaches the origin, $\delta_{\alpha \to \log 1}(x) \propto 1/|\alpha|$ diverges, whereas $\delta_{\alpha \to 1/\log 1}(x) \propto 1/|\alpha|$ vanishes. Besides the Dirac delta distribution, we can find in other branches of mathematics, such as projective geometry, and physics, such as cosmology, examples of the symbiotic bond between the prospect, not the actuality, of zero and infinity providing us with a renewed vision of the universe.

Combining both concepts leads to formidable mathematical tools and physical models. Since $\{0,\infty\} \notin [\breve{A}$, the ratio z/α exists $\forall\,\{\alpha, z\} \in \breve{A}$; $\lim_{|\alpha| \to \log 1} z/\alpha$ diverges, and $\lim_{\alpha \to 1/\log 1} z/\alpha$ vanishes. Similarly, line curvatures in the $\breve{A}$ plane are tacitly nonzero; specifically, rays are not straight. $\breve{A}$ interprets infinity as "the last point" observed from the origin at the end of every line, wrapping around a giant circle, where zero and infinity are antipodal points (180 degrees apart). This perspective allows us to identify the zeroes and poles of a modular or Möbius transformation with limiting values like 0 and ∞, around which we can define an arbitrary open region where the mapping behaves without exception regularly. Indeed, the modular group's action is the automorphism group of the "real-algebraic projective line" (mapping real-algebraic numbers to themselves and rationals to themselves), while the action of the Möbius map swaps the "algebraic projective line".

Like a Möbius map, a conformal map (e.g., the exponential function) leaves angles unchanged around every domain point or singularity and stretches equally in all directions, so the nearer a figure is, the better its shape is preserved. We also say that a manifold is conformally flat around a point if a diffeomorphism maps its neighborhood onto flat space, meaning that the angle between intersecting geodesics rather than distances is what locally matters. The proportionality (conformal) factor between the manifold and the flat metrics is physically an exponential function of a scalar potential that never dissipates. Therefore, conformality generally makes proximity a relative condition, allowing us naturally to deal with the small and the large interchangeably while fleeing flat (and infinitely curved) spaces.

Although we use zero as an ordinary number of a linear scale, the Dirac delta distribution, the Laplace transform, the Möbius group, the cross-ratio, and the conformal transformations point to handling zero and infinity together as unreachable binary limits of an algebraic sequence. This belief is evident to Descartes, who argues that "There is no imaginable extension which is so great that we cannot understand the possibility of an even greater one, and so we shall describe it as Indefinite. Again, however many parts a body is divided into, each of the parts can still be understood to be divisible, and so we shall hold that quantity is indefinitely divisible." Moreover, we must take as many steps forward from zero to get infinity as steps back from infinity to get zero. This symmetry reflects that these dual antagonists or complementary mates of a number set are two sides of the same coin.

ZERO IN PHYSICS

On why physics must distinguish the arbitrarily small from the incommensurably small and believe in a minimal natural extent.

Elusiveness

No clue so far demonstrates that zero is physical. The Third Law of Thermodynamics stays current; we cannot reduce the entropy of a closed system to its final zero. A particle's lifetime can be very near but never precisely naught. A spectral line has nonzero linewidth and extends over a range of frequencies, not a single frequency. According to Electromagnetic Theory, a photon with zero wavelength would have boundless energy. Although a photon has no rest mass, nobody has ever gauged it strictly zero. Indeed, some theoretical and exploratory studies indicate that the photon's rest mass can be nonzero. Experimentally, a strong force's gluon rest mass is < 1:3MeV. Is it an insignificant value? The answer is troublesome because "it is unclear whether the massless theory is really the limit of the massive one." Nobody has ever measured a body occupying no space, infinite mass density, or infinite charge density. The cosmological curvature parameter of the universe is very close to but not "identically zero".

Nil reckoning is intricately related to the notions of "continuum" and "infinitesimal", symbolically represented as $1/\infty$. Euclidean geometry strongly influenced Borrow and his pupil Newton. In addition, they focused on movement, which seems to be continuous. However, classical, Newtonian physics does not leave space to indetermination, against today's evidence that we cannot precisely localize objects in the spacetime fabric. Since then, calculus developers used the reals rather sloppily.

Wallis introduced the modern concept of infinitesimal, later consolidated by Leibniz and Nieuwentijdt, defined as an indivisible quantity "with arbitrarily small but nonzero width". Nearly two centuries later, Cantor hated the theory of infinitesimals, which he branded as the "cholera bacillus" of mathematics, and the idea of getting them mixed with his theory of transfinite sets; from his angle, on no account can be infinity and infinitesimal understood as inverse of each other, despite traditionally, if α is infinitesimal for b, b is infinite for α.

Notwithstanding, inspired by Fechner's work on how a psychological sensation relates to the physical intensity of a stimulus, Poincaré was "who set out most clearly where debates about the real numbers were to divide mathematicians and scientists." Poincaré said (Mathematical Magnitude and Experiment in) that "the rough results of the experiments may be [...] regarded as the formula of the physical continuum." Poincaré accepted, used, and promoted the infinitesimals in what is presently known as metrology.

Regardless of how Cantor or Poincaré came to their conclusions, the fact is that infinitesimals evolved as quantities stemming from the $\mathbb{R}$ continuum examined in the field of the minimal, potentially perceptible but indistinguishable from zero. Thus, observables became a prime mover of this ultra-dense medium, although, ironically, a measurement is only accurate and precise to a degree.

The unobservability of zero remains an unsolvable pragmatical problem. For instance, are rounding errors of the initial conditions or results in any real-life numerical problem evitable? How can we ensure that an observed body's property, e.g., rotational speed, is zero? We must measure infinitely rigorously to prove a value is nil. Zero is unreachable except for the incidental zero level of the human-designed scales, such as the Julian calendar or Celsius temperatures. In general, "An infinite precise statement that there is zero change [...] is completely untestable" because "We can never measure an infinity or a zero". Additionally, the origin is invisible when we behave as observers. From our outlook, we cannot see ourselves or beyond the last visible number; we only capture what is subjectively at distances 1, 2, 3, ..., N, conditioned by our physical limitations.

Nonetheless, the reader can object that we usually run into zero as a solution or singularity of a scientific or engineering problem. As a solution, zero is either a trivial result or an outcome with a negligible magnitude. Thus, it should come with no physical units and would not strictly require a number representation. As a singularity, zero deserves further analysis.

$\mathbb{R}$-based laws of physics always break down at their conjoined singularities. Informally, a singularity is a domain value that involves some blank magnitude causing the law to behave strangely. A zero at the wrong place yields an unmanageable infinite quantity that resists an appropriate interpretation and impedes deriving new results. True singularities are hypothetically possible but also the primary source of criticism of GR. Specifically, the Penrose-Hawking singularity theorems consider singularities inescapable, although they are invisible to us outside of a black hole or even to an infalling observer. If singularities were genuine, they would perhaps reside within an elementary particle, as virtual particles, a black hole, or any other elusive form, a possibility that contrasts with their historical lack of evidence.

The surge of QFT seemed once to overcome the theoretical problem. Even though a field's value can be zero everywhere classically, our quantum universe cannot obliterate a field's uncertainty. According to QFT, which rules the Standard Model of Particle physics, quantum fields are quantities of critical features of spacetime that at every point continually fluctuate in their lowest energy state; the vacuum of a quantum field is unstable, vibrating around a ground level represented by a positive minimum-energy configuration. QFT can manage these fields separately by employing regularization and renormalization techniques based on fitting parameters, usually cutoff regulators, that attenuate singularities of observables and control self-interaction loops to absorb divergences, giving rise to "effective theories". For instance, adjustable infinitesimal distances allow calculating the particles' specific mass and charge.

Nevertheless, this "quantum fuzziness" is "not enough to deal with the $1/r^2$ singularity in the gravitational force." In other words, an effective theory of QG can be consistent but lacks predictive power because new infinities arise continually. To worsen things, the aggregation and integration of the so-called vacuum's zero-point energy density oscillations associated with every quantum field contribute to the cosmological constant with a magnitude whose theoretical value is ridiculously greater than the experimental data; the observed value of the cosmological constant beats zero slightly but significantly, begetting a mild curvature of spacetime.

The experimental elusiveness of zero and the fundamental incongruency of a singularity are enormous obstacles urging physicists to devise a theory of gravity in conformity with GR and Quantum Theory. Finitism is a must because "When Bohr tells us that quantum theory gives us the only objective description of nature of which one can possibly conceive, is he not also telling us that no description can make sense which is not founded upon the finite? [...] Encounter with the quantum has taught us, however, that we acquire our knowledge in bits; that the continuum is forever beyond our reach." This reasoning suggests a route to arrive at a QG theory that combines the functional definition of quantum action, the feasibility of countable discreteness, and a constructive and practical concept of probability.

The unifying thread of this approach is Feynman's Path Integral formulation of the Principle of Stationary Action. The probability of an event is a mean of the occurrence probabilities for the finitely countable possible ways, i.e., paths, to fulfill the event, e.g., a trajectory between two points or a transformation between two quantum objects or states. Moreover, a path's probability is a mean of the intermediate quantum objects or processes it transits through and depends on the curvature of a manifold of these transition quantum states; negative curvature (regarding the mean) would decrease occurrence probability, while positive curvature would be attractive. Specifically, when a body computes a Sum over Histories, the different nonzero wave-like amplitudes integrated over a curved space will reinforce or cancel each other to produce the quantummechanical course that the particle depicts. This least-changing action represents an asymptotic stability limit, not a concrete static path; absolute steadiness is unphysical.

Irrespective of a body's action, many observable quantities of interest in QFT can only take values in discrete sets of integers or half-integers. Discrete physics, especially discrete spacetime, wonders if the premises of $\mathbb{R}-based$ laws and principles are mistaken. The question is pertinent because the reification of zero and infinity is a longstanding controversy, pending no matter how much physicists think these are well-settled mathematical concepts. $\mathbb{R}$ is infertile and "unphysical", thwarting the most elementary change no matter how long the universe could spend processing it. Besides, a minimal size integrated into fundamental discreteness understandably exists, as we will analyze in the following subsection.

This latter issue links to a crucial question; how long is "now"? We do not know what "now" is and can only offer partial responses. Classically, a "Timing on the scale of tens of milliseconds to a few seconds" protracts the subjective stint between past and future if we admit that our consciousness emerges from the activity of neuronal nodes. Concerning Special Relativity (SR), how can a spacetime event have different non-null properties or bear a change? There must be fundamental tensed facts or relations; "if the direction of time is given by the direction of causation, and spacetime points themselves stand in causal relations, then time is, as one naturally thinks of it as being, an all-pervasive feature of the world." Likewise, some events are in our past cone by the time we watch them, and we are in the past cone of those observers with whom we can communicate. Thus, part of the past and future exist, extending behind and in front of us. This thickness, compatible with an extinct past and an unborn future, guarantees a minimum flow of information and sidesteps many paradoxical situations of the block universe. At a fundamental level, the Margolus-Levitin theorem states that a quantum system of average energy E needs at least a time $\hbar\pi/2E$ to transit between orthogonal states. In general, neither a computational process can be instantaneous nor a time interval can have zero duration. Therefore, the present is feasibly pure "becoming" conformed by tics separated by Planck time multiples.

Zero is also strange when we focus on an interaction's inherent uncertainty. Empirical results come with two essential sources of error. On the one hand, the observer effect tells us that measurement modifies the examined system, limiting "the fineness of our powers of observation and the smallness of the accompanying disturbance". We notice this prod's disruption as a measuring apparatus's systematic error, particularly as nonzero decoherence if the observed system is manifestly quantum, closely related to the measurement problem. On the other hand, Heisenberg's Uncertainty Principle, a law independent of the Schrödinger equation, claims a fundamental boundary to how well we can predict the values for a pair of complementary (wave-like) system variables. Note that the initial conditions of a transformation cannot be thoroughly specified.

Even if they were, it would be impossible to anticipate the exact value of either of the conjugate properties (Fourier transforms of one another), ensuring a minimum threshold for the product of their dispersion. Then, Ozawa's inequality aggregates the observer effect's systematic error to the Uncertainty Principle's statistical error. Fujikawa's relation combines these errors to state that the product of the inaccuracy of one variable and the subsequent fluctuation in the other is nonzero, surpassing the modulus of the commutator's expectation value of the corresponding observable operators. The idea to bear in mind is that not only are dual properties dependent on each other, but neither can disappear, which supports the thesis that zero is alien.

Zero, as currently utilized, might be unnecessary to construct most physics. Science should deal with zero as a beable projecting a property of quantitative character instead of an actual concrete value. From this standpoint, we can judge zero as a hole perfect in the abstraction of nothingness, the immaterial, the unfinished, the imminent, "the unknown" (Hindus' "sunya"), "the unthinkable", an "inaccessible number", or rather an "undetermined possibility". However, this sheer non-measurable potential can have a nonzero probability of occurrence with implications in our view of the cosmos (see the probability mass distribution of the integers.)

Extent

The old discussion about whether nature is continuous or discrete takes us to the Quest for Fundamental Length in Modern Physics. On the one hand, the structure for a continuous fabric of spacetime and matter is liable to be inexhaustible, paving the way to zero and infinity but demanding limitless resources. On the other hand, neither philosophy nor test data seem to impede fundamental discreteness or a minimum physical extent.

Descartes thought that "the nature of a body consists just in extension" (2:4) and "nothingness cannot have any extension" (2:18), albeit "a body can be divided indefinitely" (1:26). In Hume's view, "no finite extension is infinitely divisible", a statement that embraces space, time, and abstractions (Of the Ideas of Space and Time, Book 1). In string theory, one cannot compress a circle below a minimal stretch given a fundamental string tension, which suggests that "smaller distances are not there". Smolin argues that nature cannot contract distances ad infinitum. Hossenfelder warns that there might not be a minimal length, just a minimal length scale, as a lower bound on the product of spatial and temporal extensions, for instance.

Furthermore, a minimal length scale would not necessarily appear as a spatial resolution limit but could be noticeable at any layer. If this is the case, the recursive depth could be a predetermined value or limited to several possible values given by an inwards quantum number, much as the location of a particle confines itself to a countable number of spatial positions. IT leads to a similar judgment; "Whether the inevitable limit on precision is simply a limit on the number of bits that can be invoked in physics or is more complex and statistical is unclear". Whether absolute or relative, the deepest zoom-in would ultimately reach a discontinuity, an indivisible dull spot, an uncertainty bubble, or a demarcated lattice cell.

Mind that discreteness and finite resolution of spacetime are often mistakenly coupled. Discreteness is insufficient to justify an absolute minimum length because we could always find a lower and lower scale as a lattice spacing approaches zero (e.g., as a fractal), and vice versa, we can have a structural resolution limit without discreteness (e.g., in String Theory). So, to guarantee that natural extents are nonzero, a quantum theory of spacetime should state an additional assumption or deduce that a fuzzy or crisp granularity exists. This approach is the most common when research unites GR and QFT. String theory, Loop Quantum Gravity, Asymptotically Safe Gravity, and non-commutative geometries, nowadays the most renowned roads to QG, feature a minimal length scale. Ultimately, the physicality of a minimum observable extent weakens the notions of locality and coherence, a model for spacetime foam associates the minimal length scale with decoherence in terms of nonlocal interactions to explain quantum gravitational effects.

Another issue is whether the minimum natural distance is the Planck length. We do not know if the Planck scale will play an essential role in a robust theory of QG or whether it will involve another minimal size, surely invariant. The Planck length stands for the diameter of the minor possible black hole, i.e., where a mass' Compton wavelength and Schwarzschild radius coincide, or the minimum ball of spacetime accuracy. Consequently, GR and QFT interact intensely at the Planck scale to produce gravitation. Some physicists argue that putting the Planck length and the minimal length on the same level implies modifying SR; otherwise, "Planck-pixels" would violate Lorentz symmetry, and hence we could observe lengths contracted below the Planck supposed boundary from some inertial frame, and even electromagnetic waves squished boundlessly. Alternatively, we can turn to de Sitter Invariant Special Relativity to preserve observer-invariance. This theory exploits the constant length parameter that the de Sitter group naturally incorporates, which is the order of the Planck length. Regardless, rest-frame equivalence and a minimum size do not clash. We do not negate physics beyond the Planck scale, only that the indetermination would be so significant that thinking of the distance between two points or a spacetime topology no longer would make sense.

What is the role of the Planck length, assuming that it is a universal minimum, in our (at least) four-dimensional universe? For something to exist, it must have a nonzero quantum value of surface area (discrete multiples of the Planck length squared) and volume (discrete multiples of the Planck length cubed). This rationale leads many physicists to conceive spacetime as a network of fixed quantum states of areas and volumes that evolves sequentially to rearrange connectivity. Such a network would fluctuate as a foam if arrayed in multiple tiny ever-changing regions. This model can explain how spacetime grows but not, for example, how time dilates or a process develops. The time dimension should preexist mated to the spatial regions, furnishing the network instances with a nonzero temporality to enable the relational cohabitation of its elements. Our universe indeed hosts beings with nonzero hypervolume.

Moreover, the patch of spacetime occupied by every being in the universe should be large enough to allow local interaction; otherwise, the being would be incapable of yielding a minor action, staying unconnected from the presence of other bodies. This view points to the Planck length as the minimum distance (or equivalently, the Planck time as the minimum duration) needed to carry out a move, deformation, or modification to reality in general. Since the Planck constant $6\times10^{-34}\,N.m.s$ is the minimum force integrable over a spacetime interval in one dimension, we can formulate the Uncertainty Principle as $\sigma_{st}\sigma_F \geq \hbar/2$, where σ_{st} and σ_F are the standard deviations from the length-time interval and force means. So, the more precise the strength applied to a body, the less predictable its position from initial conditions. Vice versa, the narrower the interval for action, the stronger the push to generate a change, i.e., our endeavor will be in vain if the push span is too short. Even another way to express this idea is that, in one dimension, a transformation requires the capacity of a "natural" quantity of work $N.m$ and impulse $N.s$. Therefore, a being is necessarily fourdimensional if it interacts with its environment in all three spatial dimensions. Zero-dimensional elements are passive structural pieces to make up a being; hence their existence is only potential.

ZERO INFORMATION

On why zero has no information content and needs no repressentation.

Gap

Adopting a minimal length scale is also an old idea of quantum IT to provide a minimal discrete model of QG. A minimal length as a fundamental level of resolution implies a finite bandwidth and density of degrees of freedom for IT-reconstructable physical fields, agreeing with the fact that "theories formulated on a differentiable spacetime manifold can be completely equivalent to lattice theories" even when the radius of curvature is minuscule. At the opposite end, spacetime tends to flatten on large scales. Since the strength of the correlations is inversely proportional to the distance, quantum entanglement phenomenology progressively evaporates as the universe expands, albeit not entirely. Quantum IT tells us that we gain detail in the observed object by descending enough to the appropriate layer of reality. The other way around, we embrace a sense of continuity by ascending sufficiently, i.e., scaling up to a suitable coarse grain of spacetime. Validation of the quantum-information linkage at both ends might mean that a natural information flow ranges between a minimum and maximum, and spacetime is a discrete computational framework handling neither zeroes nor infinities.

The physicality of information has been commonplace in scientific research during the last century. In this vein, we claim that something with no quantummechanical degrees of freedom constitutes an information gap. Can an informational system encode, maintain, and decode data without physical support? Can information gathered, transformed, and disseminated in material media have an immaterial substratum? Can data be ethereal? Whatever that information is, it is "encoded in the state of a physical system". Besides, why would a quantum system keep matter or energy not serving an informational purpose? What good would an artificial interface between matter or energy and information be at a fundamental level? Can moving energy be anything other than information flow? "The particle passing by you is really a bit, or group of bits, moving along a set of interlinked logic units", or simply going through a medium. In this bidirectional relationship between physical process and information transformation, matter tells information how to organize, and information tells matter how to proceed.

A simple but astonishing fact is that the unit of classical information is indivisible. The transformation of a qubit also gives us a classical binary digit as output. If the information we can obtain from a system is what we still ignore about its properties, we can also measure a system's uncertainty reduction in bit units. Thus, information (and dually uncertainty) is a discrete physical quality impossible to cancel because the most elemental computational system provides us with one bit of information. Moreover, we cannot count the information that the hidden variables of a system transmit. If incompleteness is not the only source of uncertainty, what information can we quantify (and quantize)? Can we discern between classes of (nonzero) uncertainty? We present the following list of significant steps in this exertion.

The Hartley function measures the difficulty of choosing among N alternatives as log N; we only find specificity, i.e., no complication at all, in single sets when there is no choice (N = 1), but genuine systems entail generality implemented as degrees of freedom ($N \geq 2$). Von Neumann entropy measures the extent to which the eigenstates of a quantum-mechanical system interact with each other, and the entropy of entanglement calculates the correlation between a bipartition of the system. The reduced density matrix for the composite's subsystems comprises a quantum hybrid state where partway separability preponderates. Maximum entropy corresponds to indistinguishable eigenstates of this matrix, so we cannot correctly speak of subsystems. In contrast, zero entropy signals a pure quantum state where the subsystems are uncorrelated, i.e., we have no system but the set of subsystems. Shannon's Theory of Communication meant a gigantic leap to defining the essential elements of a communication system. A message's (expected) information content depends on how the codewords' occurrence probabilities friction with each other; the information a message conveys cannot be nada, but log 2 at least. Zadeh's fuzzy logic manages knowledge clearness via vague membership to sets or properties, showing that truth is not absolute. In summary, "nonspecificity", "confusion", "conflict", and "fuzziness" are the informational aspects that we can measure in bits and never disappear.

At first sight, the resemblance between information entropy and thermodynamical entropy is unavoidable, but physicists found it opaque how these measures can engage with each other. In principle, the latter evaluates an object's energy unable to do work. Thermal energy, under steady conditions, is fixed, untransferable, with no transformation capability, while outside the equilibrium, it moves as heat and can be productive. However, we cannot cool a body to absolute temperature zero or nullify its heat radiation. If the information is physical and the laws of thermodynamics rule information, thermal energy, heat, work, and temperature are comparable with information content, flow, communication, and PN radix (see following subsection), and zero is untouchable from either perspective. Let us recall a squad of masterworks that corroborate this angle.

Leó Szilárd's engine is a refinement of the famous Maxwell's demon scenario. Szilárd's demon exchanges information by mechanical work; as a corollary, no work implies no flux of information. Erwin Schrödinger's negative entropy measures the statistical divergence from normality (a Gaussian signal), i.e., a capacity for entropy increase that parallels the thermodynamic potential by which we can increase the entropy of the system without changing its internal energy or augmenting its volume. Since the entropy of an isolated system spontaneously evolving cannot decrease, we can infer that its negentropy (or free enthalpy) never runs out, so the perfect normal distribution and the absolute chemical equilibrium are inaccessible. Léon Brillouin states that for information to be stored, processed, transmitted, and retrieved factually, it must obey the principles and laws of physics. More specifically, changing a bit value of a system at absolute temperature T requires at least $k_B T \log_2 2$ joules, where k_B is the Boltzmann constant; no change in the properties of a system is possible without consuming energy.

Later, Edwin T. Jaynes built a crucial bridge between statistical physics and IT (or equivalently between conserved average quantities and their structural symmetries), clarifying why thermodynamic and information entropies are equivalent, especially for the limiting case of a system at equilibrium. We can identify a system's entropy with the microstate information lost when one observes it macroscopically; while this information can be associated with free energy, detailed organizational information stays as nonusable energy by the Second Law of Thermodynamics. This view agrees that a being balances one type of energy against the other throughout its existence, neither ever becoming zero. Similarly, from the perspective of IT, when we observe a system, we gather information about its global parameters such as temperature, pressure, and volume; mere observation evades a total lack of knowledge. Vice versa, if we narrow the central focus of attention, we gain information about a system's internal structure, while holistic information gets imprecise. Thus, a being handles only imperfect information and struggles between the specific and the general. In particular, a living being is never wholly ignorant or aware of its environment.

This argument is also why we can strictly discard neither the null nor the alternative hypothesis in statistics. The most we can do is to minimize presumptions about a system, such as configurational preconditions and restrictions. The maximum entropy principle, or minimal information principle, claims that "the probability distribution appropriate to a given state of information is one that maximizes entropy subject to the constraints imposed by the information given", a bet on the less informative a priori distribution possible and hence the most sensible of all "fair" random a priori distributions. This principle "affords a rule of inductive inference of the widest generality", a giant step toward formalizing Occam's razor as a rule, for instance, to choose a prior probability distribution in Bayesian thinking. We always believe something before considering the evidence, i.e., presuming nada is impossible. Likewise, the posterior distribution is never thoroughly informative; we cannot cancel its information entropy. These are new signs that we live in a countable and orderly world.

Information interchange, discussed above, preserves heat. However, what can we learn from a dissipative process? Information loss is a physical process intricately linked with irreversibility or unpredictability, so a being must constantly acquire new information to postpone death. Landauer's erasure principle tells that any phase-space compressing irreversible computation, such as erasing information from memory, "would result in a minimal nonzero amount of work converted into heat dumped into the environment and a corresponding rise in entropy". Then, a computational system continually creates entropy, and constant entropy means no computation takes place.

Can a system have no entropy? Defining "entropy" as the logarithm of the number N of equally probable microstates of a system, its entropy vanishes when the system has only one microstate, i.e., one component (N = 1) with no freedom degrees, whence closed; it would not be a system per se, but just a beable, if any. Can a system be simplistic in full? Suppose we measure the complexity of a being by the length of the computer program in a predetermined formal programming language that produces such a system as output. In that case, the only possibility of possessing no complexity is no entropy, i.e., no information content. Thus, the representation of the void, i.e., no freedom and no complexity, is log 1. Moreover, speaking of entropy and complexity is nonsense when a system evaporates, which mathematically translates as the undefined logarithm of zero.

We finish this subsection by commenting on a few paramount investigations that have consolidated the thermodynamics-information connection. Shannon's Source Coding Theorem limits the reliability of data coding; an ordinary (noisy) channel cannot transfer error-free data if the codewords are finite in length, meaning that error is natural. Bell's theorem unveils that locality is a condition vaguer than classically assumed; we had better regard phenomena as neither absolute local nor global. IT Evans' fluctuation theorem generalizes the Second Law of Thermodynamics to microscopic systems, with a nonzero probability that their entropy might spontaneously decrease. Zeilinger's principle of equivalence between mechanical and information quantization certifies that an elementary system bears just one bit of information because it can only give a definite result in one specific quantum query, so that measurement turns out to be an indivisible action as well! Sagawa and Ueda's research extends the Second Law of Thermodynamics to explicitly incorporate the information, showing that physics must treat information, entropy, and energy equally. We can conclude that no reservoir of information, entropy, or energy can be categorically empty, and logically, incommensurable densities of any type are also unphysical. In this sense, honorable mention deserves Bekenstein's upper bound on the entropy in a region of space, and the assertion that "there is no infinite amount of information in any finite space volume".

Economy

The content of the previous subsection suggests that if a being's properties and its statistical or thermodynamical microstates are bound, we can think of information as a series of data structures carved into the matter or generating radiation or sustaining the spacetime weave, as quantum-mechanical degrees of freedom. Are these equiprobable? Would an evolutionary universe operate ubiquitous information, i.e., some of those forms of energy, kept in a linear mode? No, it would not, undoubtedly due to inefficiency. Without a thrifty sense, nature could not even carry out a single transformation in a finite spacetime interval. Considering this central premise, let us review how the logarithmic scale of a positional number system uses the digit zero.

Place-value notation responds to the Principle of Position, which consists in giving a digit symbol "a value that depends not only on the member of the natural sequence it represents but also on the position it occupies with respect to the other symbols of the group." ("The Empty Column"). The contribution of the digit symbol to a numeral's value is the product of the digit by the radix (or base) raised to the power of the digit's place, e.g., $10 = 2^3 + 2^1 = 1010_2$. In turn, the radix (that we here mark as a decimal subscript after the represented number) is the cardinality of the set of unique symbols PN uses to represent numerals. Standard PN includes the digit 0 in such a set.

PN uses the radix r to encode and decode numbers using the base-r logarithm and the powers of r, respectively. Standard PN excludes the unary system (r = 1); we cannot correctly speak of a code in this case because it only uses the unit symbol to represent natural numbers. PN is plausibly universal because it agrees with the observation that ours is not only linear but also a logarithmic (or harmonic) world [19]. The minor numbers of the linear scale match the bulkiest ones of the logarithmic scale through the hyperbola so that products, quotients, powers, and roots translate into sums, differences, multiplications, and divisions.

Note that the logarithmic scale is deceptive in the following respect. Consider the standard ternary numeral system, which provides the best "radix economy". Many people might associate offhand the unit interval with a concatenation of three segments corresponding each to a symbol 0, 1, and 2. However, this impression is incorrect and caused by our acquaintance with the decoded world. The encoded version of a radix-3 number shows that the unit is not divided into equal intervals but shrunk logarithmically, so 0 dematerializes, and 1 occupies more space than 2. Specifically, when a position requires a nonzero, standard ternary writes "1" if the fractional part of the logarithm is less than $\log_3 2 = {}^{In2}\!/_{In3}$ (63:1% of the unit space); otherwise, it writes "2" (occupying $1 - \log_2 3 = \log_3 {}^{3}\!/_{2}$, 36:9% of the unit space). For instance, $35 \equiv 1022_3$ starts with "1" because of ${}^{35}\!/_{3^3} = 1.296$ and $.296 < \log_3 2$.

PN gives us the "characteristic" (the integer part) of the logarithm of a
decoded number as the place of the most significant digit of the encoded number. We do not need to calculate the logarithm of the number with precision to extract its logarithm's characteristic; $[\log_2 11] = [\log_2 1011_2] = 3$, $[\log_3 58] = [\log_3 2011_3] = 3$ and $[\log_{24} 1327488] = [\log_{24} 400g0_{24}] = 4$. In other words, the logarithm calculates the number of occurrences of the same factor (the radix) in repeated multiplication ($e.g.$, $24^4 < 1327488 < 24^5$). Digit 0 conveys a "skip me and go on" order in PN. Note that we cannot express the lone zero in standard PN because it has no place in the logarithmic scale, as the fact that zero does not have a unique representation in scientific notation, an extension of standard PN, proves.

The hassle of using the symbol 0 in PN is that the representation of a natural number is not bijective; for example, 7, 07, and 0007 are possible representations of the same number. This ambiguity can be a severe problem to qualify for a universal code. To avoid running ad hoc procedures that trim the leading zeroes, we can resort to the "bijective base-r numeration", which allows writing every natural number in uniquely one way using only the symbols {1, 2, · · ·, r}. Let us enlighten the strengths of this notation.

Bijective notation requires a countable base $(r \geq 1)$, so this number system intrinsically erases the capricious zero. For a given radix (that we here indicate as an underlined decimal subscript after the represented number), there are precisely r l bijective base-r numbers of length $l \geq 1$. An ordered list of bijective base-r numbers is automatically in "shortlex" order, i.e., the shortest first and lexicographical within each length. For instance, the sequence of 12 bijective base-2 numbers

$\{22, 111, 112, 121, 122, 211, 212, 221, 222, 1111, 1112, 1121\}_{\underline{2}}$

from decimal $6 = 2 \times 2^1 + 2$ *to* $17 = 1 \times 2^3 + 1 \times 2^2 + 2 \times 2^1 + 1$, is in shortlex order and has $2^3 = 8$ numbers of length 3. We can accomplish number reversal for some calculations; e.g., the arithmetic mean of $13_{\underline{3}}$ and $31_{\underline{3}}$ is the number in the middle, namely $22_{\underline{3}}$. The representation cannot be more compact, notably more efficient than standard PN, especially for the smaller radices $r > 1$. Bijective base-1 is not sui generis, meaning the no-code case can be a permissible natural continuation of a radix reduction process.

These properties constitute an excellent advantage for processing natural numbers that our civilization affords to despise. We can add the minus sign or use a negative base for negative numerals. We can even combine the power of bijective notation with the efficiency for calculations of the signed-digit representation, say the "non-adjacent form" (also known as canonical signed digit representation) or the classic "balanced ternary" system. Let us focus on the latter, one of the best number systems regarding global computability.

Balanced ternary only uses the symbols $\{\downarrow, 0, \top\}$; for example, $\downarrow 0 \uparrow\downarrow = 3^3 + 3^1 - 3^0 = 29$ *and* $\downarrow 0 \uparrow\downarrow = -3^3 + 3^1 - 3^0 = -25$.
This system proves that we can split any integer into a positive and a negative number, a sort of double "binary" system, e.g., $-25 = -28 + 3 = [1001; 10]_3$ or $-25 = [0...01001; 0...010]_3$. To dodge the prepended zeroes problem of this notation, we can write the negative and positive components in bijective base-3 numeration, $e.g.$, $29 = \downarrow 0 \uparrow\downarrow = [1; 233]_{\underline{3}} = -1 + 30 = -1 \times 3^0 + 2 \times 3^2 + 3 \times 3^0$, $30 = \top 0 \top 0 = [; 233]_3$, and $-25 = \downarrow 0 \top\downarrow = [231; 3]_{\underline{3}} = -28 + 3 = -2 \times 3^2 - 3 \times 3^1 - 1 \times 3^0 + 3 \times 3^0$. With the convention that rationals delegate the "negative sign" to the numerator, we can write the rational ${}^{-25}\!/_{29}$ as the pair of pairs of bijective coordinates $(231; 3 / 1; 233)_{\underline{3}}$ and represent it as a unique rectangle of nonzero size in a two dimensional grid precluding the origin. Thus, this "bijective balanced ternary" representation of rationals is a zero-free, nonzero-size, unambiguous, and efficient numeral system.

Moreover, we can connect the basic code system denominated Canonical Representation for the nonzero natural numbers with bijective numeration to yield a zero-free prime-based factorized representation of the natural and rational numbers. By the Fundamental Theorem of Arithmetic, we can think of P, the set of all prime numbers, as the atoms of ℕ so that we can express every natural greater than one as a unique finite product of primes; for example, $16857179136 = 541 \times 7^2\ 3^3 \times 2^{10}$. The "arithmetic" of this prime factorization consists of binary operations such as the product, greatest common divisor, and least common multiple, whose outcome again admits a representation that depends on the operands' prime factors. To banish zero, we can express the primality order and multiplicity as a pair of components in the bijective notation; for example,

$$\frac{16857179136}{29} = \{(3131, 1)\ (31, -1)\ (23, 1)\ (11, 2)\ (2, 3)\ (1, 31)\}_{\underline{3}}$$

i.e., $p_{100}^{1} \times p_{10}^{-1} \times p_{9}^{1} \times p_{4}^{2} \times p_{2}^{3} \times p_{1}^{10}$, *where* p_n is the n-th prime number.

ZERO COUNTABILITY

On how the theory of sets struggles to endorse collections of nothing and why zero is axiomatically unnecessary

Emptiness

ST is the branch of mathematics founded by Georg Cantor that analyzes groupings of objects relevant to number theory, relational algebra, combinatorics, formal semantics, and fuzzy logic. A set is a well-determined collection of things called the set's elements or members, which can be, in turn, sets. The fiducial ST's axiomatic system is allegedly Zermelo-Fraenkel ST, along with the axiom of choice (ZFC).

ST is the branch of mathematics founded by Georg Cantor that analyzes groupings of objects relevant to number theory, relational algebra, combinatorics, formal semantics, and fuzzy logic. A set is a well-determined collection of things called the set's elements or members, which can be, in turn, sets. The fiducial ST's axiomatic system is allegedly Zermelo-Fraenkel ST, along with the axiom of choice (ZFC).

Constructive theories such as Kripke-Platek's include an Axiom of Empty Set, stating that there exists one set with no members and cardinality nil called "empty" or "null". Explicitly assume the empty set axiom. These theories usually formulate this axiom as $\exists S\, \forall y \neg (y \in S)$, where the uniqueness of { } follows from the Axiom of Extensionality. Still, most studied and used axiom systems do not include the Axiom of Empty Set (e.g., ZFC ST and its variants and extensions) but allow deriving it, for instance, through the Axiom of Specification, which states that given any set S and a logical predicate $p(x)$, the elements x ∈ S that make $p(x)$ valid form a set. Assuming that S and x exist and the Principle of Identity, a property impossible to fulfill, *i.e.*, $p(x) = x \notin S$ *or* $p(x) = x \in x \wedge x \notin x$, supposedly generates {

To begin with, it is disputable that the paradoxical expression $\{x \in S \mid x \notin S\}$ $(or \{x \notin S \mid x \in S\})$ is equivalent to { }. However, the real problem is that since the mathematical definition of a set is extensional, all that matters are the elements, so considering a void content is counterintuitive. Indeed, { } does not mirror the real world, where we cannot encounter empty collections, and it is neither apparent how it can admit relations of any type. Does this mathematical object exist? Remarkably, Dedekind excluded the empty set from his initial essay because it was unnecessary, and the same Zermelo called it "improper". According to Locke's thought ("Substance and identity"), "we should not uncritically accept the currently standard view that zero denotes the empty set, becasue it is far from clear that the notion of such a set really makes sense. [...] What is unclear is how there can be, uniquely amongst sets, a set which has no members. We cannot conjure such an entity into existence by mere stipulation [...]. So, [...] the reason why the property of being zero cannot be possessed by any object or plurality of objects is that there is no such number as zero and consequently no such property as the property of being zero."

Thus, although the null set is considered factual in practically all axiomatic systems of ST, its existence and usefulness are suspicious. Suppose the objective of an axiomatic system is to formalize the essential concept of a hereditary well founded exclusive set ("What we are describing"). In that case, one can replace (Axiom of Infinity)

$$\ldots, \{\{\{\{\}\}, \{\}\}, \{\{\}\}, \{\}\}, \{\{\{\}\}, \{\}\}, \{\{\}\}, \{\}$$

with the expression $\{1\}, \{1, \{1\}\}, \{1, \{1, \{1\}\}\}, \{1, \{1, \{1, \{1\}\}\}\}, \ldots$

The former deems { } a set, i.e., presumes that "nothing" is a countable

thing, and adopts von Neumann's interpretation that every set has as elements the sets with ordinality lesser than it so that ordinality and cardinality coincide. The latter eludes the empty set and is a Zermelo-wise interpretation where every set has two elements, i.e., 1 and the previous set. In this case, the ordinality of a set corresponds to the sum of ones or the nesting depth ascribed to the last item, *i.e.*, $Ord\{1, \{1, \{1, \{1\}\}\}\} = 4$. This account weds arithmetic to set theory more straightforwardly and is a neater semantic of individuating the natural numbers rooted in thought and language.

Moreover, all properties of sets are trivially valid for the empty set; for instance, the Axiom of Union and the Schema of Replacement claim nothing substantive. Valuable theorems refer without exception to nonempty sets, and even many axioms and theorems of ZFC specifically exclude the empty set, e.g., the Axiom of Regularity and the Axiom of Choice. What binary relation well-orders the empty set? None. What is the service of the intersection or Cartesian product of a set with the null class? None.

Users of the ST inherently assume the convenience of counting on, for example, a crate, bag, or trashcan vacated. Nevertheless, the point is that ST refers to "the" empty set. Thus, finding two empty baskets is inconvenient, a situation against the theory. What is the effectiveness of a lone bin doomed forever to hold nothing? There are infinitely many sets of any cardinality except for the empty set. The only set from which we cannot remove an element and the only one that is a (strict) subset of any set except itself! If the definition of subset requires elements in common, the statement $\{\ \} \subset S$ is false, whereas if it requires that no elements in the contained set are out of the container, the empty set is a subset of itself. Moreso, the empty set implies "vacuously true" statements that give rise to paradoxes, e.g., $\forall x\ \{\ \} \rightarrow x$ (anything is provable). Worse yet, it is a topological self-contradiction; despite representing a hole, i.e., a no-point, the empty set, closed and open simultaneously, has the property of "compactness" (neither punctures nor missing endpoints). Given these facts, we daresay that the empty set is useless, weird, or even an encumbrance.

Handling only populated sets means wriggling away from this exceptionality and gibberish. Thus, a set proved to be empty should formally disappear, an approach that agrees with Hausdorff's view when he wrote that A = {} means that A vanishes. If we needed intersections of sets even when there is no intersection, we could take such an intersection as $S \wedge \neg S$, again a dull vanishing expression. We can handle "copies of the empty set" as beables to cope with collections possibly deprived of content. For instance, consider a handful of marbles distributed in bags A and B, with $A \subset B$. How many of them are in $B - A$? We must take A as a beable, a not-being yet but a potential set. If $B - A \neq B$, A is a set; otherwise, A is empty, so we rule it out as a set, and our new universe is exclusively B. An empty beable is equivalent to handling possibly a future set. For example, one may prefer to keep a bank account where no money is left as a beable (i.e., a possibility) that sooner or later will be activated again as a set.

Futility

What is the role of zero in the essential number sets? Although the standard ISO-80000 considers zero a natural number, whether it is the origin of the natural numbers hinges on how we prefer to define a natural number, so we find reasons in the literature to include zero as much as to exclude it. According to our description, we bet on the latter because zero is exceptional, tallies nothing in nature, and has no informational value. Moreover, given that the empty set is not a well-determined collection, zero is neither ordinal nor cardinal.

One more argument is the correspondence between zero and one. Although mathematicians have agreed not to list the multiplicative unit as a prime number for the last century, the additive unit has not met the same fate. On the one hand, a prime number has exactly two distinct divisors, namely the unit and itself, whereas a composite number has more than two distinct divisors; 1 is neither prime nor composite. On the other hand, the primes define a logarithmic scale given by $p_n/(n+1)$, where n is a counting number, and $p_1 = 2$ is the first prime. Since the zeroth prime $p_0 = 1$ does not exist, neither should the zeroth counting number. Moreso, $0 \equiv \log 1$ should be the origin of the encoded naturals, much as $1 \equiv p^1/2$ is the origin of the encoded primes. To boot, $1 \in P$ would make many statements about primality verbose, some even invalid, bringing about the same difficulties zero causes to the natural numbers.

Axiomatically, we can define $\breve{\mathbb{N}} \equiv \mathbb{N} - \{0\}$ employing the Peano axioms without reference to zero; every nonzero natural number has a unique successor $S: \breve{\mathbb{N}} \rightarrow \breve{\mathbb{N}}$ sending each natural number to the next one, 1 is the natural unit and not the successor of any natural number, and the rest of the algebraic properties of the naturals utilize the method of induction. The addition of the unit defines precisely the successor, i.e., $a + 1 = S(a)$, and then $a + S(b) = S(a + b)$ for all $a, b \in \breve{\mathbb{N}}$. The multiplication by the unit defines the identity element $a \cdot 1 = a$ and then $a \cdot S(b) = a \cdot b + a$. The order relation and Euclidean division state $a > b$ if and only if $a \neq b$ and there are $q, r \in \breve{\mathbb{N}}$ such that $a = q \cdot b + r$ with $b = r$ or $b > r$ (recursively). Note that $\forall n \in \breve{\mathbb{N}}\ S(n) > n$ holds because of $S(n) = 1 \cdot n + 1$, and if $n \neq 1, n > 1$ is due to $S(n) = n \cdot 1 + 1$. For example, 8 is greater than all its predecessors since $8 = 1 \cdot 7 + 1$, $8 = 1 \cdot 6 + 2$ ($6 = 2 \cdot 2 + 2$), $8 = 1 \cdot 5 + 3$ ($5 = 1 \cdot 3 + 2$ and $3 = S(2)$), $8 = 1 \cdot 4 + 4$, $8 = 2 \cdot 3 + 2$, $8 = 3 \cdot 2 + 2$, and $8 = 7 \cdot 1 + 1$. Likewise, $8 \not> 8$, $8 \not> 9$, $8 \not> 10$, et cetera because no q satisfies the conditions of our definition. The nonzero natural numbers also satisfy the properties of closure $(\forall \{a, b\} \in \breve{\mathbb{N}}, a + b, a \bullet b \in \breve{\mathbb{N}})$ commutativity $(\forall \{a, b\} \in \breve{\mathbb{N}},\ a + b = b + a \textit{ and } a \cdot b = \cdot a)$, associativity $(\forall \{a, b, c\} \in \breve{\mathbb{N}}, a + (b +) = (a + b) + c \textit{ and } a \bullet (b \bullet c) = (a \bullet b) \bullet c)$, and distributivity of multiplication over addition $(\forall \{a, b, c\} \in \breve{\mathbb{N}}, a \cdot (b + c) = (a \bullet b) + (a \bullet c))$.

We can extend the naturals with the indeterminate element $\odot$ so that $\breve{\mathbb{N}} \equiv \breve{\mathbb{N}} + \{\odot\}$. This new element is a beable, not a number, that closes the natural line sort of projectively (see the yellow directed circle of Figure 1), allowing to extend the arithmetic of the naturals by $a + \odot = \odot \textit{ and } a \bullet \odot = \odot$. The properties of commutativity, associativity, and distributivity remain unaltered. However, $\breve{\mathbb{N}}$ cannot retain the order relation because neither $a > \odot \textit{ nor } \odot > a$ holds, given $a \in \breve{\mathbb{N}}$. For example, $2 \not> \odot$ because $2 \neq q \cdot \odot + r$. Likewise, $\odot \not> 2$ because $\odot = \odot \cdot 2 + r = 2 \cdot \odot + r$ but $\odot \neq \odot, \textit{and } \odot = q \cdot 2 + \odot$ but $2 \not> \odot$. Therefore, we cannot identify $\odot$ with ∞, let alone with 0. The indeterminate element is simply an inaccessible natural.

Well, zero is not critical for $\breve{\mathbb{N}}$. What about the integers? The eagerness to unite the negative and positive numbers was the chief reason to include zero, making Z democratic so that any point can be an objective reference frame or neutral coding source. Zero, "based on the dichotomy of source-evolution (origin and derivate), has much to do with zero as a number between negative and positive numbers" [5]. Here, evolution is progression implemented as a recursive composition so that zero is the generator of positive and negative integers; $\vec{S}^i(0) = \vec{S}^{i-1}(\vec{S}(0)) = \cdots = \vec{S}^2(i-2) = \vec{S}(i-1) = i$ and $\overleftarrow{S}^i(0) = -i$ for all $i \in \mathbb{Z}$ are the successor and predecessor operators. These operators are vital, but zero is unnecessary to generate induction.

Above everything, additive groups include zero to bridge the requirement to assign to each element in the group x another element y such that x + y = 0. However, we object that a number canceling its opposite is a vacuous expression; x + y vanishes. We can equivalently write it as $y = -x$ showing that zero is superfluous and exceptional because it is the only integer with no inverse. On the contrary, if we accept that zero opposes itself, we might as well say that it disobeys the axiom of additive inversion. Besides, the sum $0 + x = x$ and the multiplication $0 | x| = 0$ have as much sense as $\infty + x = \infty$ *and* $\infty | x | = \infty$.

The glamour of zero fools us deviating our attention. The power of fields, rings, and algebras primarily resides in the multiplicative group, focusing on invertible members, i.e., all nonzero elements. If needed, a field can represent the void by adding the multiplicative unit and its opposite. Consider the alternative definition of a ring without zero in mathematics as a set equipped with the unary operation of inversion, the binary operations of addition and multiplication, and two constants, +1 and −1, so that it holds $-x \equiv (-1)x$ and the expression $1 + (-1) \equiv 1 - 1$ vanishes. We can embrace the Peano axioms for the definition of the negative and positive integers, except that the initial case in induction proofs requires using both units as generators instead of zero, i.e., $\vec{S}^n(n+1) = 1, \vec{S}^n(-(n+1)) = -1,$ and $\vec{S}^n(-1) = -(n+1)$ for all $n \in \breve{\mathbb{N}},$ while $\vec{S}(-1)$ and $\vec{S}(1)$ vanish. Consequently, zero is axiomatically unnecessary for the integers.

Although the integers do not form a field, we can imagine a sort of "integer projective line" that splits the indetermination ⊙ into 0 and ∞; the integer line together with an idealized point at infinity, an antipodal point of zero that connects to both ends of $\mathbb{Z}$, traces a closed loop. In this set of extended integers, zero enables a germinal version of the principle of general invariance, by which fundamental physical laws must be coordinate-independent, converting zero into a factotum, an almighty angle. Nonetheless, this covariance reinforces the need for an arbitrary origin of coordinates and not the consideration of zero as an integer.

ZERO CONSISTENCY

On the zero duality; why zero is not but can be.

Singularity

Within $\mathbb{Q}$, the polarity between 0 and ∞ is even more evident. If infinity is outside the rationals, so should zero, its reciprocal counterpart. Since zero is as troublesome as infinity, the rationals should treat them on par with each other as the embodiment of abstract extreme magnitudes. However, note that "arbitrarily little" (the smallest computable rational) is not the same as "inconceivably little", which is a metaphysical claim about the nothingness opposed to "unboundedly large" and admitting no regular implementation.

The exceptionality of zero stands out in a rational context more than in $\mathbb{Z}$ because it can be a fraction’s numerator, not a fraction’s denominator. A way to sort out this algebraic disruption is the convention that a rational expression will be assumed to be vacuous if a choice of variables involves a zero denominator. An alternative solution is seeking an interpretation of "division by zero", but this approach also assumes that zero demands differentiated handling. Any strategy to give zero a global meaning is in vain; "there is no uniformly satisfactory solution". The Solomonic decision of banning the expression x/0 only if $x \neq 0$ does not work because the undefined forms 0/0 in arithmetic and 0^0 in calculus remain. Trying to figure out a meaning for these expressions leads again to absurdity. $\mathbb{Q}'s$ completions (e.g., R) and their extensions inherit these "irregularities" to the point that they are handled by computer programming languages and software libraries differently!

Inconsistencies disappear if we concede that zero is not a number of a linear scale, hence neither the numerator nor the denominator of a fraction. The nonzero rational numbers $\breve{\mathbb{Q}} \equiv \mathbb{Q} - \{0\}$ form a field because an arithmetic expression of rational variables that evaluates to zero vanishes. Moreover, we have explained that thinking of a rational number as a pair of interchangeable components is an almighty perspective, inhibited owing to our obsession with zero. Despite this potential, lightness stands a blot on the reputation of the rationals because they are negligible compared in number with the irrationals, not to mention compared with the real transcendental numbers. Specifically, $\mathbb{Q}$ has a null Hausdorff dimension in the unit segment of the reals, although it is unclear why this measure is a preference criterium to choose among number sets; for instance, the rationals have Minkowski dimension one in the unit segment of the reals.

How come is the heaviness of the $\mathbb{R}$ -line useful for calculability? The reals are so unthinkably thick that divisibility, contraction, and expansion become pointless. Quite the opposite, we claim that countability is a condition of non-rigid transformations, e.g., deformations, where "the real stuff" renders no room to contract or dilate. So, we find no solid empirical-argumentative basis to sustain this enthusiasm for the reals and wonder who abhors the vacuum between rational numbers, nature or human beings? Nevertheless, modern mathematics trusts the impeccable reals as though they were an elixir to cure almost all the flaws of "minor" incomplete number sets with cardinality infinitely countable; R "takes on many of the aspects of a religion" and we, physicists, profess this religion.

In the complex setting, the situation gets worse. One of the most severe problems in $\mathbb{C}$ is that the polar angle for the origin is undefined; new kinds of indeterminate and undefined forms appear, their values varying depending on the approaching direction to zero. For instance, consider the function $f(z) = \bar{z}$ (z's conjugate) at the origin; along the imaginary axis, $f(z)$ behaves like the function $-z$ with derivative limit $(f(z) - f(0)) / z = -1$, whereas along the real axis, f (z) equals z with derivative limit 1. $\bar{z}$ is what the theory of functions of a complex variable denominates a non-holomorphic (non-analytic) function around 0, where it does not behave regularly. More generally, complex analysis studies those points s where $\lim_{z \to s} f(z)$ *or* $\lim_{z \to s} 1/f(z)$ is undefined. If neither limit exists, s is an essential singularity for f and 1/f; for example, the origin is an essential singularity for $e^{1/z}, e^{1/z}, z^{z}$, and their reciprocal functions. To resolve a singularity of any type, we must approach it from different directions through a vanishing sequence of distances $|z-s|$.

Animosity towards zero increases with the complexity of the setting. In the quaternions H and octonions $\mathbb{Q}$, we find new undefined expressions related to zero; for example, the n-th root of quaternions q with negative $\Re[q]$ and vanishing $\mathfrak{A}[q]$, such as $\sqrt{-1}$, is undefined . Furthermore, a quaternionic and octonionic universe would be non-commutative, i.e., $\exists p, q \in H$ such that $pq - qp$ does not vanish. Likewise, an octonionic cosmos would not be even associative, i.e., $\exists p,q,r \in \mathbb{Q}$ such that $(pq)r \neq p(qr)$.

Zero does not fit the relevant number sets $\mathbb{Q}, \mathbb{R}, \mathbb{C}, H, and\ O$. It constitutes a pervasive algebraic anomaly whose intractability cannot be solved, an "oddball", and an "obnoxious bugger in a lot of ways". Zero does not exist per se, but it is a linear scale's hole, i.e., a universal singularity we must handle as a limiting value. However, as a logarithmic scale's fixed mark, zero can be.

Beability

Our proposal denies zero as an actual number of a linear scale and takes it as a potential number. Consider the following probability mass function for a random variable $Z \in \mathbb{Z}$

$$\Pr(Z) = \begin{cases} Z \in \breve{\mathbb{Z}}: & \dfrac{1}{(2Z)^2} \\ else: & 1 - \dfrac{1}{2}\varsigma(2) \end{cases}$$

(1)

where ζ (2) is the value of the Riemann zeta function at 2. It is well-defined because the probabilities sum to 1

$$\sum_{z=-\infty}^{-1} (2Z)^{-2} = \sum_{z-1}^{\infty} (2Z)^{-2} = \varsigma(2)/4$$

Since the expected value of Z is undefined

$$\hat{E}(Z) = -\hat{E}(\mathbb{Z}^{-}) + \hat{E}(\mathbb{N}) = \frac{1}{4}\left(\sum_{z=-\infty}^{-1} \frac{Z}{Z^2} + \sum_{z-1}^{\infty} \frac{Z}{Z^2}\right) = -\infty + \infty$$

picking an integer has no bias. Besides, it complies with the "minimal information principle. Thus, this distribution of probabilities is a candidate for the integers' probability mass function in nature. The case "else" is the beable not-a-number, e.g., a null value of a database's field, proving that zero can contribute to the entropy of the integers in the background. What realm zero breaths in?

We have not mentioned the functions exp (0) or 0! so far because the role that number zero plays in them differs from that in expressions involving sums, differences, products, and quotients, where zero is presumably at the center of a linear scale. In contrast, power exponents belong to the logarithmic scale, and the power of zero precisely marks the origin of this encoded algebraic space. However, zero usage is again superfluous because oof exp (log (1)) = (log (1))! = 1; the unit is the invariant "empty product" nailed by the null power, i.e., the number of ways to choose among a single item or the permutations of no elements. Indeed, "zero is the logarithm of one" indicates that zero is a pointer to a being's property data. For example, the azimuthal and magnetic quantum numbers in atomic physics are exponents, and the null values indicate the arbitrary axes chosen for the spherical coordinates. Distinguishing between the linear and logarithmic scales is essential to comprehending Lie groups and algebras, the Laplace transform, and, generally, the exponential map.

The equation $e^{x} = 0$ has a solution in no composition algebra, meaning that division algebras and split algebras dismiss null rotations. In general, zero is inaccessible to a Lie group in Lie theory. For instance, $\mathbb{R}^+$ (positive real numbers) $\breve{\mathbb{C}}$ (nonzero complex numbers), and $GL_4(\mathbb{R})$ with inverse (set of n × n matrices with nonzero determinant) are the Lie groups of the Lie algebras of $\mathbb{R}, \mathbb{C}, and\ \mathfrak{M}(n, \mathbb{R})$. Thus, the model of continuous symmetries provided by a Lie group excludes the origin, the fixed point that observes or realizes the transformation. Zero can live in a Lie algebra only.

The exponential map taking a Lie algebra into the Lie group admits a geometrical interpretation. A pseudo-Riemannian manifold is a differentiable $\mathbb{R}$-based space, locally similar enough to a vector space to do (differential) calculus, with a metric that is everywhere nondegenerate (i.e., invertible, with a nonzero determinant), allowing us to define an exponential map between the tangent space through each point and the manifold. The exponential map warps (is a diffeomorphism of) the tangent space in the neighborhood of a base point, where the tangent space linearizes the manifold. The exponential maps of the one- (straight line), two- (plane), and three-dimensional Euclidean spaces are the unit 1-sphere (circle), 2-sphere (the standard one), and 3-sphere (quaternion). In general, instances of the n-sphere are circle group members, which bans zero. The origin's exile is particularly conspicuous in the corresponding topological versions, i.e., the 1-torus, the spherical shell, and the 2-torus.

Similarly, the Laplace transform $\tau\{f(t)\}(s) = \int_0^\infty f(t)e^{-st}dt$ converts a real variable's function to a complex one through an integral on the positive reals (over the interval (0, ∞)) that involves the exponential function. The transform of an elementary stimulus at the source with non-negative delay τ is the exponential map $\tau\{\delta(t-\tau)\}(s) = e^{-\tau s}$; the universality of the exponential map is likely the outcome of a series of primal impulses. Specifically, trivial and constant uniformity $(\tau\{\delta(t)\}(s) = e^{0s} = 1)$ is unphysical in a universe like ours that rejects stark immediacy and unfailingly entails delays (τ > 0).

Moreover, a Laplace transform decodes the information of a sinusoidal wave (periodic movement) with friction or gain (exponential decay or growth) as a fractional response $(\tau\{e^{at} Sin(\omega t)u(t)\}(s) = \omega / ((s+\alpha)^2 + \omega^2)$, where u(t) is the unit Heaviside Step Function and $\alpha, \omega \in \mathbb{R}$). Zero negation, i.e., assuming $\alpha, \omega \neq 0$, implies that systems always wave and friction to some degree and consequently exhibit nonlinear dynamics without exception (see next section). The poles of a Laplace transform determine a base of nonzero waveform generators that can reproduce the dynamics of practically all linear systems, e.g., linear time-invariant systems, and many nonlinear organic systems. For instance, we conjecture that nature decodes steady ramp impulses (like the rectified linear activation function in neural networks) as the inverse-square law followed by the effects of gravitational, electric, and radiation phenomena given by the probability mass function 1, i.e., $\tau\{\frac{1}{2} tu(t)\}(s) = \frac{1}{2s^2}$.

ZERO COMPUTABILITY

On how to get around the inconsistency inherent in zero to achieve full computability through rationality and the concept of LFT.

Rationality

We have seen that various sorts of uncertainty dwell in nature's core. Specifically, we have reviewed undefined and indeterminate forms, noticing that the cases of intractability involving zero increase with the complexity of the setting, to wit, $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}, and\ \mathbb{Q}$. We have questioned the benefit of dealing with the continuum because it is an illusory description of nature, i.e., non-constructible and uncomputable by definition. Indeed, "mathematical determinism, especially if it is taken to imply (longtime) computability, is an idealization never achievable in the empirical world of actual modelling, measurements, and computations".

$\mathbb{R}$ reflects the human's naïve impression that nature is continuous and reliable but fails to cope with its inherent discontinuity, imprecision, undefinition, and undecidability. Gödel's results usher our axiomatic systems to incompleteness or inconsistency, and approximation procedures and methods rule measurement and calculus. Reality is defective, total accuracy is unattainable, and imperfection is inevitable. It is paradoxical, for instance, to model quantum nonlocality as emerging from the unphysical exactness of $\mathbb{R}^3$, the standard three-dimensional Euclidean space, E^3.

If we disavow the reals, should we not reject the complex setting too? Yes, we must because no algorithm can decide whether a polynomial $z^g - a$, where $g \in \breve{\mathbb{N}}$ and a belongs to a field, is irreducible or can be reduced to factors of a lower degree. Moreover, according to Abel-Ruffini's theorem, no algorithm can write the complex roots of some polynomials of degree five or higher in radicals, so we cannot provide a general representation of the complex numbers in terms of radicals. Furthermore, we cannot expand those polynomial roots expressed in radicals in finite time; a factorization algorithm generally finds only approximate solutions to $\alpha \in \mathbb{C}$ in form $z - a$.

Luckily we can resort to algebraic numbers. A is a countable and computable set, whence definable and arithmetical except at zero, that forms a field because the sum, difference, product, and quotient (presuming that the denominator is nonzero) of two algebraic numbers are also algebraic. Besides, A is algebraically closed because every root of a polynomial equation whose coefficients are algebraic numbers is algebraic, which we can consider the natural implementation of the Fundamental Theorem of Algebra. With this resume, an algebraic number represented as a "polyrational", i.e., a sequence of nonzero rational numbers, seems to match more realistically an observable's measurement requirements than a complex number.

Before adopting A as the leading candidate for the universal number framework, we must consider its effectiveness. An algebraic number is a root of a nonzero (non-trivial) univariate (involving one indeterminate or variable) polynomial with rational coefficients (irreducible fractions). This definition comes with a pair of caveats.

First, the representation is not unique. For example, consider the univariate polynomials

$$z^8+\frac{1}{12}z^4+\frac{1}{30}z^3+\frac{7}{2}z$$

$$2z^6+\frac{z^2}{6}+\frac{z}{15}+\frac{7}{z}$$

$$2z^7+\frac{z^3}{6}+\frac{z^2}{15}+7$$

$$z^7+\frac{1}{12}z^3+\frac{1}{30}z^2+\frac{7}{2}$$

(2)

They are somewhat equivalent. However, suppose we require the polynomial's highest degree (leading) coefficient to be the positive unit, i.e., a monic polynomial, and the lowest degree (trailing) constant to be nonzero. In that case, the only valid representation is the last one. Such a unique irreducible representation is the "minimal" polynomial, and the set of minimal polynomials forms a ring. Because the trailing constant of a minimal polynomial is a nonzero rational, neither rational numbers nor functions z^n, with n $n \in \mathbb{N}$, belong to that ring. Accordingly, zero cannot be a root of a minimal polynomial and is not algebraic. Since a rational power of a nonzero algebraic number is a nonzero algebraic, and the sum, difference, product, and quotient of two nonzero algebraic numbers are again a nonzero algebraic, $\breve{A} \equiv A-\{0\}$ is a field algebraically closed without exceptions! Note that expressions $z-z$, such as 1−1 or $2z-(3+z-1-2)-z$, vanish and do not constitute a genuine arithmetic difference.

We can set a zero-free unambiguous codification of a minimal polynomial using the bijective notation (for instance, the minimal polynomial (2) in signed bijective radix-3 notation is the "canonical" expression.

$$\{21+(1/33)3+(1/233)2+(21/2)\}_{\underline{3}}$$

with roots

$$z \approx -1.19136$$

$$z \approx 0.26902 \pm 1.15872i$$

$$z \approx 1.07653 \pm 0.52796i$$

$$z \approx -0.74986 \pm 0.93908i$$

A computable representation of this set of algebraic numbers is precisely its minimal polynomial in canonical form.

Conversely, a minimal polynomial exists for all $z \in \breve{A}$. The degrees of an algebraic number and its minimal polynomial coincide; for example, degree 0 does not exist, rational roots have degree 1 (e.g., $z-7/2$), z^2-z-1 and the golden ratio have degree 2, and the polynomial (2) and any of its seven roots have degree 7. Note that we can distinguish between rational numbers as such and rational roots of polynomials. For example, whereas the code of the rational number $7/2$ is $\{(21/2)\}_{\underline{3}}$, $7/2$ in $\{1-(21/2)\}_{\underline{3}}$ denotes the algebraic value that annuls $z-7/2$. We can use both in an algebraic expression, e.g., $7/2 = z-7/2$.

Another caveat is that if we want the minimal polynomial to be "identically zero", the expansion of a polynomial root in PN does not terminate, i.e., it is an endless calculation. To construct an algebraic number explicitly, we must relax the definition of "root" so that nullifying a minimal polynomial is calculable in A despite being undecidable in $\mathbb{C}$. Thus, a root is a value $z \in \breve{A}$ that makes its minimal polynomial P (z) vanish identically (e.g., $z=4/5$ is a root of $z^2-16/25$ or approximately as an element of the sequence converging asymptotically to $\lim_{|p(z)|\to 0} z$, where the pair of vertical bars indicates modulus (e.g., $z=-1.19136$ is an approximate root of (2), and $z=-1.19135785807$ is even a "better" solution). The inaccuracy of $\breve{A}$ is intrinsic to natural phenomena.

Although has Lebesgue ($\mathbb{R}$-based Euclidean) measure nil as a subset of $\mathbb{C}$, the property "algebraic" occupies nearly all the possibilities of the complex plane; the algebraic numbers are dense in the complex numbers, and these are algebraic numbers "almost everywhere" (or "almost surely"). Then, what is the use of complex numbers that are not algebraic? Transcendental numbers, i.e., those that belong to $\mathbb{C}-\breve{A}$, such as 0, ln $(1+\sqrt{2})$, and $3+\pi i$, are de facto virtual and indefinite entities, as inaccessible as ∞ because their "real" value is unthinkably far away.

These properties point to the nonzero algebraic numbers in canonical notation, with limiting values 0 and ∞, as a solid basis for building a universal information processing and propagation system. We can alternatively turn to the mathematical structures that allow division by zero, such as the $\mathbb{R}$-line and the $\mathbb{C}$-plane extended with "infinite points". For instance, consider $\hat{\mathbb{R}} \equiv \mathbb{R} \cup \{\infty\}$ implemented as a vast circle containing a pair of dual values, namely 0 and ∞, so that every element has a reciprocal, i.e., $1/x$ is a "total function". There is no order relation because infinity is de facto $\pm\infty$, which does not admit comparison with the rest of the elements. We have two additive constants because r + 0 = r and r+∞ = ∞, 0^0 is so undefined as ∞^∞ or ∞^0, and new indefinite expressions appear, such as ∞ + ∞, ∞ − ∞, 0 · ∞, ∞ · 0, and ∞/∞. The topology is that of a circle, but the arithmetic of intervals, especially those involving 0 or ∞, is inarticulate and still inconsistent (e.g., what is the complement to $\hat{\mathbb{R}}$?). Likewise, we must redefine critical concepts like "neighborhood" and "limit" before doing calculus in $\hat{\mathbb{R}}$. All in all, 0 and ∞ cause difficulties as real or complex numbers without a benefit.

Therefore, extended algebraic objects like $\hat{\mathbb{R}}$ and $\hat{\mathbb{C}}$ are inspiring but do not satisfy the field axioms; 0 and ∞ are antipodal points that lead to complicated arithmetic and rambling calculus. If dealing with a line at infinity is a requirement, then the proper scenario is projective geometry, where everything works with lines of sight, planes of reality, and planes of representation instead of distances. This geometry is an actual fraction-oriented framework where coordinates have the consideration of ratios. If "observational perspective", e.g., incidence, is not an issue or we need to preserve sizes or angles, an algebraic setting like $\breve{\mathbb{Q}}$ or $\breve{A}$ is the fitting choice.

Renouncing the points 0 and ∞ as numbers from the beginning, we assume that zero and infinity are unreal, limiting rather than genuine curvature values, i.e., predefined singularities. This solution achieves a consistent, rational geometry of Euclidean spaces, comparable with their dual non-metrical projective spaces. However, it does not impede using 0 and ∞ as the limit of every sequence of rational numbers whose absolute values are unboundedly decreasing and increasing, respectively. Moreover, this approach successfully extends a rational function, i.e., the ratio of two minimal polynomials, to a "continuous function" from to itself. In the following section, we analyze "conformality", a unique property of this smooth algebraic action in any dimension.

A last clarification before going on. How is Euclidean space if it is not a "real" space? A real-algebraic number is a minimal polynomial's root with an identically vanishing imaginary component (e.g., the real root of the polynomial 2). The set of real-algebraic numbers forms a field, and so do the "nonzero real-algebraic numbers". We claim that precisely this set is the one-dimensional Euclidean space E. Note that differs from E^2; while the former has a complex structure and only exiles the origin, the latter has a less elaborated structure, namely E×E, where "×" is the "direct product", banishing all the points on the plane's axes. This result agrees with experiments designed to falsify quantum mechanics based on splitting the concerned wave function into their real and imaginary parts; formulating quantum mechanics in terms of complex numbers is necessary, and complex translates into algebraic. In general, the coordinates of a point in n-dimensional Euclidean space E^n are all nonzero, and the field operations of this compartmentalized space are the coordinate-wise field operations of the nonzero real-algebraic numbers.

LFT

We have expounded why zero hinders progress in algebra and calculus. Instead, everything works fine if we concede mathematical uncertainty, i.e., managing zero as an implicit limit of a sequence of evaporating values instead of a number. For instance, we explained in the previous section that a number that belongs to $\breve{A} - \breve{\mathbb{Q}}$ inserted in its minimal polynomial provides a mere approximation to zero. Can we move this way to treat zero to other algebraic (computable) objects? Yes, we can.

A function is "univariate rational" if and only if reduced to the lowest terms is in form $P(z)/Q(z)$, i.e., a quotient of two polynomials with coefficients taken in a field F and values of the variable taken in a field G ⊃ F, where the greatest common divisor of P (z) and Q (z) (after a reduction process) is a constant. There cannot be indeterminate forms because Q (z) cannot be the null function, and zero is not a univariate rational function because P (z) cannot be either the null function.

A univariate rational function results from the sum, difference, product, or quotient of two rational functions of the same nonzero algebraic variable whose numerator and denominator are minimal polynomials over a subfield o . The zeroes and poles of a rational function and its reciprocal are rationals or limiting values in , like 0 and ∞. Thus, the set of univariate rational functions taking values in forms the closed field of fractions of the ring of minimal polynomials over the elements of $\breve{\mathbb{Q}}$ or and is a basis for generating the field of meromorphic functions over .

An LFT is a univariate rational function whose numerator and denominator are linear (degree 1) polynomials. An LFT (with coefficients) over a subfield of taking values in adjoining 0 and ∞ as projective opposite limiting values to eliminate mapping discontinuities is a Möbius transformation. The Möbius group is the automorphism group on the "algebraic projective line", of which the

Riemann sphere is a model. Since the composition of two Möbius transformations is a Möbius transformation, we can produce the effect of an ongoing smooth process over this projective line by conducting successive Möbius maps.

Given coefficients $\{a, b, c, d\} \in \breve{A}$, where ad − bc is nonzero, the rational function of one variable $z \in \breve{A}$

$$f\angle(z) = \frac{a}{c}\left(\frac{z+b/a}{z+d/c}\right) = \frac{az+b}{cz+d}$$

(3)

represents a Möbius map or permutation. The requirement $ad \neq bc$ means that the four points define a tetragon (quadrilateral or quadrangle) with a nonzero area, ensuring that this LFT has inverse

$$f^{-1}\angle(z) = \frac{d}{c}\left(\frac{z-b/d}{z-a/c}\right) = \frac{dz-b}{-cz+a}$$

If z approaches the origin $(|z| \to 0)$, we obtain $f\angle(0) \to b/d$ and $f^{-1}\angle(0) \to b/a$. If ad = bc (the parabolic transform), then $b/a = d/c$ *and* $b/d = a/c$, and hence their direct and inverse maps are (characteristic) constants of the Möbius transformation taken as the limiting values of and from infinity, i.e., $f\angle(\infty) \to a/c$ is the inverse pole and $f^{-1}\angle(\infty) \to -d/c$ is the direct pole. The requirement $\{z, f\angle(z), f_{\angle}^{-1}(z)\} \in \breve{A}$ makes these four (irreducible) rationals, namely $b/d, -b/a, a/c,$ *and* $=-d/c$, limiting values where is punctured.

Mind that eq. 3 is the composition of a translation from the direct pole (eq. 4), a conjugation (inversion and reflection) concerning the real axis (eq. 5), a nonzero homothety (scaling) plus rotation (eq. 6), and a translation to the inverse pole (eq. 7), namely

$$f_1(z) = z + d/c$$

(4)

$$f_2(z) = 1/z$$

(5)

$$f_3(z) = \frac{(bc-ad)z}{c^2}$$

(6)

$$f_4(z) = z + a/c$$

(7)

$$f_4(z) \circ = f_3(z) \circ f_2(z) \circ f_1(z) = \frac{a}{c} + \frac{(bc-ad)/c^2}{z+d/c} = f\angle(z)$$

If ad → bc, the rotation effect dissipates, so $f\angle(z) \to a/c$ (*and* $f_{\angle}^{-1}(z) \to -d/c$) as expected.

Likewise, note that a Möbius map is the Laplace transform of an impelling Dirac force (proportional to the inverse pole) and a steady exponential (decaying as the direct pole), i.e., impulse plus friction

$$\tau\left\{\frac{a}{c}\delta(t) + \frac{(bc-ad)}{c^2}e^{-\frac{d}{c}t}\right\}(z) = f\angle(z)$$

For a/c and $bc - ad$ are nonzero, no system can unfold a Möbius map without exerting a punctual shove or prod followed by growth or decline. This insight has profound implications in physics; a stimulus with reinforcement or cessation transforms the space.

The Möbius transformations of constitute the "physical" Möbius Group. It is noticeably a supergroup of the Modular Group, which we redefine as the set of LFT over $\breve{\mathbb{Z}}$ that acts transitively on the points of $\breve{\mathbb{Z}}^2$ visible from the origin, i.e., the irreducible fractions, preserving the form of polygonal shapes.

A Möbius map preserves generalized circles (lines or circles) in the complex plane. Unlike the complex plane, the plane, or Möbius plane, does not allow flat subspaces so that straight lines exist just in the limit; for instance, reflection at a line is inversion at the asymptote of a circle with diverging radius. handles a line like the real-algebraic projective line. Hence, Ă focuses on the circle only, a set of points z at radius r from a center point o, i.e., $|z - 0| = r$. After squaring, the circle becomes $z\bar{z} - \bar{o}z - o\bar{z} + o\bar{o} = r^2$, or in general, the vanishing expression $Az\bar{z} - Bz - \bar{B}\bar{z} + C$, with $A, C \in E, B \in \breve{A}$, and $B\bar{B} > AC$. A straight line corresponds to case A → 0. A circle is invariant under translations, homotheties, and rotations, while inversion at it implies the change of variable $1/z = s$, yielding the vanishing expression $Cs\bar{s} + \bar{B}s + B\bar{s} + A$, another circle, only that $AC > B\bar{B}$ in this case.

We can generalize an LFT to projective lines of rings. The LFT analog of Möbius transformations over the dual-algebraic numbers is a Laguerre transformation, which acts on the dual-algebraic plane adjoining a line of points at infinity, so topologically becoming an infinite cylinder. Much as Möbius transformations preserve the circle, parabolas are invariant under Laguerre transformations; lines are parabolas arbitrarily flattened. A Laguerre transformation maps vertical parabolas on the cylinder to vertical parabolas in the plane, just as a Möbius transformation maps circles of the Riemann sphere onto circles of the plane.

Likewise, a Minkowski transformation is an LFT over the split-algebraic numbers adjoining two lines of points at infinity that bisect the axes. This "splitalgebraic projective line" is isomorphic to the geometry of plane sections of a hyperboloid of one sheet. Minkowski maps preserve the hyperbola, which approximates a line when the curvature at its vertex vanishes. These projective lines are usually known as the Möbius, Laguerre, and Minkowski classical "planes", isomorphic to the geometry of plane sections of their corresponding three dimensional model, to wit, the sphere, the cylinder, and the hyperboloid of one sheet, from which we can accomplish "stereographic projection" onto the plane.

ZERO CONFORMALITY

On the importance of conserving concyclicity and angles as much as possible, especially locally.

Cross-ratio

Möbius, Laguerre, and Minkowski transformations are homographies, i.e., isomorphisms of projective spaces. These mappings hold the structural or configurational relations among the elements of the original space in the new space, although some information can be lost; every homography presents a new perspective or connects two perspectives of a given system. Specifically, homographies are LFT when we identify the "projective line" over a ring with its adjoining "infinite points", i.e., points at infinity.

Projective geometry deals with proportions and assumes that any two lines intersect. We can take the incidence points as projective line landmarks. Since the cross-ratio is a central rational construct that, in plain language, calculates the positioning of a pair of points concerning another, a planar homography preserves the cross-ratio of four distinct points and has vectorial form

$$(z_1, z_2; z_3, z_4) = \frac{\frac{z_3 - z_1}{z_3 - z_2}}{\frac{z_4 - z_1}{z_4 - z_2}}$$

(8)

If one of these vectors points to infinity, i.e., represents an infinite point, we erase the two differences associated with it. If $z_1 \to z_3$, *or* $z_2 \to z_4$, the cross-ratio vanishes. If $z_1 \to z_2$ *or* $z_3 \to z_4$, the cross-ratio tends to the unit. If $z_2 \to z_3$ *or* $z_1 \to z_4$, the cross-ratio diverges.

Cross-ratios are invariant under LFT over rings. In particular, $\breve{A} \cup \{0, \infty\}$ rules this projective invariance via Möbius maps according to the expression

$$(z_1, z_2; z_3, z_4) = (f\angle(z_1),\ f\angle(z_2); f\angle(z_3),\ f\angle(z_4))$$

(9)

If A, B, C, and D are four distinct points on a circle in the Möbius plane, eq. 8 reduces to $(A, B; C, D) = (|CA||DB|)/(|CB||DA|)$, where a pair of vertical bars indicates the Euclidean length of the line segment connecting the pair of points. In this form, we regard the cross-ratio as a measure of the extent to which the ratio with which point B divides AC is proportional to the ratio with which B divides AD.

Remember that the cross-ratio's imaginary part $\Im[(z_1, z_2; z_3, z_4)]$ vanishes if, and only if, the four points lie on the same circle. Since the action of the Möbius group is "simply transitive" on a triple of , the unique Möbius transformation $m_\angle$ to arrive in $\{0, 1, \infty\}$ from any triple of distinct points $\{z_3, z_2, z_4\}$ is

$$m_\angle(z) \equiv (m_\angle(z), 1; 0, \infty) = (z, m_\angle^{-1}(1); m_\angle^{-1}(0), m_\angle^{-1}(\infty)) = z, z_2; z_3, z_4\}$$

and the only Möbius transformation $m_\angle^{-1}$ to come in $\{z_3, z_2, z_4\}$ from $\{0, 1, \infty\}$

$$m_\angle^{-1}(z) \equiv (m_\angle^{-1}(z), 1; 0, \infty) = (z, m_\angle(1); m_\angle(0), m_\angle(\infty)) = (z, z_2, z_3, z_4)$$

The cross-ratio of a "concyclic" quadruple of algebraic points is a realalgebraic number because 0, 1, and ∞ are points of the real-algebraic projective line so that $(z_1, z_2; z_3, z_4)$ is real-algebraic if $m_\angle(z_1)$ *or* $m_\angle^{-1}(z_1)$ are real-algebraic.

Since a point and a line are dual and interchangeable by the "principle of plane duality" in a planar projective space, if $\underline{a}, \underline{b}, \underline{c}$, *and* $\underline{d}$ are four distinct lines emanating from a point, their cross-ratio is $(\underline{a}, \underline{b}; \underline{c}, \underline{d}) = (\sin\angle_{\underline{a}}^{c} \sin\angle_{\underline{b}}^{d}) / (\sin\angle_{\underline{b}}^{c} \sin\angle_{\underline{a}}^{d})$. In this disguise, a sine plays the role of a distance, and the interpretation of a cross-ratio is the same, i.e., a measure of how much the quadruple deviates from the ideal proportion 1. Therefore, the cross-ratio applies to points and lines equally, in agreement with the precepts of projective geometry.

If the four vectors involved in eq. 8 point to infinity, only characterized by their slopes l, m, p, and q, their cross-ratio is

$$(z_1, z_2; z_3, z_4) = \frac{\frac{p-1}{p-m}}{\frac{q-1}{q-m}}$$

We cannot expect the cross-ratio to be a real-algebraic number because these infinite points are not on a circle with a radius r ∈ A. This remark also applies to points of the Laguerre and Minkowski planes. How can this slope-based formula result in a specific algebraic, split-algebraic, or dual-algebraic number with a non-vanishing imaginary part? In the case of split-algebraic numbers, we must make $\pm\Im$ represent the diverging slopes of two lines meeting infinity. Thus

$$(l, m; \Im, -\Im) = \frac{\frac{\Im - l}{\Im - m}}{\frac{-\Im - 1}{-\Im - m}}$$

$$= \frac{(l-\Im)(m+\Im)}{(l-\Im)(m+\Im)}$$

$$= \frac{(l-\Im)^2 (m+\Im)^2}{(l^2-1)(m^2+1)}$$

$$= \frac{(1-lm)^2 + (m-l)^2 + 2\Im(1-1m)(m-l)}{(1-lm)^2 - (m-l)^2}$$

$$(*)\ \frac{1+\tanh^2 \angle_l^m}{1-\tanh^2 \angle_l^m} + \Im \frac{2\tanh \angle_l^m}{1-\tanh^2 \angle_l^m}$$

$$= \cosh(2\angle_l^m) + \Im \sinh(2\angle_l^m)$$

$$= \exp(2\Im\angle_l^m)$$

where we use the hyperbolic tangent subtraction formula $\tanh(\angle_l^m) = (m-l)/(1-lm)$ in $(*)$. Likewise, $\pm\imath$ *and* $\pm \in$ represent the diverging slopes of two pairs of lines meeting infinity in the Möbius and Laguerre planes, respectively. The reader can check $(l, m; \imath, -\imath) = \exp(2\imath\angle_m^l)$ *and* $(l, m; \in, -\in) = 1 + 2\in(\frac{1}{m} - \frac{1}{l}) = \exp(2 \in \angle_m^l)$ using the tangent subtraction formulas $\tan(\angle_l^m) = (m-l)/(1+lm)$ and $\tan p(\angle_l^m) = (m-l)/(l,)$, where tanp is the parabolic tangent.

These results confirm that we can extend the cross-ratio to rings and calculate angles using cross-ratios. Specifically, the (principal value of the) natural logarithm's imaginary part of an algebraic, dual-algebraic, or split-algebraic number of modulus one is its (double) argument. In other words, a generalized angle is half the area swept by the rotation about the origin on a segment of the unit cycle. Indeed, 0 is the area swept by the "straight" unit cycle, i.e., along the unit line segment (1-ball).

This view expands the Exponential Map, which, applied to the imaginary axis and parametrized by the generalized angle y, generates cycles of elliptic (standard) $(\exp(y\imath) = \cos y + \imath \sin y, \imath^2 = -1)$, hyperbolic $(\exp(y\Im) = \cos hy + \Im \sin y, \Im^2 = +1)$, and parabolic $(\exp(y \in) = 1 + \in y, \in^2 = \imath^2 + \Im^2)$ geometry. Any conic, e.g., a rotated ellipse, admits this interpretation of angle via the cross-ratio, in agreement with the statement "there is scarcely any part of conics to which the theory of cross-ratio is not applicable". Fig. 2 comparatively displays the geometry of the zero-free two-dimensional rings.

Moreover, we can express an LFT as a composition of cross-ratios of one variable z taking values in an extended ring; a translation $(^{-d}/_{c}, z-1; z, \infty)$ (eq. 4), a conjugation $(z-1, 0; z, \infty)$ (eq. 5), a rotation $(0, z - c^2/(bc - ad); z, \infty)$ (eq. 6), and another translation $(^{-a}/_{c}, z-1; z, \infty)$ (eq. 7). Thus, an LFT is a crossratio of cross-ratios, and we can express the cross-ratio projective invariance that generalizes eq. 9 just in cross-ratio terms as

$$(\infty, t; t+1, z) \equiv z - t$$

$$(^{-a}/_{c}, z-1; z, \infty) \circ (0, z - c^2/(bc - ad); z, \infty) \circ (z-1, 0; z, \infty) \circ (^{-d}/_{c}, z-1; z, \infty) \equiv fc(z)$$

$$(z_1, z_2; z_3, z_4) = (fc(z_1), fc(z_2); fc(z_3), fc(z_4))$$

Coding

Once described the power of the cross-ratio and its transformational approach, let us tackle the notion of conformality. Roughly, a conformal transformation preserves the angles between intersecting conics and between the cords of four points on a conic. Besides, a conformal transformation preserves conics and the cross-ratio of concyclic points, understanding concyclicity as the condition of a set of points on the same conic. For instance, the circle inversion map $1/z = s$ in the Möbius plane is "anticonformal" according to our description, meaning that angles keep their value but reverse direction. In contrast, the genuinely conformal function $^1/_{\bar{z}} = s$ preserves angles and orientation

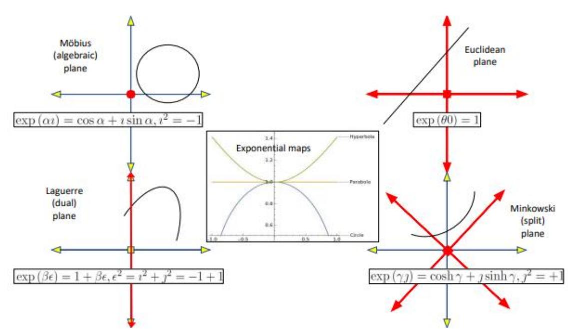

Figure 2) *We illustrate the predefined singularities of the two-dimensional rings in red. The exponential map of the generalized angle times the corresponding imaginary unit defines the associated cycle group, shown framed. The Möbius plane is on the top-left, like*

a complex plane but punctured at the origin. The geodesics of this plane are circles. The circle group uses the imaginary complex unit, which equals −1 if squared. The Laguerrian plane, on the bottom-left, is the plane of dual-algebraic numbers whose null vectors are on the imaginary axis. The geodesics are parabolas. The parabolic group uses the imaginary parabolic unit, which vanishes if squared. The Minkowskian plane, on the bottom-right, is the plane of split-algebraic numbers whose null vectors are on the diagonals. The geodesics are hyperbolas. The hyperbolic group uses the imaginary hyperbolic unit, which equals +1 if squared. The plot in the middle outlines the unit cyclic (circle, parabola, and hyperbola) rotation of the standard, dual, and split algebraic numbers as exponential maps applied at the origin of the real axis. The Euclidean plane E × E is on the top-right, the plane of the real-algebraic numbers E whose null vectors are on the axes. The geodesics are straight lines. The line group has no imaginary unit.

In two dimensions, conformal transformations in the Möbius plane are holomorphic (analytic, regular) maps such as polynomial, exponential, trigonometric, logarithmic, and power functions that preserve local angles. Any conformal mapping of a variable taking values in $\check{A}$ with continuous partial derivatives is holomorphic, and conversely, a holomorphic function is conformal at any point where its derivative does not vanish. A Möbius map has continuous derivatives except at $z = {}^{-d}\!/_{c}$; from this point, the Möbius map defined by eq. 3 is conformal throughout $\check{A}$. Any three points in the Möbius plane define a conformal map; in particular, by fixing 0 (nothing), 1 (something), and ∞ (everything), or −1 (minus something), 1 (plus something), and ∞, we can define metric-independent conformal maps that relativize the notion of "neighborhood" by adapting the size and curvature of the figures around a point to conserve their shape.

A generalized conformal map is a concatenation of LFT over rings. In addition to the elliptic rotations ascribed to , the extension contemplates the parabolic rotations in the Laguerre (right) plane and the hyperbolic cycles in the Minkowski (right) plane. Null vectors are uncomputable and hence considered limiting values; Möbius, Laguerre, and Minkowski planes ban the origin, the imaginary axis, and the diagonals ∝ (1 ± j), respectively. Conformality follows again from the fact that generalized rotations by α (f → αf), translations by β (f → f + β), and inversions (f → 1/f) preserve angles. The null map is not conformal because α and β are nonzero.

Cross-ratios are the basis of conformal geometry because we can write conformal maps in cross-ratio form, e.g., $(z-1, 0; (z+1, 0; z, \infty), \infty) \equiv z^2 - z + 1$ and $((-1, z; 1, \infty), z, 0, \infty) \equiv {}^{2}\!/_{(z-z^2)}$, where z belongs to a ring extended with (null vectors and) infinite points. The two open (for differentiability) subsets in the plane that a conformal mapping connects are essentially indistinguishable from the cross-ratio invariant projective geometry's viewpoint, so we can study functions with given properties on a somewhat complicated region by first mapping it to a simpler one, conserving the properties of the function in the two-way transference between both subsets.

This scenario suggests that we can use a subset of the domain as a region to encode the information received from the "external" world. A concyclic quadruple's cross-ratio is a real-algebraic number ranging from 1 to ∞ if the elements are consecutive, except when all the four points are infinite. Since hyperbolic functions are rational functions whose numerator and denominator are linear or quadratic on real-algebraic coefficients of the exponential function, taking the (real-algebraic) natural logarithm gives rise to inverse hyperbolic functions, such as arccos, arcsin, and arctan, where the prefix "ar-" means "area". Because the "inner" world encodes external data logarithmically, it is a "hyperbolic space", regardless of whether we model the world as a Möbius, Laguerre, or Minkowski plane. Nevertheless, these planes determine the conic invariant under LFT and somewhat the contour of the "coding space" where hyperbolic geometry applies; given two interior points and a conic, the "shortest" path connecting them must orthogonally intersect the boundary of the coding space. These paths are the geodesic lines that tie the coding space with the observable, decoded world.

The need for an efficient numeral system justifies using the logarithmic scale. While a linear scale has no curvature, so its numbers are equidistant, the logarithm shrinks angles and distances to convert (flat) Euclidean into hyperbolic spaces. For example, positional notation grows on a one-dimensional hyperbolic space, H^1, and the analog of E^2 is the hyperbolic plane, H^2, a surface of constant negative curvature. Logarithm and conformality form a strong bond because of the hyperbolic geometry's absolute relation between distance and angle (see Figure 2.5). Informally, this interdependence means that two hyperbolic lines can achieve true parallelism only at tiny distances; as they move away, one line sees the other rotate until they become perpendicular at infinity. In a coding space, parallelism and conformality are countable, i.e., nonzero and finite; conformal mapping generally preserves angles only locally and loses consistency at large distances. Pondering this fact is critical in physics problems where keeping the structural or causal links between the elements of a system or organization as much as possible is required.

Note that a coding space is doubly conformal, first, supported by and immersed in spaces whose isometries derive from conformal transformations, and second, equipped with a metric of inverse hyperbolic functions. For instance, the unit disk of the Minkowski plane preserves the angles between intersecting hyperbolas, but absolute and relative (cross-ratio) distances are Euclidean lengths. The Beltrami-Klein (Klein disk) model handles distances logarithmically but preserves no conic and distorts angles. These disk models are not doubly conformal. Additionally, we require a coding space to be a subspace of the universe; otherwise, we would not have "space" to decode the information. For instance, the hyperbola-based Gans model is doubly conformal but not a coding space because it maps H^2 onto the entire E^2.

However, the Poincaré disk (Chapter IV, The Non-Euclidean World in) is a doubly conformal model of hyperbolic geometry that projects the whole H^2 in the unit disk. Circle-preserving Möbius transformations create the isometries of the universe, . The logarithmic measure of separation between points is invariant under the subset of Möbius maps acting transitively on the unit disk, which becomes a coding space. Assuming that the disk center is at the plane's origin, points z_2 and z_3 within the disk connected by the arc of a geodesic circle perpendicularly intersecting the disk's boundary at z_1 and z_4 are at hyperbolic distance.

$$d_H(z_2, z_3) = In(z_1, z_2; z_3, z_4)$$

Physically, we can suppose that the disk is a body, and the superposition of the configurations ±1 and ±ı describes its quantum state. Since circles in $\check{A}$ tend to be straight lines when the radius of a geodesic diverges, we can take −1, 1, s, and ∞ as consecutive concyclic points of the real-algebraic projective line. Then, an object S outside the unit disk at Euclidean distance s from its boundary is at hyperbolic distance $d_H(1, s) = In(-1, 1; s, \infty) = In\,(s+1)/(s-1)$. Circle inversion $z \to 1/z$ leaves the conformal distance invariant, i.e., $0, d = 1/s, 1,$ and −1 are consecutive within the disk along a diameter satisfying $(s+1)/(s-1) = (-1, 1; d, 0) = (1+d)/(1-d)$. S is now within the disk infinitely far away from the origin and at Euclidean distance d from the boundary, and hence at conformal distance

$$d_C(d) = \frac{1}{2} d_H(1, D) = \frac{1}{2} In(-1, 1; d, 0) = \frac{1}{2} In((1+d)/(1-d) = ar\tanh(d),$$

which is in the range (0, ∞). While the sum of distances u and v is linear within this coding space, namely artanh (u) + artanh (v) (addition of two areas), it corresponds to Einstein's addition and subtraction formula (on the collinear form) of velocities $\tanh(ar\tanh(u) + ar\tanh(v)) = (u \pm v)/1 \pm uv)$ in the external, decoded world.

The Poincaré (conformal) distance is a measure of relativistic speed, i.e., the hyperbolic area that separates two frames of reference in relative motion, ergo the punctured Poincaré disk is a rapidity space that encodes what happens in the Möbius plane. Alternatively, one sheet of a two-dimensional hyperboloid of revolution embedded in the Minkowskian three-space serves as another model of H^2.

Remember that the Möbius, Laguerre, and Minkowski planes are flat, and so is the Minkowski spacetime. Minkowskian spaces handle hyperbolic angles, but the distance between two vectors is the norm of their difference, i.e., Euclidean, while the embedded hyperboloids measure hyperbolic distances along geodesics. In other words, only "the non-Euclidean style of Minkowskian relativity" is doubly conformal, while the algebraic ambient spaces are not.

According to Liouville's theorem, a conformal map in n ≥ 3 dimensions between two open regions of Euclidean space is equivalent to a composition of n-dimensional Möbius transformations, namely homotheties, translations, rotations, and inversions in the (n − 1)-sphere. Using the exponential map, we can interpret a conformal transformation as a diffeomorphic mapping between two manifolds implemented as a Möbius map in an open neighborhood of each point. If f is a bijective mapping of an open set S in E^n onto f (S) with vanishing differential $df_{\mathbf{x}}$ nowhere in x ∈ S, then f is conformal if and only if $\langle df_x\mathbf{u}, df_x\mathbf{v}\rangle = e^{2\omega(x)}\langle \mathbf{u}, \mathbf{v}\rangle$ for all $\mathbf{u}, \mathbf{v} \in E^n$ **,**where the brackets denote the inner product; ω (x) is a rational-valued function that is nonnull if the angles $\angle_u^v$ *and* $\angle_{df_x\mathbf{u}}^{df_x\mathbf{v}}$ differ. Conformal transformations of E^n map m-spheres to m-spheres (m < n) without exception because degenerate flat subspaces do not exist.

We can employ the unit ball of E n centered at the origin as a coding space having as a boundary the unit (n − 1)-sphere. Depending on the conformal transformation, the open side (infinity) can be the origin or the boundary. Using the Euclidean norm || ||, the Poincaré ball model defines the conformal distance d_C between two points z_2 and z_3 in the n-dimensional unit ball, connected by the geodesic that intersects the unit (n − 1)-sphere at the ideals z_1 and z_4, as

$$d_C(\mathbf{z}_2, \mathbf{z}_3) = \frac{1}{2}\left| In \frac{\frac{\| z_3 - z_1\|}{\| z_3 - z_2\|}}{\frac{\| z_4 - z_1\|}{\| z_4 - z_2\|}} \right|$$

and Brouwer studied faithful continuum representations based on discrete elements. As collateral damage, the "real" zero rolled up to present-day science; today, mathematics and physics use it habitually. This inertial frenzy combines with professional pressure to put aside the foundational issues necessary to settle a sturdy QG theory that condemns zero in the name of reality.

The lure of zero is due to its halo of mystique, and none questions the magic of this mathematical object. Science seems afraid of taking on the uncharted lands of a vacuous digit, a missing quantity, an information gap, a concept denoting "what remains from the total". One can hardly browse results about its intrinsic indeterminacy, and essays about the zero's ontological meaning are rare, primarily referring to absence or negation. In this metaphysical context, the threesome {something, everything, nothing} reasonably symbolizes a part of the universe, the whole of it, and the whole's complement, represented by {1, ∞, 0}.

Animal species seem to have a system for approximating numerosity that includes zero, albeit probably taking emptiness as a simple conception of refuted presence. Since zero represents "nothing" and has no actual numerical weight, humans deem it unfamiliar, as the history of humanity proves. We cannot observe zero in nature; if anything, we encounter many clues that it is a cosmological ghost. Because it plays a dull role in many mathematical branches and lacks naturalism, we have posited that zero is often futile and, sometimes, a nuisance.

Zero is nowhere. In statistics and probability theory, every sequence of items has a first and last but no zeroth element, and no Bayes reasoning assigns null a priori probabilities. In logic and computer science, nullary (null arity) functions, i.e., operations with no arguments, always have some hidden input as global variables or contextual properties of the system in question (e.g., state of memory or network time). In fractal theory, fractional dimension nil is impossible; otherwise, we could contract and dilate nothingness. The separation between two distinct points (distance) or lines (angle) cannot be zero in geometry. Physical states, processes, and transformations have a nonnull finite duration, and the interplay between systems is nil in no circumstances. A being balances free energy (macro-information, the general, the global) against organizational energy (micro-information, the specific, the local) throughout its existence, and neither ever becomes zero. Moreso, a being is never thoroughly open or closed, black holes included. In particular, nobody is wholly isolated or fully aware of reality, and our introspection capability implies a nonzero finite consciousness. A quantum system cannot achieve complete coherence due to the inevitable environmental friction, while overly "noisy" surroundings lead to its destruction before reaching plain decoherence; these extreme states are unreachable, so a quantum state is always a mixed ensemble of basis states. In quantum information theory, we cannot perfectly copy or delete an unknown quantum state (no-cloning and no-deleting theorems). Forces and interactions are invariably somewhat dissipative, so Noether's first theorem referring to the correspondence between differentiable symmetries and conserved quantities is an evasive limit case in practice. While we can anticipate the behavior of a system to a degree, "perfect predictability is not achievable, simply because we are limited in our resolving power" (The Theoretical Minimum by Susskind). Likewise, our explanatory capacity of the past is nonzero but also restricted at a fundamental level because a quantum measurement outcome has an aleatory character impeding inferring the system's initial state accurately. In game theory, no game gives null chances to one of the participants. Change has a nonzero cost in daily life; "There is no such thing as a free lunch".

Advance in QG has encountered severe obstacles in the last century. Physics has traditionally identified zero and infinity as "immeasurably small" and "immeasurably great", disregarding that both concepts are untouchable. Classical Newtonian physics does not leave space to indetermination, against the fact that we cannot precisely localize objects in the spacetime fabric. Similarly, something rather than nothing must sustain the universal assets that propagate information causally and quantum-mechanically, conveying that the beings the spacetime accommodates must retain a minimum hypervolume, energy-momentum, curvature, and torsion. On top of this, as Smolin remembers, our physical theories should predict a system's dynamics avoiding predetermined mathematical structures as much as possible, especially without reference to a background metric or asymptote. Neither classical nor quantum physics are background-independent theories; the assumption that the background is noninteractive is an idealization. Accordingly, QG must account for a countable, i.e., nonzero and finite, world and pursue a coordinate-free and zero-free model where all the actors, especially the background, sidestep inaction by locally enabling arbitrary freedom degrees.

The suspicion about zero has driven many physicists to believe there may be a universal minimal length scale at approximately the Planck length. This threshold determines the scale beyond which measurements of spacetime intervals are impossible. It is likely but not necessarily the definite scale limit of the universal weave. Anyhow, a fundamental granularity is a universe's motivation for countable discreteness because the physical implementation of a lattice with endless tiers would require unlimited energy to be built, transformed, adapted, and maintained. A finite assemblage is the only way through if the cosmos has an evolutionary sense via a logic of economy and efficiency. This claim agrees with our experience, which "seems to controvert at every step the concept of something unending", as Parkhurst and Kingsland remark.

The thermodynamics of quantum information also banishes zero and pivots on the vital notion of entropy. Kelvin was the first to recognize the significance of "a universal tendency in nature to the dissipation of mechanical energy." Roughly, entropy measures the number of possible configurations the atoms in a system can have. Hence, a low entropy value means fewer ways to rearrange microscopic things to create a macroscopic structure. Beings' activity contributes to an enduring growth of thermodynamical entropy, foretelling that we gradually approach a cosmic death characterized by a limiting value of maximum entropy of the observable universe; in particular, quantum systems are prone to decohering mechanically, with decreasing entanglement. Simultaneously, for information is physical, computation naturally raises global entropy through free energy consumption, possibly at an increasingly lower rate as the universe ages. This proclivity to reduce information gaps with every interaction means that uncertainty standardization is unidirectional too. Besides, physical inaction would signify no data generation, encoding, transmission, or decoding, i.e., no time passage. Still, we see that time flows. Can we deduce that computation is pervasive and has always been? Yes, we can. Can we conclude that the universe in its original state had a null thermodynamical or informational entropy? We can only confirm that it was minuscule immediately after the Big Bang (the "past hypothesis"). If zero is unreal, as we declare, and if the arrows of entropy and time run in parallel, the origin of time, i.e., absolute time zero, does not exist.

What a being cannot change is not being's information. Something unable to change is not a being. Therefore, a being necessarily possesses nonzero information. Specifically, a being is a physical system that can change the statistical or quantum-mechanical degrees of freedom that somehow harbor its information. Such information must have a positional code if nature owns a pragmatic touch. In this respect, we have posited that standard PN is artificial due to inefficiency; it only aggregates the nonzero terms, and the leading zeroes introduce ambiguity. We have shown that signed (or signed-digit) bijective notation provides zero-free, cost-saving, and unique representations of nonzero rational numbers and their extensions.

Observation of nature led mathematics to introduce and formalize the thought of grouping beings with the same properties. Unfortunately, ST deviated from reality, starting to deal with intricate concepts such as the logic of large cardinals. Surprisingly, this mathematical branch has explored the null class much less than any infinite set. We have explained why managing collections without content makes no sense, concluding that ST should circumscribe to inhabited and functional sets. The empty set is fictitious; neither the cardinal nor the ordinal zero exist. If a group of things is possibly void, it is a beable yet not a set, i.e., a potential collection that precludes the current universe of discourse by default and temporarily. This "settable" implies abandoning the ideal of completeness because we cannot anticipate its valuation, renouncing the close-world assumption.

We have argued that removing zero from the natural numbers is acceptable and necessary. An additive constant is not a great deal, especially if we can define 50 it as the sum of two multiplicative constants, one for the negative and one for the positive integers. We can take -1 as a precursor of the negative integers, much as 1 is an inductor of the positive ones, both fixed under reciprocation, and $0 \equiv 1 - 1$ vanishes. Alas, our mathematics adopts 0 to make $5 + 0$ and $2 - 2 = 0$ well-defined arithmetic expressions at the expense of dealing with undefined forms such as $x/0$; what is good about it? We have not found any compelling reason to involve zero in the arithmetic of the sets N, Z, and Q. Similarly, fields and rings based on Q-completions such as R, C, H, and O are troublesome because zero has no multiplicative inverse, and evaluating to zero is uncomputable. Algebraically, ignoring division by zero and conceding that a ring is a domain free of null vectors is not a drama. In this case, the difference between field and ring blurs; the mathematical definitions of field and domain lose their sense, and only commutative and non-commutative rings stay. Every member of a zero-free ring has a polar decomposition.

The critical problem with zero is rooted in the ideas of infinity and ultradensity, which are very far from the (quantum) reality. Physics increasingly discovers new evidence of the cosmos' compartmentalized character, although the impossibility of endless divisibility at the lower scales will remain untestable for decades. Empirically, 0 is as exceptional as ∞ is, only acquiring significance as the limiting value of an asymptotically vanishing sequence. As Liangkang puts it, "zero represents the horizon of metaphysics: we can forever approach it, but we cannot ultimately arrive at it." Moreover, the irresistible instinct to insert a fulcrum that fills the hole between a line's negative and positive sides has a simple explanation as a sheer manifestation of the "horror vacui".

With all respect to Brahmagupta, antiquity was correct in denying zero as a number of the linear scale. Zero initially seemed to provide us with new ken rudiments but has been disappointing afterward. Although a significant part of the effort of the last two centuries in physics pivots around the number zero, the outcome has been less fruitful than disturbing. Today, zero covers mathematics and physics in mud, making them less easy to teach and learn. Because zero is exceptional, vacuous, and inoperable, science must reconsider its meaning and how to utilize it.

Zero's future

Nonetheless, zero is possible and has a reasonable probability of existing as a reference value in prospect. From this outlook, we can give zero a geometrical meaning; zero symbolizes flatness, i.e., the straight line constant, much like P, e, and π, constants of the perfect parabolic, hyperbolic, and elliptic cycles. The continuum, 0, P, e, π, and ∞, only exist virtually in the offing. Algebraically, zero is the transcendental number $0.0\cdot\cdot\cdot$, whose expansion in standard PN is zero between the decimal point and "the end of the sequence". Consequently, although mathematics has adopted zero as the abstraction of "inconceivably minute", physics must take it as a computable "arbitrarily small" number or the smallest nonzero rational in hand. No special apparatus is needed to carry out this actualist idea, for computers already make us keep our feet on the ground, giving us enough power and precision in real-time to attack and solve real-life problems far before reaching the natural boundaries.

Real numbers devouring memory and time are nonsensical in a cosmos eager for productivity. "Therefore, all of classical continuum mathematics, normally invoked in our formulation of the laws of physics, is not really physically executable. [...] If we cannot distinguish π from a terribly close neighbor, then all the differential equations that constitute the laws of physics are only suggestive; they are not really algorithms that allow us to calculate to the advertised arbitrary precision", affirms Landauer. Although we build our physical theories upon $\mathbb{R}$, "real" is a misnomer in an inaccurate universe. Nature bears and propagates moderate quantities of imprecision, whence controlled randomness, to minimize the missing information while balancing simplicity and complexity, a comparative and incremental task that ensures sustainable computability and evolution. The rationals fit better in a relational universe where uncertainty is everywhere and ratios predominate over absolute distances. We surmise that measuring a being's property is a refinement process of successive rational approximation to the relation between the observer instrument and the observed, as the convergents of an irrational's continued fraction do.

Furthermore, we must accept imperfection as essential. Given that some indeterminacy is unavoidable, a natural positional number system must operate with constructible entities so that calculating the exact root of a polynomial does not consume the universe's lifetime. We claim this universal computational framework manipulates properties as though they were variables taken in the nonzero rational numbers, $\mathbb{Q}$, concatenating arithmetic operations to resolve polynomials only approximately. Ultimately, a list of nonzero rationals univocally defines a minimal polynomial whose roots give rise to the algebraic numbers punctured at the origin, $\breve{A}$, and to the nonzero real-algebraic numbers, the actual Euclidean space **E** in one dimension.

The mathematical structures allowing division by zero, such as the projectively extended $\mathbb{R}$-line and $\mathbb{C}$-line, are partly helpful but do not satisfy the field axioms because they handle zero and infinity as points. Recanting zero from the beginning is a cleaner solution to the problem of undefined expressions. We propose to replace these extended structures with mathematical objects that interpret zero and infinity as the limit of a sequence of nonzero rational numbers whose absolute values vanish and diverge, respectively. This bet on a rationaloriented geometry of Euclidean spaces is crucial when we compare them with projective spaces, where coordinates have the consideration of ratios.

That zero without infinity makes no sense is crystal clear when we study $\mathbb{Q}$- based modular maps and the more general setting of a Möbius transformation. A Möbius map is an LFT that defines a pure (triply-)transitive group action on the 2 × 2 invertible matrices with elements of $\breve{A}$, which fixes 1 (the basic something) plus the predefined limiting values 0 (nothing) and ∞ (everything). A Lie group, the exponential map of a Lie algebra, aggregates these infinitesimal transformations successively through the ongoing action of its multiplication operator. Laplace transform, whose integral uses the exponential as a weighting function, also illustrates that nature is twofold, simultaneously handling a linear and a logarithmic scale. Specifically, the poles of a Laplace Transform synthesize the critical parameters of the generating signal, net information coding. The universality of the exponential function is even more apparent when we consider maps of linear fractions over the nonzero dual and split-algebraic numbers via concatenation of translations, conjugations, and rotations. In these rings, the LFT's singularities are limiting values that adjoin the plane, like 0 and ∞. A generalized LFT preserves the angle (area) between two intersecting conics of the same type, hence the ring's conformal structure (e.g., a figure's shape).

When $n \geq 3$, an n-dimensional Möbius transformation is a domain's most general conformal isometry, ensuring smooth translation moves, dilations, contractions, rotations, and inversions. The main invariant of a conformal transformation over a ring is the cross-ratio, the relative distance that separates a pair of points (or lines) from another, precisely the extent to which two ratios of differences deviate from a proportion. The cross-ratio is a universal construct that ensures angle invariance, contemplates an exponential map as a generalized rotation, fosters recursion to generate all conformal maps, and qualifies coding spaces ruled by hyperbolic geometries. Conformality enhances the thesis that our universe handles (at least) two scales, and zero only fits in the logarithmic scale of coding spaces. Thus, we have arrived at the suggestive insight of conformality departing from the redefinition of rationality deprived of zero.

Our finitistic arguments have meaningful implications in physics. Zero and infinity are unphysical opposite extreme magnitudes, so every lower bound must have its counterpart on the large as a maximum value. For example, we cannot quantify a particle position as incommensurably imprecise because the probability density function of its dual single-moded plane wave is a uniform distribution with diverging standard deviation. Consequently, we can state the Dual Uncertainty Principle, which limits to what extent conjugate variables can lose (instead of retaining) their approximate meaning by renouncing simultaneously ill-defined (instead of well-defined) complementary properties expressed by a single value; $\sigma_x \sigma_p / \hbar \leq \triangleright\triangleleft$, where 1 is a dimensionless absolute scaling limit. This "certainty principle" hinders foretelling a magnitude with arbitrary uncertainty and results indiscernible on the microscopic scales humans notice compared to the size of the universe; it blocks infinity much as the Uncertainty Principle blocks zero. A region demarcated by the two hyperbolas corresponding to the minimal and maximal product of the variances σ_x^2 *and* σ_p^2 confines our realm of indetermination.

Likewise, our theory predicts that a minimum speed exists; none can observe that another frame of reference is static, i.e., experimentally measuring zero relative speed would require infinite energy. Additionally, if a discrete and finite universe is rational-oriented as we sustain, nature would implement irrational numbers as computable algebraic expressions whose main constants, e.g., $\sqrt{2}, \sqrt{3}$, *and* $\sqrt{5}$, might be built-in.

In the long-term future work, we point to a couple of additional issues that would deserve further investigation; first, the prospect of a universal double logarithmic scale (i.e., primality + linear + logarithmic), second, whether the logarithmic scale connects with a "natural" probability mass function for the rationals (and algebraic numbers), and third, a discrete model of spacetime based on a regular space-filling tessellation (or honeycomb) of the four-dimensional Euclidean or hyperbolic space. A premise for these research topics is to put zero aside.

All in all, this research on zero has helped us recognize that the primordial master duality is nothing-everything (zero-infinity) and that nature yields geometric and arithmetic series managed by linear and logarithmic scales onto which "something" is respectively decoded and encoded. Precisely, the logarithmic scale supports PN. Zero is mathematically a null power rather than a number, philosophically a beable instead of a being. However, a beable introduces indefiniteness to a degree; otherwise would be nada. Therefore, zero is budding information implemented as a limiting value that indicates the commencement of spacetime, much as infinity suggests the end of it. We must treat zero and infinity on par and interpret them as interchangeable nascent possibilities.

POSTCRIPT

This apopemptic chapter includes additional information about our activity's circumstances before and during the essay's development.

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We include an extensive literature list of relevant works related to this research. However, the scope of the work is so broad that we deem it incomplete and always feel like ignoring something or someone laudable. Browsing the Web is exhausting, and the information available is overwhelming; we apologize to all those working on related matters that were ignored or not studied as they deserved.

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STATEMENTS AND DECLARATIONS

The author (ORCID 0000-0003-3980-5829) asserts that only scientific rigor, significance, and clarity drive the high-level goals of this work. It contains no known minor or significant incongruencies, errors, or inaccuracies.

The author did not receive support from any organization for the submitted work and declares that he has no competing financial or non-financial interests directly or indirectly related to the work submitted for publication. No personal relationships have influenced the content of this work.

This work is an original one-piece that must be kept unsplit into several parts. It has not been published elsewhere in any form or language, partially or in full (no self-plagiarism).

The author claims to have committed no ethical wrongdoing related to this paper on purpose, including plagiarism, far-fetched self-citations, conflict of interest, inaccurate authorship declarations, and unacceptable biases concerning the references. This work respects third parties rights such as copyright and moral rights.

Disclosing our background is appropriate to realize the milieu of investigation. "The Zero Delusion" is not the product of a sudden revelation or epiphany but a process that has taken decades. The author is a physicist specializing in computational science. He has comprehensive experience in artificial intelligence, fuzzy logic, computational science, and general physics. For the last decade, he has thoroughly studied algebra, topology, relativity, and quantum physics to fathom zero's troubles. This research is part of "Universe Intelligence", a personal project about mathematical physics, the physicality of information, quantum gravity, and the Computational Universe Hypotheses, motivated by the conjecture that physics emerges from mathematics.

To minimize the cognitive bias, the author has bet on the long-term, on his own, and out of the programs ruled by budgets and subventions. He has kept a distance from certain scientific circles during the development of this work to avert group thinking; it is a radical strategy to prevent the excessive enthusiasm produced by social reinforcement.

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