## THEORY

# Theory of the four-dimensional electromagnetic universe, part II: Temporal waves as the foundation of the creation and expansion of the universe 

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#### Abstract

Maglione D. Theory of the four-dimensional electromagnetic universe, part II: Temporal waves as the foundation of the creation and expansion of the universe. J Mod Appl Phys. 2024; 7(1):1-17.


#### Abstract

In this second part of the theory, by enunciating three additional postulates, and relative corollaries, we characterise the unique element that constitutes the true universe in its entirety: the Temporal Waves (TWs). They are stationary electromagnetic waves that oscillate along the diameter of the 4D universe which is its time dimension. They appear as quanta of matter in the 3 D part of the 4 D universe, corresponding to dark matter. The first four TWs emerged from a quantum fluctuation within an unspecified region (the Big Bang event) of an always-existing quantum vacuum. The negative radiation pressure exerted on the local quantum vacuum by these emerging temporal waves has led to the creation and expansion of the 4D universe. Therefore, the negative radiation pressure exerted by all TWs represents the dark energy $(\Lambda)$ of the $\Lambda$ CDM model. Subsequently, a resonancelike phenomenon occurred between these initial TWs and the newly


spacetime generated by expansion led to the creation of adjacent new TWs, alternatively in phase or phase-shifted by $180^{\circ}$. Based on the "restricted holographic principle" postulate 4 states that this only two possible phases of the TWs are perceived as positive and negative electric charges in the 3D part of the 4D universe.

In this work, we derive the equations of the TWs energy and calculate the radius, energy, and equivalent mass of the entire 4D universe. In particular, the total universe energy is always equal to zero. In addition, thermodynamic equations are developed including those to determine the temperatures of the 4 D universe at various red shift $(\mathrm{Z})$ that agree with those published. Finally, we calculate the amplitude of the TWs electric and magnetic fields and obtain that the electric and magnetic forces acting between two TWs have the same value.

Key Words: Cosmology theory; Four-dimensional universe; Privileged quantities; Restricted holographic principle; Radiation pressure; Dark energy; Dark matter

## INTRODUCTION

In the first part of this theory, two postulates and their related corollaries have been stated, of which a summary follows [1]:
Postulate 1 (On the Truly 4D Universe) and its Corollaries 1 and 2. They assert that the real universe is a true 4 D hypersphere, with three spatial dimensions and a fourth spatial dimension that appears to us as time. Furthermore, these postulates assert that in the true 4D universe, everything is in motion due to the expansion of the time dimension at the speed of $c$, according to the equation:

$$
\begin{equation*}
d R=c \cdot d t \tag{1}
\end{equation*}
$$

Through them, a privileged reference system centered on the Big Bang event has been defined, with coordinates ( $0,0,0,0$ ), representing the centre of the 4D universe.

Postulate 2 (Restricted Holographic Principle) and relative Corollaries

1 and 2. They assert that any physical phenomenon, with its physical quantities that measure and characterize it, which occurs partially or totally along the time dimension must be perceived and measured in the 3 D part of the 4 D universe, where we live, in a qualitatively different but quantitatively proportional way, always coherently with the phenomenon itself. Therefore, mass is nothing more than the energy carried by the temporal component of an Electromagnetic Wave (EMW) moving in the 4D universe with wavelength given by de Broglie's formula $(\lambda=h / p)$. Consequently, in the 4D universe, there are no physical objects, that is, entities with mass, but merely electromagnetic waves whose temporal component manifests as mass within the 3 D portion of the 4 D universe.

Based on these postulates and corollaries, have been reached the following main conclusions:

[^0]1. In the 4 D universe there are only EMWs that move at a spacetime speed between $c$ and $c \sqrt{2}$. The vector $\left(\overrightarrow{\boldsymbol{v}_{\boldsymbol{S T}}}\right)$ consists of a temporal component having a constant magnitude equal to c and a spatial one $\leq \mathrm{c}$.
2. The temporal component of the spatiotemporal energy appears as a mass in the 3D part of the 4D universe, where we live.
3. The wave-like behaviour of what we observe as EMWs in the 3D part of the 4D universe, is due to their spatial component which accounts for half of their overall energy.
4. The particle-like behaviour of what we observe as EMWs in the 3 D part of the 4 D universe, is attributed to their temporal component which corresponds to the remaining half of their total spatiotemporal energy.
5. Like mass, acceleration, and the associated fields, as well as the physical quantities derived from it (such as force and work), exist only in the 3D part of the 4D universe. Therefore, the 3D universe can be considered as a mass hyperspherical shell of the 4D universe which, not exerting gravity on itself, cannot decelerate the expansion of the 4D universe.
Furthermore, in the cited paper, 'Temporal Waves' (TWs) have been defined as waves that oscillate only along the time dimension of the 4D universe [1].

Since in this theory the universe is a real four-dimensional hypersphere, it is useful, for the subsequent calculation, to highlight some of its characteristics. A 4D hypersphere, also referred to as a 4 -sphere, has the following features [2]:

1. 4D volume. Also indicated by V4, analogous to the volume (3D) of a three-dimensional sphere, calculated by the following equation:

$$
\begin{equation*}
V_{4}=0.5 \pi^{2} \mathrm{R}^{4} \tag{2}
\end{equation*}
$$

2. 3D hypersurface. Also denoted by S3, analogous to the surface of a sphere, calculated by the following equation:

$$
\begin{equation*}
S_{3}=2 \pi^{2} \mathrm{R}^{3} \tag{3}
\end{equation*}
$$

Where $R$ is the radius of the 4D-sphere.
Below, all the indicated quantities (Time, space, etc.) are to be considered as privileged quantities, unless otherwise specified. In this work, three additional postulates will be stated, through which a new perspective on the origin and expansion of the real 4D universe will be provided.

## POSTULATE 3

## (On the Temporal Waves)

A Temporal Wave (TW) is a stationary electromagnetic wave that oscillates along the entire time dimension of the 4D universe (its diameter), between two extremes, corresponding to the antipodes of the 3 D part of the 4 D universe. It possesses the minimum possible energy, which means a wavelength equal to four times the radius of the 4 D universe.

$$
\begin{equation*}
\lambda_{t w\left(R_{t}\right)}=4 R_{t} \tag{4}
\end{equation*}
$$

Where $\lambda_{t w\left(R_{t}\right)}$ represents the wavelength of the individual TW and $R_{t}$ denotes the radius of the 4D universe at a privileged time $t$ (i.e., the time elapsed after the Big Bang).

Two varieties of TWs can exist, differing by a phase shift of $180^{\circ}$. As
per postulate 2 (restricted holographic principle), these phases are perceived in three-dimensional space as positive electric charge, and magnetic North Pole, (phase $=\pi / 2$ ) or negative electric charge and magnetic South Pole (phase $=-\pi / 2$ ) [1].

## Corollary 1 to the postulate 3

The wavelength, and thus, the energy of the TWs $\left(h f_{t}\right)$, see Eq. 36 in, decreases as the radius (real temporal dimension) of the 4D universe increases according to Eq.1. This is schematically depicted in Figure 1 [1].


Figure 1) The figure schematically depicts the increase in wavelength of the TWs (dashed lines) as the radius of the 4D universe rises. In figure are represents two epochs, $t_{1}$ with radius $R_{t 1}$ (Black), and $t_{2}$ with radius $R_{12}($ Red $)$, where $t_{2}>t_{1}$.

## Corollary 2 to the postulate 3

According to the restricted holographic principle, the energy of TWs appears in the 3D part of the 4D universe as mass [1]. Consequently, the equivalent mass of a TW corresponds to a "quantum of matter". When these matter quanta are not aggregated, they correspond to the Dark Matter.

## POSTULATE 4

## (On the Origin of Temporal Waves and the Expansion of the 4D Universe)

In the absence of the universe, only a quantum vacuum existed [3]. Within an unspecified region of this vacuum (that will correspond to the Big Bang event), characterized by a "privileged" measure equal to the Planck's length, denoted as $l_{p l}$, four Temporal Waves (TWs) emerged. Each of these initial TWs was perpendicular to the others but phase-shifted by $180^{\circ}$ between each pair, resulting in opposite electric charges and magnetic poles (postulate 3). The radiation pressure exerted by these emerging TWs on the local quantum vacuum has given rise to the 4D universe starting its expansion along the temporal dimension.

Subsequently, a resonance-like phenomenon occurred between these TWs and the new spacetime generated by the expansion. This resonance led to the creation of adjacent new TWs, alternatively in phase or phase-shifted by $180^{\circ}$ with those that had originated them. In this way, total charge of the 3D part of the 4D Universe is always zero. The pressure exerted by all TWs along the time dimension on the 3D part of the 4D universe continues to drive its expansion.

## Considerations on Postulate 4

The initial existence of 4 TWs was postulated since, in a 4D universe, this is the minimum number of TWs capable of exerting evenly distributed negative pressure on the 3D component (Figure 2).


Figure 2) In Figure A, the 2D surface of the sphere represents the two spatial dimensions and is analogous to the 3D portion of the 4D Universe. To achieve homogeneous negative radiation pressure (thick arrows) on the 2D spatial part (spherical surface), three TWs along three perpendicular temporal trajectories are required. Similarly, to achieve homogeneous pressure on the 1 D part (circumference) of a hypothetical 2D universe (Figure B), two perpendicular TWs are required. By extrapolation, it can be inferred that in the 4D universe, four initial TWs perpendicular to each other are necessary.

## Corollary 1 to the postulate 4

From postulate 4 follows that the four spatial dimensions, of which one is the radius of the 4D universe corresponding to the time dimension, are quantized, meaning that only finite increments equivalent to the $\left(l_{p l}\right)$, occur. In other words, the spatial quantum assumes only discrete and constant values equal to Planck's length.
Due to this quantization, the differential relative to the radius of the 4D universe $d R t=c d t$ (Postulate 1), reduces to its smallest finite difference $\Delta R t=c \Delta t=l p l$. Consequently, in all cases where we will have a function f of $R t((R))$, it will a discrete function, and the integral will correspond to a summation of n times $l p l$, where n in this case is equal to:
$n=\frac{R_{t}}{l_{p l}} \quad$ (5)

## POSTULATE 5

## (Constant Density of the TWs)

The density of TWs remains constant, equivalent to that of the initial universe, which is 4 TWs within a 4D hypersphere with a radius equal to Planck's length. Because the volume of a four-dimensional hypersphere, also called a 4 -sphere, is equal to Eq.2, then the constant density of TWs the following equation gives it:

$$
\begin{equation*}
D_{t w}=\frac{4}{0.5 \pi^{2} l_{p l}^{4}} \tag{6}
\end{equation*}
$$

Where $D_{t w}$ indicates the TW-density expressed as the number of TWs per $m^{4}$, and $l_{p l}$ represents the Planck's length (about 1.616 . $\left.10^{-35} \mathrm{~m}\right)$.

## RADIUS OF THE 4D UNIVERSE AT ANY TIME

The Hubble constant at any privileged time $H_{t}$ is equal to:

$$
H_{t}=\frac{d a}{a_{t} d t}
$$

Where $a_{t}$ is the scale factor of the 3D part of the 4 D universe at privileged time $t, d a$ and $d t$ indicate the differential of the scale factor " $a$ " and of the privileged time " $t$ ", respectively.
In this theory, we can substitute the scale factor (a) with the radius of the 4 D universe $\left(R_{t}\right)$, which represents the real-time dimension. Consequently, the previous equation becomes:

$$
H_{t}=\frac{d R}{R_{t} d t}
$$

Replacing the Eq. 1 to $d R$, we get:

$$
H_{t}=\frac{c d t}{R_{t} d t}=\frac{c}{R_{t}}
$$

We isolate $R_{t}$ by obtaining:

$$
\begin{equation*}
R_{t}=\frac{c}{H_{t}} \tag{7}
\end{equation*}
$$

That is to the so-called Hubble radius at a generic privileged time " $t$ ". As discussed in Appendix C of this paper, for our calculations we will use the local value of $H_{0}$ results from the final full-mission Planck measurements, equal to $67.4 \pm 0.5 \mathrm{~km} / \mathrm{s} / \mathrm{Mpc}$. Expressing it in $\mathrm{Sec}^{-1}$, allows us to calculate the current radius of the universe, $R_{0}$ :

$$
\begin{equation*}
R_{0}=\frac{c}{H_{0}} \cong(1.37 \pm 0.01) \cdot 10^{26} m \tag{8}
\end{equation*}
$$

From Eq.B. 5 (Appendix B) and given that the value of $Z \approx 1100$ (Eq. B.3.2), we can calculate $R_{C M B}$, i.e., the radius of the 4 D universe at the time of the Cosmic Microwave Background (CMB) emission.

$$
\begin{equation*}
R_{C M B}=\frac{R_{0}}{\left(Z_{C M B}+1\right)} \cong(1.25 \pm 0.009) \cdot 10^{23} \mathrm{~m} \tag{9}
\end{equation*}
$$

## ENERGY AND EQUIVALENT MASS OF A TW

From the postulate 3 and using the Planck's equation for electromagnetic wave energy, $U=h f$, we formulate the equations for the energy $\left(U_{t w}\right)$ of a single TW.

$$
U_{t w\left(R_{t}\right)}=h f_{t w\left(R_{t}\right)}=\frac{h c}{\lambda_{t w\left(R_{t}\right)}}
$$

Where $U_{t w\left(R_{t}\right)}$ is the Energy of a TW when the radius of the 4D universe equals $R_{t}, h$ is Planck's constant and $f_{t w\left(R_{t}\right)}$ and $\lambda_{t w\left(R_{t}\right)}$ are, respectively, the frequency and wavelength of a TW at radius of the 4D universe equal $R_{t}$.
By substituting $\lambda_{\left(R_{t}\right)}$ with Eq.4, we obtain the energy of one TW when the radius of the 4 D universe is $R_{t}$

$$
\begin{equation*}
U_{t w\left(R_{t}\right)}=\frac{h c}{4 R_{t}} \tag{10}
\end{equation*}
$$

From which, by utilising the mass-energy equation, one derives the total mass equivalent to the energy of a temporal wave $m_{t w\left(R_{t}\right)}$ when the radius of the 4D universe is $R_{t}$ :

$$
m_{t w\left(R_{t}\right)}=\frac{U_{t w\left(R_{t}\right)}}{c^{2}}=\frac{h c}{4 R_{t} c^{2}}
$$

And by simplifying:

$$
\begin{equation*}
m_{t w\left(R_{t}\right)}=\frac{h}{4 c R_{t}} \tag{11}
\end{equation*}
$$

It is important to observe that a single TW manifests as mass simultaneously in two opposite zones (antipodes) within the 3D portion of the 4D universe (see postulate 4 and figure 1 ). Hence, when necessary, equations 10 and 11 will be divided by 2. From Eq. 10 and Eq. 11 , it is evident that the energy and corresponding mass of a TW decrease as the radius of the 4D universe increases.
In other words, the TW is 'stretched' by the expansion of the universe, meaning its wavelength increases (Eq.4), resulting in a decrease in its energy. Using Eq. 10 and Eq. 11, let's calculate the today, both energy and equivalent mass, of one TW.

$$
\begin{gather*}
U_{t w(0)}=\frac{h c}{4 R_{0}} \cong(3.62 \pm 0.027) \cdot 10^{-52} \mathrm{~J}  \tag{12}\\
m_{t w(0)}=\frac{h}{4 c R_{0}} \cong(4.03 \pm 0.029) \cdot 10^{-69} \mathrm{Kg} \tag{13}
\end{gather*}
$$

## CURRENT AGE OF THE 4D UNIVERSE AND AT THE TIME

 OF THE COSMIC MICROWAVE BACKGROUND EMISSIONBy knowing the current radius of the 4D universe (Eq.8), and using Eq.1, we can calculate the age of the current 4D universe ( $t_{0}$ ), as privileged time.

$$
\begin{equation*}
t_{0}=\frac{R_{0}}{c} \cong(4.58 \pm 0.034) \cdot 10^{17} s \tag{14}
\end{equation*}
$$

Corresponding to:

$$
\begin{equation*}
t_{0} \cong(14.51 \pm 0.011) \cdot 10^{9} \text { years } \tag{15}
\end{equation*}
$$

Similarly, we calculate the age, in privileged time, of the 4D universe at the time of the Cosmic Microwave Background (CMB) emission:

$$
\begin{equation*}
t_{C M B}=\frac{R_{C M B}}{c} \cong(4.16 \pm 0.031) \cdot 10^{14} s \tag{16}
\end{equation*}
$$

Corresponding to:

$$
\begin{equation*}
t_{0} \cong(13.17 \pm 0.098) \cdot 10^{6} \text { years } \tag{17}
\end{equation*}
$$

## UNCERTAINTY PRINCIPLE AND STABILITY OF TWs

Postulate 4 states that at the beginning of the formation of Universe $4 D$, four TWs were generated from the existing quantum vacuum, each with an energy given by Eq. 10 .
In order for these four initial TWs to become real, that is stable, the temporal duration (here understood as privileged quantity) of their life must satisfy the principle of uncertainty in the energy/time form:

$$
\begin{equation*}
\Delta U \cdot \Delta t \geq \frac{h}{4 \pi} \tag{18}
\end{equation*}
$$

Where $\Delta$ indicates minimal uncertainty and h is Planck's constant. In this theory the minimum uncertainty of the radius of the 4 D universe is equal to $l_{p l}$, so $\Delta R=l_{p l}$. Replacing the previous one ( $\Delta R=l_{p l}$ ) in Eq.1, we get the minimum uncertainty of the privileged time:

$$
\begin{equation*}
\Delta t=\frac{l_{p l}}{c} \tag{19}
\end{equation*}
$$

The energy of all 4 initial TWs is given by the number 4 multiplied by Eq. 10 , in which $R_{t}$ is replaced by $l_{p l}$.

$$
\begin{equation*}
\Delta U_{t w}=4 \frac{h c}{4 l_{p l}}=\frac{h c}{l_{p l}} \tag{20}
\end{equation*}
$$

Using the Eq. 18, we calculate how long these first four TWs can exist.

$$
\begin{gathered}
\Delta t \leq \frac{h}{4 \pi} \cdot \frac{l_{p l}}{h c} \\
\Delta t \approx \frac{l_{p l}}{4 \pi c}
\end{gathered}
$$

Hence, by replacing Eq. 19 for $\Delta t$, we have:

$$
\frac{l_{p l}}{c} \approx \frac{l_{p l}}{4 \pi c}
$$

And by simplifying we get a false result.

$$
1 \approx \frac{1}{4 \pi}
$$

This would mean that the four TWs of postulate 4, created by the initial quantum vacuum, could not stabilize by transforming from "virtual" to "real" in the privileged time interval equal to $\Delta t=\frac{l_{p l}}{c}$. Instead, it is the uncertainty relationship used (Eq.18) that in the context of this theory is not suitable. Indeed, Paul Busch states that there is no unique universal relationship of uncertainty between time and energy, as for the relationship of uncertainty between position and moment, but there are several types that can be inferred in specific contexts [4]. In the same paper, it is reported that the different equations of energy-time uncertainty are also due to the ambiguity of the role of time in quantum theory. The author writes: The conundrum of the time energy uncertainty relation is related to an ambiguity concerning the role of time in quantum theory.
In this theory, (Theory of the Four-dimensional Electromagnetic Universe) time is defined as a real fourth spatial dimension corresponding to the radius of the 4 D universe divided by c . We can, therefore, deduce that the energy-time uncertainty relation valid for this theory must be that which transforms the false result of $1 \approx \frac{1}{4 \pi}$, in a true one. In other words, we must get: $1=1$.

Below we will demonstrate that this sought relationship is that of BohrWigner (see chapter 3.4.3 in [4]):

$$
\Delta U \cdot \Delta t \approx h
$$

It's very interesting to note that this equation is used by Tryon to hypothesize the origin of the universe from quantum vacuum within the Quantum Field Theory [3].
Proceeding as before we get:

$$
\Delta t \approx h \cdot \frac{1}{4 U_{t w}}
$$

and

$$
\Delta t \approx h \cdot \frac{l_{p l}}{h c}=\frac{l_{p l}}{c}
$$

From which, replacing $\Delta t$ with the Eq. 19 , we have:

$$
\frac{l_{p l}}{c}=\frac{l_{p l}}{c}
$$

That mean:

$$
1=1
$$

In conclusion, applying the Bohr-Wigner uncertainty relation we deduce that the first four TWs are stable. Subsequently, the expansion of the 4D universe induced by them, causes the passage of their wavelength from the initial quantum values to macroscopic values for which the uncertainty principle no longer has significant effects. In other words, all the initial and subsequent TWs are stable. Finally, the above does not go against the conservation principle of energy because, as we will see later, the overall energy of the universe is always equal to zero.

## TOTAL ENERGY AND MASS OF THE 4D UNIVERSE AT A GENERIC TIME $T$, TODAY AND AT THE TIME OF THE CMB EMISSION

For these calculations, it is necessary to know the number of TWs $\left(_{(R)}\right)$ existing in the 4D universe at a generic radius $R_{t}$. This number can be calculated based on Postulate 4 and the density in several TWs per hyperspherical 4D hypervolume (Eq. 6 and Postulate 5).

$$
N_{t w\left(R_{t}\right)}=D_{t w} V_{4 D\left(R_{t}\right)}=\frac{4}{0,5 \pi^{2} l_{p l}^{4}} 0,5 \pi^{2} R_{t}^{4}
$$

And simplifying, we obtain the sought-after equation:

$$
\begin{equation*}
N_{t w\left(R_{t}\right)}=\frac{4 R_{t}^{4}}{l_{p l}^{4}} \tag{21}
\end{equation*}
$$

By multiplying the number of TWs (Eq.21) by the energy of a single TW (Eq.10), we obtain the total energy present in the 4D universe having radius $R_{t}$.

$$
U_{R_{t}}=N_{t w\left(R_{t}\right)} E_{T W\left(R_{t}\right)}=\frac{4 R_{t}^{4}}{l_{p l}^{4}} \cdot \frac{h c}{4 R_{t}}
$$

From which, simplifying:

$$
\begin{equation*}
U_{R_{t}}=\frac{h c R_{t}^{3}}{l_{p l}^{4}} \tag{22}
\end{equation*}
$$

Then, we derive the equation for the total equivalent mass:

$$
\begin{equation*}
M_{R_{t}}=\frac{U_{R_{t}}}{c^{2}}=\frac{\mathrm{h} R_{t}^{3}}{l_{p l}^{4} c} \tag{23}
\end{equation*}
$$

From Equations 22 and 23, we can calculate the total equivalent energy and mass present in the 4 D universe today $\left(R_{0}\right)$, and at the time of the emission of the $\mathrm{CMB}\left(R_{C M B}\right)$.

$$
\begin{gather*}
U_{R_{0}}=\frac{h c R_{0}^{3}}{l_{p l}^{4}} \cong(7.53 \pm 0.167) \cdot 10^{192} \mathrm{~J}  \tag{24}\\
M_{R_{0}}=\frac{h R_{0}^{3}}{l_{p l}^{4} c} \cong(8.37 \pm 0.186) \cdot 10^{175} \mathrm{Kg} \tag{25}
\end{gather*}
$$

And at the time of the emission of the $\mathrm{CMB}\left(R_{C M B}\right)$.

$$
\begin{equation*}
U_{R_{C M B}}=\frac{h c R_{C M B}^{3}}{l_{p l}^{4}} \cong(5.64 \pm 0.126) \cdot 10^{183} \mathrm{~J} \tag{26}
\end{equation*}
$$

$$
\begin{equation*}
M_{R_{C M B}}=\frac{\mathrm{h} R_{C M B}^{3}}{l_{p l}^{4} c} \cong(6.27 \pm 0.140) \cdot 10^{166} \mathrm{Kg} \tag{27}
\end{equation*}
$$

## RADIATION PRESSURE OF TWs AT THE BASE OF 4D UNIVERSE EXPANSION

The fourth postulate asserts that the creation and expansion of the 4D universe are caused by the radiation pressure exerted by TWs along the temporal dimension and perpendicular to the 3D portion of the 4D universe. Below, we will derive the equations related to this radiation pressure. However, in our specific case, we have a momentum that acts not on a surface, but on a volume, specifically a hypersurface, see Eq.3, ( $S_{3}=2 \pi^{2} R^{3}$ ), represented by the 3D part of the 4D universe. Therefore, we will term this pressure "3D pressure" and denote it with the symbol $\Pi$ (Uppercase Pi).
As discussed in Chapter 9.1, acceleration, force, work, and all other fundamental physical quantities derived from them are phenomena that occur solely in the 3D portion of the 4D universe [1]. Therefore, the pressure causing the expansion of the 4D universe is not due to a force but to the TWs momentum $\left(p_{t w}\right)$.
This amount of momentum is transferred and divided by the TW onto the two ends (antipodes) of the 3D portion of the 4D universe. However, since the TW is a stationary electromagnetic wave (Postulate 3 ), it is fully reflected by the 3D portion of the universe where it impinges perpendicularly. Consequently, the momentum transferred to a single side of the 3D portion of the Universe is doubled. Thus, the halving of the momentum of a TW is compensated for by the doubling of the same due to total reflection.
Finally, the total momentum transferred from a TW to the spatial part of the 4D universe, at the privileged time " $t$ ", is given by the following equation:

$$
p_{t w\left(R_{t}\right)}=\frac{h}{\lambda_{t w\left(R_{t}\right)}}
$$

The wavelength of the TW has been postulated to have the minimum energy for a stationary wave, namely equal to $\lambda_{t w\left(R_{t}\right)}=4 R_{t}$ (Eq.4). Thus, the equation describing the total momentum transferred by the single TW onto the 3D portion of the 4D Universe is:

$$
\begin{equation*}
p_{t w\left(R_{t}\right)}=\frac{h}{4 R_{t}} \tag{28}
\end{equation*}
$$

Where $R_{t}$ denotes the radius of the 4D universe at a privileged time " t ", $\lambda_{t w\left(R_{t}\right)}$ represents the wavelength of the individual TW, " $h$ " is the Plank's constant, and $p_{t w\left(R_{t}\right)}$ is the momentum of a TW when the radius of 4D universe is $R_{t}$.
In general, the radiation pressure ( $P$ ) expressed as a function of momentum is given by the equation:

$$
\begin{equation*}
P=\frac{p}{s \cdot d t} \tag{29}
\end{equation*}
$$

Where S is a surface. We don't use differentials because, in this theory, $(\Delta t)_{\min }=\frac{(\Delta R)_{\min }}{c}=\frac{l_{p l}}{c}$ is a discrete quantity since the function of $R_{t}$, which is also discrete.
Since TWs are fully reflected from the 3D part of the 4D universe, the radiation pressure on each extreme will be doubled. From this consideration and the previous equation, we derive the 3 D pressure $\Pi_{t w\left(R_{t}\right)}$ exerted by a single TW on the 3D hypersurface, when the radius of the universe $R_{t}$ :

$$
\Pi_{t w\left(R_{t}\right)}=2 \frac{p_{t w\left(R_{t}\right)}}{(\Delta t)_{\min } \cdot S_{3\left(R_{t}\right)}}=2 \frac{p_{t w\left(R_{t}\right)}{ }^{c}}{l_{p l} \cdot S_{3\left(R_{t}\right)}}
$$

Where $S_{3\left(R_{t}\right)}$ is the 3D hypersurface (Eq. 3 ) when the radius of the 4D universe is $R_{t}$.
Substituting $p_{t w\left(R_{t}\right)}$ of the Eq. 28 and $S_{3\left(R_{t}\right)}$ of Eq. 3 into previous
equation we obtain:

$$
\begin{equation*}
\Pi_{t w\left(R_{t}\right)}=\frac{h c}{4 \pi^{2} R_{t}^{4} l_{p l}} \tag{30}
\end{equation*}
$$

To have the overall pressure exerted by a single TW on the 3D portion of the 4 D universe with a radius of $R_{t}$, one must perform the following summation.

$$
\Pi_{t w\left(R_{t}\right)}=\frac{h c}{4 \pi^{2} R_{t}^{4}} \cdot \frac{1}{\sum_{n=l_{p l}}^{\frac{2 R_{t}}{l_{p l}}}\left(l_{p l}\right)}
$$

This summation is equal to $2 R_{t}$ (see also Eq.A.1), obtaining the sought relationship:

$$
\begin{equation*}
\Pi_{t w\left(R_{t}\right)}=-\frac{h c}{8 \pi^{2} R_{t}^{5}} \tag{31}
\end{equation*}
$$

Where $S_{3\left(R_{t}\right)}$ is the 3D hypersurface (Eq.3) when the radius of the 4D universe is $R_{t}$.
This pressure is considered negative because it causes an expansion. By multiplying the previous expression by the total number of TWs $\left(N_{t w\left(R_{t}\right)}\right)$ existing in the 4D universe when its radius is Rt , we derive the overall pressure exerted by all TWs along the time dimension, perpendicular to the 3D part of the 4D universe, where we reside.

$$
\Pi_{R_{t}}=-N_{t w\left(R_{t}\right)} \frac{h c}{8 \pi^{2} R_{t}^{5}}
$$

By substituting Eq. 21 for $N_{t w\left(R_{t}\right)}$, we obtain:

$$
\Pi_{R_{t}}=-\frac{4 R_{t}^{4}}{l_{p l}^{4}} \frac{h c}{8 \pi^{2} R_{t}^{5}}
$$

From which, simplifying:

$$
\begin{equation*}
\Pi_{R_{t}}=-\frac{h c}{2 \pi^{2} R_{t} l_{p l}^{4}} \tag{32}
\end{equation*}
$$

Where $\Pi_{R_{t}}$ is total 3D pressure when the radius of the 4 D universe is $R_{t}, h$ is Planck's constant, $c$ is the speed of light in vacuum and $l_{p l}$ indicates the Planck's length.
It is important to note that this negative pressure corresponds to the dark energy ( $\Lambda$ ) of the $\Lambda$ CDM model.
Based on Eq.32, we determine the value of the 3D pressure when the Universe originated by expanding from the initial quantum vacuum. This means that $R_{t}=l_{p l}$, and thus, the Eq. 32 becomes:

$$
\begin{equation*}
\Pi_{r_{p l}}=-\frac{h c}{2 \pi^{2} l_{p l}^{5}} \cong-9.12 \cdot 10^{147} \frac{\mathrm{~N}}{\mathrm{~m}^{3}} \tag{33}
\end{equation*}
$$

Instead, the current 3 D pressure is equal to:

$$
\begin{equation*}
\Pi_{0}=-\frac{h c}{2 \pi^{2} R_{0}^{4} l_{p l}} \cong-(1.07 \pm 0.008) \cdot 10^{87} \frac{\mathrm{~N}}{\mathrm{~m}^{3}} \tag{34}
\end{equation*}
$$

## THERMODYNAMICS OF THE 4D UNIVERSE

The expansion of the 4D universe can be regarded as an adiabatic thermodynamic process with $d Q=0$, meaning no heat $(Q)$ exchange with the external (given that there is no 'external'). Furthermore, in this theory, being the 4D universe composed exclusively of Temporal Electromagnetic Waves (TWs), it is logical to consider it as a "gas of photons". First, we determine the change in internal energy.

## Internal Energy of the 4D Universe

The variation in potential energy " $U$ " of an adiabatic thermodynamic process is considered negative and equal to the opposite of the expansion work "W":

$$
\begin{equation*}
-d U=d W=d(F \cdot R) \tag{35}
\end{equation*}
$$

Where $d(F \cdot R)$ represents the differential of the product between the force " F " causing the expansion and the value of the expansion " R ".
Since momentum of the 4D universe is a discrete function of the 4D universe radius (R), will be used summation as specified in Corollary 1 to Postulate 4 (se also Eq.A. 2 in appendix A). Therefore, the Eq. 35
transform in:

$$
\begin{equation*}
-\Delta U=\Delta W=\Delta(F R)=F_{f} R_{f}-F_{i} R_{i} \tag{36}
\end{equation*}
$$

Where the indices ' $f$ ' and ' $i$ ' indicate, respectively, a final and an initial state.
Since in the 4D universe there are no forces (see discussion in [1]), the expansion work is not caused by a force but is due to the total momentum ( $p$ ) of all TWs being transferred perpendicularly against the 3D part of the 4D universe. Consequently, given that $F=\frac{p}{t}$, by substituting Eq. 1 (see "Introduction") for $t$, we have:

$$
\begin{equation*}
F=\frac{p c}{R_{t}} \tag{37}
\end{equation*}
$$

Thus, Eq. 36 becomes the following:

$$
\Delta W=\Delta(F R)=F_{f} R_{f}-F_{i} R_{i}=\left(\frac{p_{f} R_{f} c}{R_{f}}-\frac{p_{i} R_{i} c}{R_{i}}\right)
$$

From which, simplifying, we have:

$$
\begin{equation*}
\Delta W=c\left(p_{f}-p_{i}\right) \tag{38}
\end{equation*}
$$

The total momentum of all TWs $\left(p_{\text {tot. }(t w)_{R_{t}}}\right)$ is given by Eq. 28 multiplied by the total number of TWs when the radius of the 4D universe is equal to $R_{t}$ (Eq.21):

$$
p_{t o t .(t w)_{R_{t}}}=\frac{h}{4 R_{t}} N_{t w(R)}=\frac{h}{4 R_{t}} \frac{4 R_{t}^{4}}{l_{p l}^{4}}=\frac{h R_{t}^{3}}{l_{p l}^{4}}
$$

From which we have that the expansion work from $R_{i}$ to $R_{f}$ is:

$$
\Delta W=c\left(p_{f}-p_{i}\right)=\frac{c h R_{f}^{3}}{l_{p l}^{4}}-\frac{c h R_{i}^{3}}{l_{p l}^{4}}
$$

From which, simplifying:

$$
\begin{equation*}
W_{\left(R_{i} \rightarrow R_{f}\right)}=-U_{\left(R_{i} \rightarrow R_{f}\right)}=\frac{h c}{l_{p l}^{4}}\left(R_{f}^{3}-R_{i}^{3}\right) \tag{39}
\end{equation*}
$$

From the previous equation, we can estimate the change in internal energy of the universe from the beginning, i.e., $R_{i}=l_{p l}$, to today $R_{0}$ (Eq.8):

$$
\begin{equation*}
U_{0}=-\frac{h c}{r_{p l}^{4}}\left(R_{0}^{3}-l_{p l}^{3}\right) \cong(-7.53 \pm 0.167) \cdot 10^{192} J \tag{40}
\end{equation*}
$$

Comparing the previous result with the total energy today, due to TWs (Eq.24), it is seen that the overall energy of the current 4D universe is equal to zero. This result holds for any radius of the 4D universe (data not shown).

## General Law of Ideal Gases Applied to the 4D Universe

In this theory, the universe exhibits 4 real dimensions, thus the "known" thermodynamic equations must be reformulated because:

1. The volume is a 3D hypersurface of a 4D hypersphere (See Eq.2).
2. Pressure is a 3D pressure (Force/3D hypersurface) denoted by the symbol $\Pi$.
3. The number of moles of TWs (n) is not constant but increases with the expansion of the 4D universe. The number of moles equals to the number of TWs $\left(N_{t w\left(R_{t}\right)}\right.$, see Eq.21) divided by Avogadro's number. That is:

$$
\begin{equation*}
n_{t w\left(R_{t}\right)}=\frac{N_{t w\left(R_{t}\right)}}{A}=\frac{4 R_{t}^{4}}{A l_{p l}^{4}} \tag{41}
\end{equation*}
$$

Where $R_{t}$ denotes the radius of the 4 D universe at a privileged time " $t$ ", $n_{t w(R)}$ represents the number of moles of TWs when the radius of the 4D universe is equal to $R_{t}, \mathrm{~A}$ is Avogadro's number, and $l_{p l}$ is the Planck's length.
From the above, the equation of state of ideal gases in the 4D Universe becomes:

$$
\begin{equation*}
\Pi V_{4}=n R_{4 D} T \tag{42}
\end{equation*}
$$

Where $\Pi$ is the 4 D pressure, $V_{4}$ is the hypervolume $4 \mathrm{D}, n$ represents the number of moles of TWs, $R_{4 D}$ represent the universal gas constant
for the 4D universe.
This equation, for a generic transformation from an initial stage $(i)$ to a final state $(f)$, considering that $n$ also varies, becomes:

$$
\begin{align*}
\Pi_{i} V_{4 i} & =n_{i} R_{4 D} T_{i}  \tag{43}\\
\Pi_{f} V_{4 f} & =n_{f} R_{4 D} T_{f} \tag{44}
\end{align*}
$$

As the gas constant $(R)$ has been experimentally determined, we will do the same for the one valid for the TWs gas, which underlies the 4D universe ( $R_{4 D}$ ). To achieve this, we will utilize the inverse formula of Eq. 42 and the theoretical and experimental data of the thermodynamic variables T, $\Pi$, and $V_{4}$.
So, knowing that:

1. Current temperature $\left(T_{0}\right)$ of the CMB is $2.725^{\circ} \mathrm{K}$ (See Appendix B);
2. Current 3D pressure is $\Pi_{0}=-1.07 \cdot 10^{87} \frac{N}{m^{3}}$ (Eq. 34).
3. Current 4D volume $\left(V_{4(0)}=1.75 \cdot 10^{105} \mathrm{~m}^{4}\right)$ calculated by Eq. 2 .
4. Current number of TWs moles $\left(n_{0}=3.45 \cdot 10^{220}\right.$, calculated by Eq.41).
We can calculate the mean value of $R_{4 D}$ :

$$
\begin{equation*}
R_{4 D}=\frac{\Pi_{0} V_{4(0)}}{n_{0} T_{0}}=1,998712 \cdot 10^{-29} \frac{\mathrm{~J}}{\mathrm{~mol} .{ }^{\circ} \mathrm{K}} \tag{45}
\end{equation*}
$$

In Appendix D are displayed equations for isochoric, isothermal, and isobaric transformations for the 4D universe.

Relationship between Temperature and Radius of the 4D Universe This relationship is obtained using the general equation of perfect gases for the 4D universe (Eq. 42), in which we substitute:

1. At the 3 D pressure, the Eq. $32\left(\Pi_{R_{t}}=-\frac{h c}{2 \pi^{2} R_{t} l_{p l}^{4}}\right)$;
2. At the 4 D hypervolume, the Eq. $2\left(V_{4}=0.5 \pi^{2} R_{t}^{4}\right)$;
3. To the number of moles, the Eq. $41\left(n_{t w\left(R_{t}\right)}=\frac{4 R_{t}^{4}}{A l_{p l}^{4}}\right)$

So, starting from:

$$
\Pi V_{4}=n R_{4 D} T
$$

And proceeding with the substitution, we obtain:

$$
\frac{h c}{2 \pi^{2} R_{t} l_{p l}^{4}} \frac{1}{2} \pi^{2} R_{t}^{4}=\frac{4 R_{t}^{4}}{A l_{p l}^{4}} R_{4 D} T
$$

Simplifying:

$$
\frac{h c}{4 R_{t}}=\frac{4}{A} R_{4 D} T
$$

And isolating T, we have the sought-after relationship:

$$
\begin{equation*}
T=\frac{A h c}{16 R_{t} R_{4 D}} \tag{46}
\end{equation*}
$$

This thermodynamically obtained equation corresponds to equation B.3.1 $\left(T_{Z}=(Z+1) T_{0}\right)$ (data not shown).

## Temperature of the CMB (of the 4D Universe) at the Beginning

 and at Various EpochsCosmic Microwave Background (CMB) is radiation emitted when the universe was about 13.2 billion "privileged" years (Eq.17). Today, that same radiation has a blackbody temperature of $2.72548 \pm 0.00057^{\circ} \mathrm{K}$. Now, we verify that the equation established earlier (Eq.46) enables us to ascertain the temperature of the CMB at the time of its emission, which must be approximately $3,000^{\circ} \mathrm{K}$ (see Appendix B). To do this, we insert the value of the 4 D universe radius at the time of the CMB emission (Eq.9) into the Eq.46, obtaining, indeed, the peak temperature of the CMBR when it was emitted ( 13.2 million of privileged years since the creation of the 4D universe):

$$
T_{C M B}=\frac{A h c}{16 R_{C M B} R_{4 D}} \cong 3,000^{\circ} \mathrm{K}
$$

By substituting the Planck length $\left(R_{t}=l_{p l}\right)$, we obtain an estimate of the temperature of the 4D Universe at its birth $\left(T_{B B}\right)$, i.e., at time zero:

$$
T_{B B}=\frac{A h c}{16 l_{p l} R_{4 D}} \cong 2.31 \times 10^{61 \circ} \mathrm{~K}
$$

Furthermore, by utilizing Eq.B.3, we can estimate the value of Z at the Big Bang level:

$$
Z_{0}=\frac{T_{B B}}{T_{0}}-1 \cong 8.49 \times 10^{60}
$$

TABLE 1
Comparison between CMB temperatures measured and predicted at different $Z$

Calculated Radius of 4D

| Z | Universe ${ }^{\text {a }}$ $\boldsymbol{R}_{(z)}=\frac{R_{o g g i}}{Z+1}(\mathrm{~m})$ | Predicted Temperature $\left({ }^{\circ} \mathrm{K}\right)$ | Measured <br> Temperature |
| :---: | :---: | :---: | :---: |
| 0.89 | $(7,26 \pm 0.054) \cdot 10^{25}$ | $\begin{aligned} & 5,15 \pm 0,038^{\mathrm{b}} \\ & 5,15 \pm 0,001^{\mathrm{c}} \end{aligned}$ | $5,08 \pm 0,{ }^{\circ} \mathrm{K}^{\text {d }}$ |
| 3.025 | $(3,41 \pm 0.025) \cdot 10^{25}$ | $\begin{aligned} & 10,97 \pm 0,081^{\mathrm{b}} \\ & 10,97 \pm 0,002^{\mathrm{c}} \end{aligned}$ | $8,4 \div 16,2^{\circ} \mathrm{K}^{\mathrm{e}}$ |
| 6.34 | $(1,87 \pm 0.014) \cdot 10^{25}$ | $\begin{aligned} & 20,00 \pm 0,148^{\mathrm{b}} \\ & 20,00 \pm 0,0044^{\mathrm{c}} \end{aligned}$ | 16,4 $-30,2^{\circ} \mathrm{K}^{\mathrm{f}}$ |

${ }^{(2)}$ Equation derived from Eq.B.5; ${ }^{(b)} \mathrm{Eq} \cdot 46 ;{ }^{(c)} \mathrm{Eq}. \mathrm{B.3.1}.{ }^{(d)}[5] ;{ }^{(e)}[6] ;{ }^{(t)}[7]$.
Note that the two temperature errors differ because for the Eq. 46 we use the error of the $4 D$ universe radius, while temperature from Eq.B.3.1 is calculated based on the error of the current $C M B$
temperature [8].
Table 1 above reports experimentally determined temperatures of the CMB at three different redshifts (Z), along with those predicted based on Eq. 46 and its equivalent formula (Eq.B.3.1). These calculated temperatures are consistent with published values, providing evidence in support of this theory.

Thermodynamic determination of the internal energy and work for any transformation in the 4D universe
To determine both, internal energy, and work for any transformation in the 4D universe, it is necessary to calculate the values of the specific heat at constant volume $\left(C_{v(4 D)}\right)$ and the specific heat at constant pressure $\left(C_{p(4 D)}\right)$ in the 4D universe. To do this, we can utilize the kinetic theory of ideal gases by recalculating the degrees of freedom which increase in a 4D Universe compared to its 3D portion. The Table 2 concisely illustrates how various thermodynamic parameters are calculated for different types of gases in the 3D region of the 4D universe. For example, molecules of a polyatomic gas possess 3 degrees of freedom for translational motion (movements possible in the 3 dimensions $x, y$, and $z$ ), plus another 3 degrees of freedom for rotational motion around their centre, for a total of 6 degrees of freedom. The value of $C_{v}$ is then equal to the product of several degrees of freedom by the gas constant R divided by 2 .

TABLE 2
Thermodynamics parameters for various species of gas in the 3 D portion of the 4 D universe

| Gas Species | Translational Degrees of Freedom | Rotational <br> Degrees of Freedom | Total Degrees of Freedom | Internal Energy (U) | Specific Heat at Constant Volume $\left(C_{v}\right)$ | Specific Heat at Constant Pressure $\left(C_{p}=C_{v}+R\right)$ | $\gamma=\frac{C_{p}}{C_{v}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Monoatomic | 3 | 0 | $3+0=3$ | $U=n C_{v} T$ | $C_{v}=\frac{3 R}{2}$ | $\frac{3}{2} R+R=\frac{5}{2} R$ | $\gamma=\frac{5}{3}$ |
| Diatomic | 3 | 2 | $3+2=5$ | $U=n C_{v} T$ | $C_{v}=\frac{5 R}{2}$ | $\frac{5}{2} R+R=\frac{7}{2} R$ | $\gamma=\frac{7}{5}$ |
| Polyatomic <br> or <br> Photon gas | 3 | 3 | $3+3=6$ | $U=n C_{v} T$ | $C_{v}=\frac{6 R}{2}=3 R$ | $3 R+R=4 R$ | $\gamma=\frac{4}{3}$ |

Since the 4D universe is composed exclusively of Temporal Waves (TWs), from a thermodynamic point of view it is logical to consider it as a "photon gas". Therefore, for the 4D universe, the degrees of freedom for the TWs became equal to 8 , comprising 4 translational
(movements possible along the $x, y, z$ and $t$ axes) and 4 rotational (rotations around the 4 axes). Thus, the thermodynamic parameters become those listed in Table 3.

## TABLE 3

Thermodynamics parameters for analogous of photon gas (TWs) in the 4D universe

| Gas Species | Translational <br> Degrees of <br> Freedom | Rotational <br> Degrees of <br> Freedom | Total <br> Degrees of <br> Freedom | Internal <br> Energy <br> $(\mathbf{U})$ | Specific Heat at <br> Constant Volume <br> $\left(\boldsymbol{C}_{\boldsymbol{v}(4 D)}\right)$ | Specific Heat at Constant <br> Pressure <br> $\left(\boldsymbol{C}_{\boldsymbol{p}(4 D)}=\boldsymbol{C}_{\boldsymbol{v ( 4 D )}}+\boldsymbol{R}_{4 D}\right)$ | $\boldsymbol{\gamma}=\frac{\boldsymbol{C}_{\boldsymbol{p ( 4 D )}}}{\boldsymbol{C}_{\boldsymbol{v ( 4 D )}}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| TWs* | 4 | 4 | $4+4=8$ | $U=n C_{v(4 D)} T$ | $C_{v}=\frac{8 R_{4 D}}{2}=4 R_{4 D}$ | $4 R_{4 D}+R_{4 D}=5 R_{4 D}$ | $\gamma=\frac{5}{4}$ |

* Here considered as analogous of photon gas.

In chapter titled "Internal energy of the 4D Universe", we computed the variation in the internal energy of the 4D universe as the opposite of the work done by the radiation pressure of the TWs. Below, we will calculate it thermodynamically. The formula is that anticipated in Table 3, which is identical to the "classical" one. What changes is the value of $C_{v}$.

$$
\begin{equation*}
U=-n C_{v(4 D)} T \tag{47}
\end{equation*}
$$

The value is negative as it represents an expansion.
Furthermore, the change in internal energy of the 4D Universe also depends on the variation in the number of moles ( $n$ ), given that in this theory, $n$ varies with the radius of the 4 D universe (see Eq.41). That is:

$$
U_{\left(R_{i} \rightarrow R_{f}\right)}=-c_{v} \Delta(n T)
$$

From which:

$$
\begin{equation*}
U_{\left(R_{i} \rightarrow R_{f}\right)}=-c_{v}\left(n_{f} T_{f}-n_{i} T_{i}\right)=c_{v}\left(n_{i} T_{i}-n_{f} T_{f}\right) \tag{48}
\end{equation*}
$$

The internal energy of the universe today, that is, its variation from the beginning to today, is equal to:

$$
U_{0}=c_{v(4 D)}\left(n_{B B} T_{B B}-n_{0} T_{0}\right) \cong-(7.53 \pm 0.225) \cdot 10^{192} J
$$

Where the subscript " 0 " indicates the current value, while the subscript "BB" indicates the initial (Big Bang) value. The value obtained thermodynamically is equal to that obtained using the momentum of the TWs (Eq.40), demonstrating the correctness of the reasoning made
earlier.
Let's now consider the equation for work in an adiabatic process in the 4D Universe. We apply the first law of thermodynamics, which is:

$$
U=Q-W=0
$$

But since we are dealing with adiabatic processes where $Q=0$, the previous equation becomes:

$$
U=-W
$$

Substituting Eq. 47 for U , the previous equation transforms into:

$$
W=-n C_{v_{4 D}} T
$$

Recalling that $R_{4 D}=C_{v_{4 D}}\left(\gamma_{4 D}-1\right)$, from which $C_{v_{4 D}}=\frac{R_{4 D}}{\left(\gamma_{4 D}-1\right)}$, the previous equation became:

$$
W=-n \frac{R_{4 D}}{\left(\gamma_{4 D}-1\right)} T
$$

And then:

$$
W\left(\gamma_{4 D}-1\right)=-n R_{4 D} T
$$

And since $n R_{4 D} T=\Pi V_{4}$ (general gas law, Eq.42), we obtain the general equation for work (W) in an adiabatic process in the 4D universe:

$$
\begin{equation*}
W=\frac{\Pi V_{4}}{1-\gamma_{4 D}} \tag{50}
\end{equation*}
$$

The formula is formally identical to the 'Classical' one, however, the gamma factor is referred to the photon gas in the 4D universe (see Table 3). Therefore, the work done by TW (in the form of negative pressure) to expand the universe from a generic initial state (i) to a final one ( f ) is given by:

$$
\begin{equation*}
\Delta W=W_{f}-W_{i}=\frac{\Pi_{f} V_{4 f}-\Pi_{f} V_{4 f}}{1-\gamma_{4 D}} \tag{51}
\end{equation*}
$$

The work performed by the pressure generated by all TWs to expand the 4D universe from the Big Bang to today is the follow:

$$
\begin{equation*}
W_{0}=-U_{0}=\frac{\Pi_{0} V_{40}-\Pi_{B B} V_{4 B}}{1-\gamma_{4 D}}=7.53 \cdot 10^{192} \mathrm{~J} \tag{52}
\end{equation*}
$$

Result in accordance with the equations 40 and 49.

## Determination of the entropy of the 4D universe

The equation that allows calculating the variation of entropy of the 4D universe, as its radius increases from an initial value $R_{i}$ to a final one $R_{f}$ is the follows:

$$
\begin{equation*}
\Delta S_{4 D}=\frac{16 R_{4 D}}{A l_{p l}^{4}}\left[\frac{R_{f} R_{i}\left(R_{f}^{3}-R_{i}^{3}\right)}{\left(R_{f}-R_{i}\right)}+\left(R_{f}^{4}-R_{i}^{4}\right)\right] \tag{53}
\end{equation*}
$$

Refer to Appendix E for how the above equation has been obtained. Let's calculate the variations of entropy of the 4D Universe from:

1. the beginning ( BB ), when $R_{i}=l_{p l}$, and the epoch of the emission of the CMB , that is $R_{f}=R_{C M B}$ (Eq.9):

$$
\begin{equation*}
\Delta S_{B B \rightarrow c m b} \cong 1.88 \cdot 10^{180} \frac{\mathrm{~J}}{{ }^{\circ} \mathrm{K}} \tag{54}
\end{equation*}
$$

2. the beginning and today, when $R_{f}=R_{o}$ (Eq.8):

$$
\begin{equation*}
\Delta S_{B B \rightarrow \text { today }} \cong 2.76 \cdot 10^{192} \frac{\mathrm{~J}}{{ }^{\circ} K} \tag{55}
\end{equation*}
$$

As observed, the entropy of the universe increases with the increase of its radius, confirming that its expansion is an irreversible adiabatic expansion. Furthermore, the entropy has increased by a factor of $10^{12}$ from the epoch of recombination to the present day.

## RELATION BETWEEN MAGNETIC AND ELECTRIC FIELD OF A TEMPORAL WAVE

In this chapter, we will study a highly simplified system, represented by

2 TWs, corresponding to two quanta of matter of opposite charge in the 3D portion of the 4D universe, placed at a constant distance $r$ from each other.
These two TWs of opposite charge attract each other according to Coulomb's law, and this electric attraction is equal to the overall electric force between the same two TWs extended along the time dimension (Fig.3). The previous can be fully extended to a system of 2 TWs having the same electric charge. Obviously, in this case, we will have a repulsion force that always acts along the 3 D part of the 4 D universe.
The quanta of electrically charged matter, corresponding to the TWs in the 3 D portion of the 4 D universe (Figure 3), are charged particles moving at the speed of light $c$, that is the expansion velocity of the 4D Universe along its temporal dimension (radius) (Postulate 1 in [1]). Therefore, they can be considered as an analogous system to a steady current of charged particles moving along a wire of a length equal to the diameter of the 4D universe. Since the two considered TWs correspond, in the 3 D part, to material quanta having opposite charges, there is an attraction between them.


Figure 3) Schematic representation of two TWs with opposite phases $( \pm \pi / 2)$ appearing at the antipodes in the $3 D$ portion of the $4 D$ universe as quanta of matter with opposite electric charges (circles with inside +or - ). The two TWs are positioned at a constant distance $r$ from each other. The magnitude of the electric field (symbols + or - ) is maximum at the centre of the 4D universe and decreases gradually as one approaches the 3D portion of the 4D universe. Yellow arrows represent the magnetic attraction force between the 2 TWs along the time dimension. The blue arrows represent the electric attraction force between the two quanta of matter corresponding to the 2 TWs. The summation of all magnetic forces (Yellow arrows) is equal to twice the electric force acting between the oppositely charged matters quanta placed at the antipodes of the 4D universe. The magnetic field of the TWs is not depicted

The equation describing the magnetic field $B$ generated by a steady current of intensity $I$, moving at velocity $v$ along a conductor wire at point $r$ is the Biot-Savart law. Below is the equation for the magnitude only:

$$
B=\frac{\mu_{0}}{4 \pi} \int \frac{I(d \vec{l} x \vec{r})}{r^{3}}
$$

Where " $I$ " is the steady current intensity, $\mu_{0}$ is the magnetic
permeability in a vacuum, $d \overrightarrow{\boldsymbol{l}}$ is the differential element of length (correspond to $l_{p l}$ ), $\overrightarrow{\boldsymbol{r}}$ is the vector connecting the current element $d \boldsymbol{l}$ to the point where the magnetic field is being measured, $r$ is the distance between $d l$ and the point where the magnetic field is being measured (see also Fig.3).
By definition, the electrical current intensity $I$ is given by $I=\frac{d q}{d t}$. In our case, since $q_{t w}$, the total charge of a corresponding mass quantum in the 3D part of the 4D universe is constant for a radius $R$, we have: $I=\frac{q_{t w}}{t}$. In addition, $t$ (from Eq.1) corresponds to the ratio of the 4D universe diameter $(2 R)$ divided by $c$, then we have:

$$
I=\frac{q_{t w}}{t}=\frac{q_{t w} c}{2 R}
$$

Replacing the last equation in the previous Biot-Savart's one, we get:

$$
B=\frac{\mu_{0}}{4 \pi} \int \frac{q_{t w} c(d \vec{l} x \vec{r})}{2 R r^{3}}
$$

In our case, the vector $\overrightarrow{\boldsymbol{r}}$ can be considered always perpendicular to the TW itself. It follows that the vector product $(d \vec{l} \boldsymbol{l} \overrightarrow{\boldsymbol{r}})$ will be equal to the scalar product $(d l \cdot r)$. Thus:

$$
\begin{equation*}
B_{t w}=\frac{\mu_{0}}{4 \pi} \int \frac{q_{t w} c r d l}{2 R r^{3^{2}}}=\frac{\mu_{0}}{4 \pi} \int \frac{q_{t w} c d l}{r^{2}} \tag{56}
\end{equation*}
$$

Always in our case $r$ is substantially constant along the entire TW (see Fig. 3) as well as R. Therefore, they come out of the integral obtaining:

$$
B_{t w}=\frac{\mu_{0} c q_{t w}}{4 \pi r^{2} 2 R} \int d l
$$

Integrating over the entire length of the TW, that is, between $-R$ and $+R(2 R)$, the previous one becomes:

$$
B_{t w}=\frac{\mu_{0} c q_{t w}}{4 \pi r^{2} 2 R} \int_{-R}^{R} d l=\frac{\mu_{0} c q_{t w}}{4 \pi r^{2} 2 R} 2 R
$$

Hence, simplifying:

$$
\begin{equation*}
B_{t w}=\frac{\mu_{0} c q_{t w}}{4 \pi r^{2}} \tag{57}
\end{equation*}
$$

Notice that $d l=l_{p l}$ thus, even applying the summation, we obtain the same result of 2R.
In fact:

$$
\sum_{n=\frac{-R}{l_{p l}}}^{\frac{R}{l_{p l}}} l_{p l}=2 R
$$

We must also consider, as done for the TW equivalent mass, that a single TW appears as charged matter on opposite sides of the 3D part of the 4D universe (see Fig.3). Therefore, the magnitude of the magnetic field ( $B_{t w}$ ) relative to a single antipode of the 4D Universe must be halved. Thus Eq. 57 transforms in:

$$
\begin{equation*}
B_{t w}=\frac{\mu_{0} c q_{t w}}{8 \pi r^{2}} \tag{58}
\end{equation*}
$$

If we now consider a second TW with an opposite charge placed at a distance $r$ from the previous one (see Fig.3), we find that the magnitude of the magnetic field generated by it is exactly equal to the previous one (Eq.58). Consequently, the magnitude of the total magnetic field generated by the two TWs is equal to their sum.
That is:

And simplifying:

$$
B_{t w(t o t)}=2 \frac{\mu_{0} c q_{t w}}{8 \pi r^{2}}
$$

$$
\begin{equation*}
B_{t w(t o t)}=\frac{\mu_{0} c q_{t w}}{4 \pi r^{2}} \tag{59}
\end{equation*}
$$

Coulomb's law describes the magnitude of the electric field generated by point charges $(q)$ at a distance $(r)$. That is:

$$
E=\frac{K q}{r^{2}}
$$

Where K is the Coulomb's constant.
Now let's apply the same law to a TW corresponding to a quantum of
matter with an electric charge in the 3D part of the 4D universe:

$$
E_{t w}=\frac{K q_{t w}}{r^{2}}
$$

As done previously, also in this case the magnitude of the electric field ( $E_{t w}$ ) relative to a single antipode of the 4D Universe must be halved. Therefore:

$$
\begin{equation*}
t w=\frac{K q_{t w}}{2 r^{2}} \tag{60}
\end{equation*}
$$

If we now consider a second quantum of matter with an electric charge opposite to the previous one, we will find that the total electric field will be twice.
That is:

$$
\begin{equation*}
E_{t w}=\frac{K q_{t w}}{r^{2}} \tag{61}
\end{equation*}
$$

Finally, since the Coulomb constant is equal to $K=\frac{\mu_{0} c^{2}}{4 \pi}$, substituting it into Eq. 61 we obtain:

$$
\begin{equation*}
E_{t w(t o t)}=\frac{\mu_{0} c^{2} q_{t w}}{4 \pi r^{2}} \tag{62}
\end{equation*}
$$

By dividing Eq. 62 by Eq.59, we obtain the previously established relationship between the electric and magnetic fields which, consequently, demonstrates applicability to TWs as well.

$$
\begin{equation*}
E_{t w(t o t)}=B_{T W(t o t)} \cdot c \tag{63}
\end{equation*}
$$

Notice that since the minimum distance in this theory is $l_{p l}$ (corollary 1 to postulate 4), the TWs have a "thickness" equal to $l_{p l}$. This ensures that the intensity of the electric or magnetic field ( E or B ) at the level of the TW itself is not $\pm \infty$ as $r=0$, but rather equal to:

$$
\begin{equation*}
B_{t w}=\frac{\mu_{0} c q_{t w}}{4 \pi l_{p l}^{2}} \tag{64}
\end{equation*}
$$

And

$$
\begin{equation*}
E_{t w}=\frac{\mu_{0} c^{2} q_{t w}}{4 \pi l_{p l}^{2}}=\frac{K q_{t w}}{l_{p l}^{2}} \tag{65}
\end{equation*}
$$

## EQUIVALENCE OF ELECTRIC AND MAGNETIC FORCES BETWEEN TWO TEMPORAL WAVES

In general, the magnetic force exerted on a current-carrying conductor immersed in a magnetic field is called Lorentz's force. The following equation gives its magnitude:

$$
F_{m}=|q| v B \sin (\theta)
$$

Where $|q|$ is the magnitude of charge moving at velocity $v$ along the conductor, $B$ is the magnetic field, and $\theta$ represents the angle between the velocity vector and the magnetic field.
Continuing the analogy with TWs, we find that $\theta$, which in our case represents the angle between the velocity vector $\overrightarrow{\boldsymbol{v}}$ of a TW and the magnetic field, is $90^{\circ}$. Therefore, the total magnetic force exerted between two TWs of opposite charge placed at distance $r$ from each other (see Fig.3) is equal to:

$$
\begin{equation*}
F_{m \cdot(t o t)}=q_{t w} c B_{t w(t o t)} \tag{66}
\end{equation*}
$$

And substituting Eq. 59 with the previous one (Eq.66), we have:

$$
F_{m .(t o t)}=q_{t w} c \frac{\mu_{0}\left|q_{T W}\right| c}{4 \pi r^{2}}
$$

From which:

$$
\begin{equation*}
F_{m .(t o t)}=\frac{\mu_{0} c^{2} q_{t w}^{2}}{4 \pi r^{2}} \tag{67}
\end{equation*}
$$

The electric force between two point charges ( $q_{1}$ and $q_{2}$ ) of opposite sign is given by Coulomb's equation:

$$
F_{e}=\frac{K q_{1} q_{2}}{r^{2}}
$$

From the previous equation, we derive the electric force of attraction between two quanta of matter with opposite electric charges,
corresponding to two TWs in the 3D part of the 4D universe:

$$
\begin{equation*}
F_{e(T W)}=\frac{K q_{t w}^{2}}{r^{2}} \tag{68}
\end{equation*}
$$

Since K, the Coulomb constant, equals $\frac{\mu_{0} c^{2}}{4 \pi}$, by substituting it into Eq.68, we obtain:

$$
\begin{equation*}
F_{e(t w)}=\frac{\mu_{0} q_{q w}^{2} c^{2}}{4 \pi r^{2}} \tag{69}
\end{equation*}
$$

Equation identical to the Eq.67, namely:

$$
\begin{equation*}
F_{e(t w)}=F_{m(t w)} \tag{70}
\end{equation*}
$$

Note that these forces only act within the 3D part of the 4D universe. Indeed, as discussed, along the temporal dimension there is no acceleration and physical quantities derived from it [1].
In conclusion, between two TWs out of phase by $180^{\circ}$, there exists only one type of force, which acts perpendicularly along the entire time dimension, corresponding to what we know as the magnetic force.
According to the restricted holographic principle, this force manifests in the 3D part of the 4D universe as the electric force between the two matters quanta corresponding to TWs with opposite charges (see also Fig.3) [1].

## ELECTRIC AND MAGNETIC FIELDS OF A SYSTEM COMPOSED OF TWO TEMPORAL WAVES

In general, an attraction or repulsion process, for example, between charges of the same sign or opposite sign, occurs because it leads the system to the state of minimum energy.
Repulsion. Using, as an example, a generic system consisting of two charges with the same sign, there will be repulsion because the system tends to reach a stable state, namely, a state of minimum potential electrostatic energy. This state will be reached when their distance tends to $+\infty$. Indeed:

$$
\begin{equation*}
U_{E(\min .)}=\lim _{r \rightarrow+\infty} \frac{K q_{1}\left(-q_{2}\right)}{r}=0 \tag{71}
\end{equation*}
$$

Attraction. Using as an example a generic system consisting of two charges of opposite signs, this system reaches a stable state, namely, a state of minimum energy, when the distance between the charges tends to zero. That is:

$$
\begin{equation*}
U_{E(\min )}=\lim _{r \rightarrow 0^{+}} \frac{K q_{1}\left(-q_{2}\right)}{r}=-\infty \tag{72}
\end{equation*}
$$

In this theory, since space, and its time equivalent, are quantized (corollary 1 to postulate 4), the quanta of matter corresponding to TWs in the 3 D part of the 4 D universe, have a "characteristic dimension" equal to $l_{p l}$. In addition, the quantization of space implies that even the quanta of matter cannot be point-like but will have a minimum diameter equal to $l_{p l}$. This means that two TWs, cannot cancel each other out by destructive interference, because they would have to be at a distance equal to zero that is not possible. Furthermore, a TW and, consequently, the corresponding 3D particle, will have an electric field distributed on itself equal to:

$$
\begin{equation*}
E=\frac{K q}{l_{p l}^{2}} \tag{73}
\end{equation*}
$$

From the above statement, the maximum approach between the two TWs will be equal to $2 l_{p l}$ i.e., the minimum allowed distance $\left(l_{p l}\right)$ plus the "characteristic dimension" of the TWs $\left(l_{p l}\right)$. (Note that the separation between TWs occurs only along the 3D part of the 4D universe).
Regarding the electric fields of the two TWs, they do not cancel out but decrease when the two TWs are at a minimum distance $\left(2 l_{p l}\right)$. It will be given by the algebraic sum of the electric field on itself (Eq.73), and the electric field exerted by the second TW on the first. That is:

$$
E_{t w(1)(2-s y s)}=E_{t w(1)}+E_{t w(2)}=\frac{K q_{t w}}{l_{p l}^{2}}+\frac{K\left(-q_{t w}\right)}{\left(2 l_{p l}\right)^{2}}=\frac{3 K q_{t w}}{4 l_{p l}^{2}}(74)
$$

The same result for the second TW:

$$
E_{t w(2)(2-s y s)}=E_{t w(2)}+E_{t w(1)}=\frac{K\left(-q_{t w}\right)}{l_{p l}^{2}}+\frac{K q_{t w}}{\left(2 l_{p l}\right)^{2}}=-\frac{3 K q_{t w}}{4 l_{p l}^{2}}
$$

Where $q_{t w}$ is the absolute electric charge of the isolated TW, and the subscript " 2 -sys" indicates the electric field relative to the binary system. Therefore, we have a $3 / 4$ reduction of the electric field intensity of both TWs respect to the isolated TWs, and the binary system has an overall zero electric charge.
The same reduction to $3 / 4$ of the initial intensity is observed for the magnetic field intensity $B_{t w(2-s y s)}$.
Since K and $2 l_{p l}$ are constants, the obtained result indicates that in the system composed of two TWs, the charge of each TW decreases compared to the charge of an isolated TW, while their sum is always zero. Suppose, for example, we want to evaluate the electric charge of a single TW by measuring the electric field of one of the two TWs inside the aforementioned binary system. To do this, we use the following general equation:

$$
q=E \frac{r^{2}}{K}
$$

In our specific case, since $r=l_{p l}$ and $E=E_{t w(2-s y s)}$ which is given by Eq. 74 , the previous one becomes:

$$
q_{t w(2-s y s)}=E_{t w(2-s y s)} \frac{l_{p l}^{2}}{K}=\frac{3 K q_{t w(i s o l a t e d)}}{4 l_{p l}^{2}} \cdot \frac{l_{p l}^{2}}{K}
$$

From this, simplifying we obtain:

$$
q_{t w(2-s y s)}=\frac{3}{4} q_{t w(i s o l a t e d)}
$$

And then:

$$
q_{t w(\text { isolated })}=\frac{4}{3} q_{t w(2-s y s)}
$$

This demonstrates that the electric charge of an isolated TW is greater than the electric charge of a TWs binary system.
Extrapolating, and very generically, we can say that in a system composed of $n$ TWs, the absolute charge of individual TWs inside the system, compared to that of an isolated TW, decreases proportionally to the number of TWs composing the system itself. As we will see in more detail in the chapter entitled "Electric Charge of Isolated Temporal Wave", the fundamental electric charge associated with elementary particles, electrons and protons, will be lower than that of an isolated TW.
This is because, in this theory, these particles constitute a system composed of numerous quanta of matter with opposite charges, corresponding to TWs out by phase of $180^{\circ}$.
Now we will consider the energy implications of the above, always in the case of two TWs of opposite charge. The minimum electrostatic potential energy must be calculated using the general equation associated with the electric field.
That is:

$$
U_{E}=q \cdot E \cdot r
$$

From which:

$$
U_{t w(2-s y s)}=-q_{t w} \frac{3 K q_{t w}}{4 l_{p l}^{2}} 2 l_{p l}
$$

And simplifying:

$$
\begin{equation*}
U_{t w(2-s y s)}=-\frac{3 K q_{t w}^{2}}{2 l_{p l}} \tag{75}
\end{equation*}
$$

Important to note that the effect of space quantization on the electrostatic potential energy of the system consisting of two oppositely charged TWs, besides preventing their cancellation, reduces it respect to a classical system where this energy would have been:

$$
U_{E}=-\frac{K q^{2}}{l_{p l}}
$$

This energy reduction represents the binding energy of the binary TWs system, and is provided by the following equation:

$$
U_{E}-U_{t w(2-s y s)}=-\frac{K q^{2}}{2 l_{p l}}-\left(-\frac{3 K q_{t w}^{2}}{2 l_{p l}}\right)=\frac{K q^{2}}{2 l_{p l}}(3-1)=\frac{K q^{2}}{l_{p l}}
$$

In conclusion, in this simple system consisting of TWs out of phase by $180^{\circ}$ (corresponding to two quanta of matter with opposite charge), they can approach the minimum distance allowed by this theory $\left(2 l_{p l}\right)$, reaching a stable state of minimum energy given by Eq. 75 . Furthermore, their electric and magnetic fields are reduced to $3 / 4$ of their initial values. Also, the absolute value of the electric charge of the two TWs in the binary system decreases by $3 / 4$ compared to that of a TW in an isolated system.

## TEMPORAL WAVES AS STATIONARY ELECTROMAGNETIC WAVES

The third postulate states that the TW is a stationary electromagnetic wave perceived by us in the 3D part of the universe as mass with a positive charge if the phase shift is $-90^{\circ}$ degrees and negative if it is $+90^{\circ}$ degrees.
Below, we will determine the equation of a TW understood as a stationary electromagnetic wave with a wavelength equal to 4 times the radius of the 4D universe.
The equation of a generic stationary wave is as follows:

$$
\begin{equation*}
y=2 A \sin \left(\frac{\pi x}{\lambda}+\varphi\right) \cos (\pi f t+\varphi) \tag{76}
\end{equation*}
$$

Where $y$ represents the amplitude of the wave at a specific point $x, \mathrm{~A}$ represents the absolute maximum amplitude of the wave, while $\varphi$ indicates the phase shift.
Since $\lambda=c / f$ and, in this theory, $x=c t$, by substituting into Eq. 76 and simplifying, we have:

$$
\begin{equation*}
y=2 A \sin (\pi f t+\varphi) \cos (\pi f t+\varphi) \tag{77}
\end{equation*}
$$

Since $2 \sin (\alpha) \cos (\alpha)=\sin (2 \alpha)$ and calling $\alpha=\pi f t+\varphi$, then Eq. 77 transforms into the following one:

$$
\begin{equation*}
y=A \sin (2 \pi f t+2 \varphi) \tag{78}
\end{equation*}
$$

Knowing that $f=c / \lambda, \quad t=x / c, \lambda_{t w(t)}=4 R_{t}$ (Eq.4), and substituting them into Eq.78, we obtain:

$$
\begin{equation*}
y=A \sin \left(\pi \frac{c x}{2 R_{t} c}+2 \varphi\right) \tag{79}
\end{equation*}
$$

setting $y=E_{t w(x)}, A=E_{0\left[t w\left(R_{t}\right)\right]}$, which represents the absolute maximum electric field, we derive the equation of the electric field of a generic TW in the 4D universe:

$$
E_{T W\left(x, R_{t}\right)}=E_{0\left[t w\left(R_{t}\right)\right]} \sin \left(\frac{\pi x}{2 R_{t}}+2 \varphi\right)
$$

Given that $2 \varphi= \pm 90^{\circ}$ (Postulate 3) and knowing from trigonometry that $\sin (\alpha \pm \pi / 2)=\mp \cos (\alpha)$, the previous equation simplifies into the sought-after one:

$$
\begin{equation*}
E_{T W\left(x, R_{t}\right)}= \pm E_{0\left[t w\left(R_{t}\right)\right]} \cos \left(\frac{\pi x}{2 R_{t}}\right) \tag{80}
\end{equation*}
$$

Where $x$ indicates a point along the time dimension (of amplitude $l_{p l}$ ), ranging from the privileged coordinates $-R_{t}$ to $+R_{t}, E_{t w\left(x, R_{t}\right)}$ indicate the intensity of the electric field of a TW at point $x$ (along the radius $R_{\mathrm{t}}$ of the 4D universe), and $E_{0\left[t w\left(R_{t}\right)\right]}$ the maximum absolute amplitude of the electric field.
Given the known relationship between electric and magnetic fields
$(E=B \cdot c)$, we can derive the equation of the magnetic field of a generic TW:

$$
\begin{equation*}
B_{T W\left(x, R_{t}\right)}= \pm B_{0\left[t w\left(R_{t}\right)\right]} \cos \left(\frac{\pi x}{2 R_{t}}\right) \tag{81}
\end{equation*}
$$

Figure 4 outlines a TW, with its electric and magnetic fields, in the 4D universe.


Figure 4) Schematic representation of electric and magnetic fields of a single TW

## MAXIMUM AMPLITUDE OF ELECTRIC AND MAGNETIC FIELDS OF TWs

To determine the equation that allows us to calculate the amplitude of the electric and magnetic fields of TWs, it is necessary to know its energy density and the 3D volume (3D hypersurface, or S3, see Eq.3) in which one TW appears "enclosed" in the 3D part of the 4D universe. Corollary 1 of postulate 4 states that 4D space is quantized, and each quantum of space is equal to a Planck length $\left(l_{p l}\right)$. Consequently, the apparent 3D volume of each TW at any epoch, or radius of the 4D universe, will be equal to:

$$
\begin{equation*}
S_{3(T w)}=2 \pi^{2} l_{t w}^{3} \cong 8.33 \cdot 10^{-104} m^{3} \tag{82}
\end{equation*}
$$

Energy density of a TW $\left(u_{t w_{3 D}}\right)$ is given by the ratio of its energy (Eq.10) to its 3D volume (Eq.82). That is:

$$
u_{t w_{3 D}}=\frac{U_{t w\left(R_{t}\right)}}{S_{3(T w)}}=\frac{h c}{4 R_{t}} \cdot \frac{1}{2 \pi^{2} l_{p l}^{3}}
$$

From which:

$$
\begin{equation*}
u_{t w_{3 D}}=\frac{h c}{8 \pi^{2} R_{t} l_{p l}^{3}} \tag{83}
\end{equation*}
$$

The TW is a temporal stationary electromagnetic wave. For all electromagnetic waves we know that their instantaneous energy density $\left(u_{t}\right)$ is also given by the following equation, which utilizes the electric field at instant $t\left(E_{t}\right)$ :

$$
\begin{equation*}
u_{t}=\varepsilon_{0} E_{t}^{2} \tag{84}
\end{equation*}
$$

Where $\varepsilon_{0}$ indicates the vacuum permittivity constant.
In this theory, the instant $t$ corresponds to a point $x=c t$ along the time dimension. Therefore, in the case of the TWs, Eq. 85 becomes the following:

$$
\begin{equation*}
u_{t w\left(x, R_{t}\right)}=\varepsilon_{0} E_{x\left[t w\left(R_{t}\right)\right]}^{2} \tag{85}
\end{equation*}
$$

The index $R_{t}$ appears because the energy of TWs varies as the radius of the universe changes (Eq.10).
Recall that $x$ is not a point but is a discrete and constant value equal to $l_{p l}$. Therefore, the total energy density of a TW is given by Eq. 83 , which corresponds to the summation of Eq. 86 for $n$ ranging from $-R_{t} / l_{p l}$ to $+R_{t} / l_{p l}$, representing the total number of space quanta $\left(l_{p l}\right)$ that composing the entire diameter of the 4D Universe (from $-R_{t}$ to $+R_{t}$ ).
That is:

$$
\begin{equation*}
\varepsilon_{0} \sum_{n=-\frac{R_{t}}{l_{p l}}}^{\frac{R_{t}}{l_{p l}}} E_{n\left[t w\left(R_{t}\right)\right]}^{2}=\frac{h c}{8 \pi^{2} R_{t} l_{t w}^{3}} \tag{86}
\end{equation*}
$$

By substituting Eq. 80 for $E$, we have:

$$
\begin{equation*}
\varepsilon_{0} \sum_{n=-\frac{R_{t}}{l_{p l}}}^{\frac{R_{t}}{l_{p l}}} E_{0\left[t w\left(R_{t}\right)\right]}^{2} \cos ^{2}\left(\frac{\pi l_{p l}}{2 n}\right)=\frac{h c}{8 \pi^{2} R_{t} l_{t w}^{3}} \tag{87}
\end{equation*}
$$

Since $E_{0}$ is constant respect to $R_{t}$ it comes out of the summation and the Eq. 87 becomes:

$$
\begin{equation*}
\varepsilon_{0} E_{0\left[t w\left(R_{t}\right)\right]}^{2} \sum_{n=-\frac{R_{t}}{l p l}}^{\frac{R_{t}}{l_{p l}}} \cos ^{2}\left(\frac{\pi l_{p l}}{2 n}\right)=\frac{h c}{8 \pi^{2} R_{t} l_{p l}^{3}} \tag{88}
\end{equation*}
$$

Because:

$$
\sum_{n=-\frac{R_{t}}{l_{p l}}}^{\frac{R_{t}}{l_{p l}}} \cos ^{2}\left(\frac{\pi l_{p l}}{2 n}\right) \cong \frac{2 R_{t}}{l_{p l}}
$$

The equation 88 becames:

$$
\begin{equation*}
\varepsilon_{0} E_{0\left[t w\left(R_{t}\right)\right]}^{2} \frac{2 R_{t}}{l_{p l}}=\frac{h c}{8 \pi^{2} R_{t} l_{p l}^{3}} \tag{89}
\end{equation*}
$$

Then, we isolate $E_{0}$ :

$$
E_{0\left[t w\left(R_{t}\right)\right]}^{2}=\frac{h c l_{p l}}{8 \pi^{2} \varepsilon_{0} R_{t} l_{p l}^{3} 2 R_{t}}=\frac{h c}{8 \pi^{2} \varepsilon_{0} R_{t} l_{p l}^{2} 2 R_{t}}
$$

From which:

$$
\begin{equation*}
E_{0\left[t w\left(R_{t}\right)\right]}=\frac{1}{4 \pi R_{t} l_{p l}} \sqrt{\frac{h c}{\varepsilon_{0}}} \quad\left[\frac{N}{C}\right] \tag{90}
\end{equation*}
$$

From equation 91 we can calculate $E_{0}$ of the TWs:

1. Today:

$$
\begin{equation*}
\mathrm{E}_{0\left[\mathrm{tw}\left(\mathrm{R}_{0}\right)\right]}=\frac{1}{4 \pi \mathrm{R}_{\mathrm{o}} \mathrm{l}_{\mathrm{pl}}} \sqrt{\frac{\mathrm{hc}}{\varepsilon_{0}}}=5.373 \pm 0,040 \frac{\mathrm{~N}}{\mathrm{C}} \tag{91}
\end{equation*}
$$

2. At CMB emission:

$$
E_{0\left[t w\left(R_{e}\right)\right]}=\frac{1}{4 \pi R_{e} l_{p l}} \sqrt{\frac{h c}{\varepsilon_{0}}}=(5.916 \pm 0,044) \cdot 10^{3} \frac{\mathrm{~N}}{\mathrm{C}}
$$

3. At beginning (Big Bang):

$$
\begin{equation*}
E_{0\left[t w\left(R_{b b}\right)\right]}=\frac{1}{4 \pi l_{p l}^{2}} \sqrt{\frac{h c}{\varepsilon_{0}}}=4.563 \cdot 10^{61} \frac{\mathrm{~N}}{\mathrm{C}} \tag{93}
\end{equation*}
$$

where $R_{0}, R_{\mathrm{e}}$ and $R_{b b}=l_{p l}$ represent the radius of the 4 D universe today, at the time of CMB emission, and at the beginning (Big Bang), respectively.
Finally, using Eq. 63 it is possible to calculate the amplitude of the magnetic field $B_{0\left[t w\left(R_{t}\right)\right]}$.
Note that the maximum amplitudes $E_{0}$ of the magnetic and electric fields of TWs at various epochs or radii of the 4D universe occur exactly at the origin of the privileged reference system (the Big Bang). This is simplistically illustrated in Fig. 4.

## ELECTRIC CHARGE OF AN ISOLATED TEMPORAL WAVE

The equation 80 describes the electric field of the TW.

$$
E_{t w\left(x, R_{t}\right)}= \pm E_{0\left[t w\left(R_{t}\right)\right]} \cos \left(\frac{\pi x}{2 R_{t}}\right)
$$

By substituting Eq. 91 for $E_{0\left[t w\left(R_{t}\right)\right]}$, we obtain:

$$
E_{t w\left(x, R_{t}\right)}=\frac{1}{4 \pi R_{t} l_{p l}} \sqrt{\frac{h c}{\varepsilon_{0}}} \cos \left(\frac{\pi x}{2 R_{t}}\right)
$$

Using the previous equation, we calculate the total electric field of a TW along the entire diameter of the 4D universe.

$$
\begin{equation*}
E_{t w\left(T o t, R_{t}\right)}=\frac{1}{4 \pi R_{t} l_{p l}} \sqrt{\frac{h c}{\varepsilon_{0}}} \sum_{n=-\frac{R_{t}}{l_{p l}}}^{\frac{R_{t}}{l_{p l}}} \cos \left(\frac{\pi l_{p l}}{2 n}\right) \tag{94}
\end{equation*}
$$

Since:

$$
\sum_{n=-\frac{R_{t}}{l_{p l}}}^{\frac{R_{t}}{l_{p l}}} \cos \left(\frac{\pi l_{p l}}{2 n}\right) \cong \frac{2 R_{t}}{l_{p l}}
$$

The equation 94 became:

$$
E_{t w\left(T o t, R_{t}\right)}=\frac{1}{4 \pi R_{t} l_{p l}} \sqrt{\frac{h c}{\varepsilon_{0(4 D)}}} \cdot \frac{2 R_{t}}{l_{p l}}
$$

Hence, simplifying:

$$
\begin{equation*}
E_{t w\left(T o t, R_{t}\right)}=\frac{1}{2 \pi l_{l p}^{2}} \sqrt{\frac{h c}{\varepsilon_{0}}} \tag{95}
\end{equation*}
$$

Knowing that the total electric field of a TW along itself $E_{t w\left(T o t, R_{t}\right)}$ is given by Eq. 65, then the previous one becomes:

$$
\frac{K q_{t w}}{l_{p l}^{2}}=\frac{1}{2 \pi l_{l p}^{2}} \sqrt{\frac{h c}{\varepsilon_{0}}}
$$

Simplifying and isolating the charge $q_{t w}$, we have:

$$
q_{t w}=\frac{1}{2 \pi K} \sqrt{\frac{h c}{\varepsilon_{0}}}
$$

The previous equation must be divided by 2 as it represents the charge on only one side (antipode) of the 4D universe obtaining:

$$
\begin{equation*}
q_{t w}=\frac{1}{4 \pi K} \sqrt{\frac{h c}{\varepsilon_{0}}} \tag{96}
\end{equation*}
$$

Let's substitute K with the known relationship involving $\varepsilon_{0}$ :

$$
K=\frac{1}{4 \pi \varepsilon_{0}}
$$

Obtaining:

$$
q_{t w}=\frac{4 \pi \varepsilon_{0}}{4 \pi} \sqrt{\frac{h c}{\varepsilon_{0}}}=\varepsilon_{0} \sqrt{\frac{h c}{\varepsilon_{0}}}
$$

By inserting $\varepsilon_{0}$ within the square root, and simplifying, we obtain the equation that allows to calculate the absolute charge of a TW:

$$
\begin{equation*}
q_{t w}=\sqrt{h c \varepsilon_{0}}=1.326 \cdot 10^{-18}[\text { Coulomb }] \tag{97}
\end{equation*}
$$

It is important to note that, unlike all other physical quantities studied here, which depend on the radius $R$ of the 4 D universe, the electric charge of a TW, and therefore, the corresponding 3D amount of charged matter, remains constant for any R.
The value obtained is approximately 8.28 times the fundamental charge $\left(1,602 \cdot 10^{-19} C\right)$ in agreement with what was anticipated in the chapter named "Electric and magnetic fields of a system composed of two Temporal Waves". In fact, in this chapter has been stated that the electric charge of an isolated TW is always greater than that of a system of TWs, such as an elementary particle, like the electron or the proton.
Consequently, it is logical to hypothesize that the interaction between all the TWs constituting a charged particle (electron, proton, quark etc) will result in a reduction of the electric charge by approximately 8.28 times compared to that of a single TW.

## CONCLUSION

This theory drastically simplifies the existing universe and explains its origin without resorting to a singularity. In summary, the universe originated from a quantum vacuum that has always existed, through the spontaneous generation of four initial TWs (Temporal Waves), which, by exerting radiation pressure, caused the creation of 4D space and its continuous expansion. This radiation pressure represents the dark energy $(\Lambda)$ of the $\Lambda$ CDM model.
The overall energy balance of the universe always remains zero, as was zero at its origin (Big Bang).
The 4D Universe consists of four-dimensional space and stationary electromagnetic waves oscillating at the lowest possible frequency along one of the four spatial dimensions that we perceive as time. According
to the restricted holographic principle (postulate 2 in [1]), in the 3D portion of the universe where we reside, we have that:

1. The energy of TWs corresponds to mass, and thus each TW corresponds to a quantum of matter.
2. There are only two types of TWs with phases of $-90^{\circ}$ and $+90^{\circ}$ corresponding to negative and positive charge, respectively.
3. The electrical force exerted along the 3D part of the 4 D universe between the electric fields of two adjacent TWs is exactly equal to that exerted between the magnetic fields of these TWs.

In this work, we developed the thermodynamic equations, including those to determine the temperatures of the 4D universe at various redshifts ( $Z$ ). These calculated temperatures all agree with those published for Z of $0.89,3.025$ and 6.34.
In this theory, the quanta of matter corresponding to TWs as they appear to us in the 3D portion of the 4D universe, represents to the so-called dark matter. These quanta, which form around already formed and rotating cosmic masses, would constitute the so-called halos of dark matter present around galaxies. They follow the rotation of the galactic mass itself, thus without "falling" towards its centre and without the possibility of aggregating to the mass around which they rotate.

On the other hand, since these quanta are very small (with a diameter equal to the Planck length), they can only be observed and interact with electromagnetic waves having wavelengths equal to, or shorter than the Planck length, the only ones with such resolving power. Radiation of such high energy essentially does not exist in nature, which would explain why these quanta of matter (That is, dark matter) would not be observable, despite maintaining gravitational action. In fact, the most energetic gamma-ray emission ever detected is reported from the Vela pulsar [9]. It reaches at least 20 teraelectronvolts, corresponding to a wavelength of approximately $10^{-20} \mathrm{~m}$, much higher than the Planck length that is $\sim 10^{-35}$ meters.

The TWs act simultaneously in two opposite zones of the 4D universe (see for Example Fig.3). From this it follows that the 4D universe is specular, meaning that at the antipodes there could exist another Earth, another solar system, identical to ours. This is because, in this theory, time is simply the radius of the 4D universe, so the evolutionary process that led to the aforementioned formations must be identical on both antipodes. In other words, to the antipodes to this earth, there could be another me writing this paper.

## 4D multiverse

In the chapter titled "Uncertainty Principle and Stability of TWs," we have seen that all the TWs are stable. Through the Bohr-Wigner Uncertainty Principle relation and using the energy of the first 4 TWs (see Eq.10),

$$
4 U_{T W\left(l_{p l}\right)}=\frac{h c}{l_{p l}}=1.228 \cdot 10^{10} \mathrm{~J}
$$

we can determine the average statistical frequency $\bar{f}$ with which the 4 TWs can be generated, initiating the creation of a 4D universe from the quantum vacuum. Since in this case the uncertainty on time indicates the average frequency at which the event occurs, we have:

$$
\begin{equation*}
\bar{f} \sim \frac{1}{\Delta t}=\frac{\Delta U}{h}=4 \frac{h c}{4 l_{p l} h}=\frac{c}{l_{p l}} \sim 1.85 \cdot 10^{43} \mathrm{~s}^{-1} \tag{99}
\end{equation*}
$$

Equation 99 indicates that the spontaneous creation of 4 TWs capable of initiating a new 4D universe within a Planck length in the quantum vacuum, can occur on average once every $1.85 \times 10^{43} s=5.88 \times$ $10^{35}$ years. Converting time in space, by using Eq. 1 , we have that, on average, each:
$5.56 \times 10^{51} \mathrm{~m}$
a 4D universe can be generated. This is schematized in Figure 5.


Figure 5) The figure schematizes the formation of $4 D$ universes distributed in the always-existing quantum vacuum. Within a very small region (Planck length), according to the Bohr-Wigner uncertainty principle, 4 TWs can be generated, from which new 4D universes arise

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## APPENDIX A

In this appendix, there are some examples of summation calculations for the discrete functions of $R$. For example, $R d R$ transform in $R(\Delta R)_{\text {min }}=R \cdot r_{p l}$.
Where $(\Delta R)_{\min }$ indicates the minimum variation of R that is $(\Delta R)_{\min }=r_{p l}$
So, while:

$$
h_{(R)}=\int_{0}^{R} R d R=\frac{R^{2}}{2}
$$

In our case, being $l_{p l}$ constant it comes out of the summation and the resulting function $h_{(R)}$ can be calculated by the following equation:

$$
\begin{equation*}
h_{(R)}=R \sum_{n=r_{p l}}^{\frac{R}{p_{p l}}} l_{p l}=R l_{p l} \sum_{n=l_{p l}}^{\frac{R}{l_{p l}}} 1=R l_{p l} \cdot \frac{R}{l_{p l}}=R^{2} \tag{A1}
\end{equation*}
$$

Another general example is the integration of the differential of two functions products:

$$
\int d\left(f_{(R)} g_{(R)}\right)=\int f_{(R)} d g_{(R)}+\int f_{(R)} d f_{(R)}
$$

which can be replaced by the variation "delta ( $\Delta$ )" from the final state " $f$ " and the initial state " $i$ ", if they are function of $R$ :

$$
\begin{equation*}
\Delta\left(f_{(R)} g_{(R)}\right)=f_{(R)_{f}} g_{(R)_{f}}-f_{(R)_{i}} g_{(R)_{i}} \tag{A.2}
\end{equation*}
$$

For example, from Eq.A. 2 we derived that the definite integral of $\operatorname{Rd}\left(\frac{1}{R^{2}}\right)$ from $R_{i}$ to $R_{f}$ is:

$$
\Delta h_{(R)}=\int_{R_{i}}^{R_{f}} R d\left(\frac{1}{R^{2}}\right)=\int_{R_{i}}^{R_{f}}-\frac{2 R}{R^{3}} d R=2\left(\frac{1}{R_{f}}-\frac{1}{R_{i}}\right)
$$

While, based on Eq.A.2, and considering $f_{(R)}=R$, and $g_{(\mathrm{R})}=\frac{1}{\mathrm{R}^{2}}$, we have:

$$
\begin{equation*}
\Delta h_{(R)}=R_{f} \frac{1}{R_{f}^{2}}-R_{i} \frac{1}{R_{i}^{2}}=\frac{1}{R_{f}}-\frac{1}{R_{i}} \tag{A.3}
\end{equation*}
$$

In fact, in terms of summation, we have:

$$
\begin{aligned}
& \Delta h_{(R) \min .}=R \Delta\left(\frac{1}{R_{\text {min. }}^{2}}\right)=R \Delta\left(\frac{1}{l_{p l}^{2}}\right) \\
& \Delta h_{\left.R_{f} \rightarrow R_{i}\right)}=\frac{R}{\Delta\left(\sum_{n=l_{p l}}^{\frac{R}{l_{l}}} l_{p l}\right)^{2}}= \\
& =\frac{R}{\Delta\left[l_{p l}^{2}\left(\sum_{n=l_{p l}}^{\frac{R}{p_{p l}}} 1\right)^{2}\right]}=\frac{R}{\Delta\left[l_{p l}^{2} l_{p l}^{R_{p l}^{2}}\right]}
\end{aligned}
$$

Therefore:

$$
\frac{R}{\Delta R^{2}}=\Delta\left(\frac{1}{R}\right)=\frac{1}{R_{f}}-\frac{1}{R_{i}}
$$

## APPENDIX B

## (On the Redshift of the Cosmic Microwave Background)

The redshift due to the expansion of the Universe is indicated by the letter Z and can be calculated based on the following equation:

$$
\begin{equation*}
Z=\frac{\lambda_{0}-\lambda_{e}}{\lambda_{e}}=\frac{\lambda_{0}}{\lambda_{e}}-1 \tag{B.1}
\end{equation*}
$$

Where $\lambda_{0}$ and $\lambda_{e}$ indicate, respectively, the wavelength observed today from Earth and the wavelength of the electromagnetic wave when ha been emitted.
From Wien's law we know that for blackbody radiation, the product of the maximum wavelength and the temperature " T " of a blackbody is equal to a constant, denoted as " $b$ ":

$$
\begin{equation*}
\lambda=\frac{b}{T} \tag{B.2}
\end{equation*}
$$

Substituting Eq.B. 2 into Eq.B. 1 we have Z expressed as a function of the temperature.

$$
\begin{equation*}
Z=\frac{T_{Z}}{T_{0}}-1 \tag{B.3}
\end{equation*}
$$

From Eq.B. 3 we isolate $T_{z}$ obtaining the equation that allows us to calculate the maximum temperature of the cosmic microwave background (CMB) at various Z , knowing the current peak temperature $\left(\mathrm{T}_{0}=2.72548 \pm 0.00057^{\circ} \mathrm{K}\right)[8]$.

$$
\begin{equation*}
T_{z}=(Z+1) T_{0} \tag{B.3.1}
\end{equation*}
$$

The redshift attributed to the CMB is approximately 1,100 . The temperature at which hydrogen atoms become stable, making the universe transparent to radiation, is approximately $3,000^{\circ} \mathrm{K}$. The current temperature $(\mathrm{Z}=0)$ of the CMB is $\sim 2.725^{\circ} \mathrm{K}[8]$.

Thus, the value 1,100 can be obtained by substituting in Eq.B. 3 to $\mathrm{T}_{z}$ the value $3,000^{\circ} \mathrm{K}$ and to $\mathrm{T}_{0}$ the value $2.725^{\circ} \mathrm{K}$ obtaining:

$$
\begin{equation*}
Z=\frac{3000}{2.725}-1 \cong 1,100 \tag{B.3.2}
\end{equation*}
$$

Z can also be calculated based on the scale factor "a" of the 3D part of the 4D universe. It represents the size of the universe at a certain time $\left(a_{t}\right)$ relative to the current one $\left(a_{0}\right)$.
The latter is equal to 1 .

$$
\begin{equation*}
Z=\frac{a_{0}-a_{t}}{a_{t}}=\frac{a_{0}}{a_{t}}-1=\frac{1}{a_{t}}-1 \tag{B.4}
\end{equation*}
$$

In this theory the real universe 4D has its own ray (postulate 1 in [1]), so we can replace the scale factor with the radius of the universe 4D by obtaining:

$$
\begin{equation*}
Z=\frac{R_{0}-R_{t}}{R_{t}}=\frac{R_{0}}{R_{t}}-1 \tag{B.5}
\end{equation*}
$$

Where $R_{t}$ denotes the radius of the 4D universe at privileged time $t$ and $R_{o}$ is the radius of the 4 D universe today.

## APPENDIX C

## (Hubble Tension and Hubble Constant)

The precise value of the Hubble constant $H_{0}$ has again become one of the most debated topics in cosmology. This debate has been fuelled by the apparent disagreement between local, direct measurements of $\mathrm{H}_{0}$ $\sim 73 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{MPC}^{-1}$, and that estimates derived from the Cosmic Microwave Background (CMB) which give $67.4 \pm$ $0.5 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{MPC}^{-1}[10,11]$. Hence, the measurements of the Hubble constant in the late universe and early universe exhibit a significant non-negligible discrepancy. This issue is commonly known as the Hubble tension.
There are several hypotheses attempting to "resolve" this tension. For example, Haslbauer et al. have shown that a high local Hubble constant naturally arises due to gravitationally driven outflows from the observed Keenan-Barger-Cowie (KBC) super void into a large local region of approximately 300 MPC around the local group with underdensity [12].

Furthermore, analysing recent literature reveals that several articles are "easing" the Hubble tension by employing new methods for local measurements, such as the inverse distance ladder technique, through which the authors obtain an estimated value of $\mathrm{H}_{0}$ of $67.8 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{MPC}^{-1}$ [13]. Another method involves measuring the time delay between multiple images of a supernova subject to gravitational lensing. In this second case, the authors obtain an estimated value of $\mathrm{H}_{0}$ of $66.66_{-3.3}^{+4.1} \mathrm{~km} \mathrm{~s}^{-1} M P C^{-1}$ [15]. Both data agree with that derived from the CMB $\left(67.4 \pm 0.5 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{MPC}^{-1}\right)$

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[11]. Lastly, Zhang et al., utilizing a cosmological model-independent radial basis function neural network to describe the Hubble parameter as a function of redshift, obtain a value of $\mathrm{H}_{0}$ of $67.1 \pm$ $9.7 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{MPC}^{-1}$, which is more noise-resistant and fits the $\Lambda \mathrm{CDM}$ model at high redshifts better [14]. So, it appears that this discrepancy between measurements of $\mathrm{H}_{0}$ in the late universe (at the time of the CMB emission) and the recent universe is poised to be resolved.

In addition, for this theory, the "Hubble tension" is merely apparent because matter is perceived as such only in the 3D part of the 4D universe and cannot slow down the expansion of the 4D universe (see discussion in [1]). Thus, the expansion of the entire 4D universe cannot neither accelerate nor decelerate in terms of Hubble constant compared to today.
The entire 4D universe expands along the temporal dimension at the constant speed "c," which is a true velocity since it is measured in meters per second. Therefore, the Hubble tension is attributable to the 3D portion alone of the 4D universe.
Finally, it's worth noting that the Hubble constant does not measure a true velocity. Indeed, when converting MPC (Megaparsecs) to kilometres, it is expressed in second ${ }^{-1}$.
For all the reasons listed above, we will employ the cosmological parameter results from the final full-mission Planck measurements, which are equal to $67.4 \pm 0.5 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{MPC}^{-1}$ equivalent to $\sim(2.184 \pm 0.0162) \cdot 10^{-18} s^{-1}[11]$.

## APPENDIX D

## (Isochoric, Isothermal, and Isobaric Transformations for the 4D Universe)

As described in Chapter entitled "General Law of Ideal Gases Applied to the 4D Universe", in the 4D universe, no state variable is constant. The equation that relates all thermodynamic variables between an initial and final state is obtained by dividing equations 44 and 43:

$$
\frac{\Pi_{f} V_{4 f}}{\Pi_{i} V_{4_{i}}}=\frac{n_{f} R_{4 D} T_{f}}{n_{i} R_{4 D} T_{i}}
$$

And simplifying:

$$
\begin{equation*}
\frac{\Pi_{f} V_{4 f}}{\Pi_{i} V_{4_{i}}}=\frac{n_{f} T_{f}}{n_{i} T_{i}} \tag{D.1}
\end{equation*}
$$

The law for isochoric transformations in the 4D universe is obtained from the previous one by setting $V_{4 f}=V_{4 i}$. It is as follows:

$$
\begin{equation*}
\frac{\Pi_{f}}{n_{f} T_{f}}=\frac{\Pi_{i}}{n_{i} T_{i}} \tag{D.2}
\end{equation*}
$$

For isobaric transformation, since $\Pi_{\mathrm{f}}=\Pi_{\mathrm{i}}$, Eq.D. 1 becomes:

$$
\begin{equation*}
\frac{V_{4_{f}}}{n_{f} T_{f}}=\frac{V_{4 i}}{n_{i} T_{i}} \tag{D.3}
\end{equation*}
$$

Finally, for isothermal transformations in the 4D universe, since $T_{f}=$ $\mathrm{T}_{\mathrm{i}}$, we obtain:

$$
\begin{equation*}
\frac{\Pi_{f} V_{4 f}}{n_{f}}=\frac{\Pi_{i} V_{4_{i}}}{n_{i}} \tag{D.4}
\end{equation*}
$$

Where, $\Pi, V_{4}, n$ and $T$, indicate, respectively, 3D Pressure, 4D hypervolume, number of moles, and temperature. $i$ indicate an initial state, while $f$ the final one.

## APPENDIX E

(Derivation of the Entropy Equation for the 4D Universe) In general, the variation of entropy from an initial state $(i)$ to a final state $(f)$ is given by the following integral:

$$
\begin{equation*}
\Delta S=\int_{i}^{f} \frac{d Q}{T} \tag{E.1}
\end{equation*}
$$

Where, S is entropy, Q indicate the heat and T the temperature.
In a reversible adiabatic process, since $d Q=0$, the change in entropy is zero. However, in an irreversible adiabatic process, as the expansion of the universe, according to the third law of thermodynamics, the change in entropy is always greater than zero. In this case, the equation that can be used for an irreversible adiabatic process is the following:

$$
\begin{equation*}
\Delta S_{\text {irrev.adiabatic }}=\Delta S_{\text {rev. isochoric }}+\Delta S_{\text {rev.isothermal }} \tag{E.2}
\end{equation*}
$$

Where "irrev." means irreversible and "rev." reversible.
We therefore determine the two general equations for the variation of entropy in reversible isothermal and isochoric processes in the 4D universe.

## Entropy Variation in a Reversible Isochoric Transformation in the 4D Universe

Recalling the first law of thermodynamics, which states:

$$
\begin{equation*}
d U=d Q-d W \tag{E.3}
\end{equation*}
$$

In an isochoric transformation, we have that $d W=0$, and substituting the internal energy (U) with Eq. 47, we obtain:

$$
\begin{equation*}
d Q=d U=-C_{V} d(n T) \tag{E.4}
\end{equation*}
$$

Since, in this theory, the number of moles, $n$, and the temperature $T$ are both functions of the radius $R$ of the 4D universe (see Eq. 41 and Eq. 46, respectively), and based on what is specified in Appendix A, we can consider deltas and summations instead of differentials and integrals. Therefore, we have:

$$
\begin{equation*}
\Delta S_{\text {isochoric }}=\int_{i}^{f} \frac{\Delta Q}{T}=\frac{\Delta Q}{\Delta T}=-C_{V} \frac{\Delta(n T)}{\Delta T}=-C_{V} \frac{\left(n_{f} T_{f}-n_{i} T_{i}\right)}{\left(T_{f}-T_{i}\right)} \tag{E.5}
\end{equation*}
$$

By substituting Eq. 46 for ( $T$ ), and Eq. 41 for ( n ), we obtain:
$\Delta S_{\text {isochoric }}=-C_{V}\left[\frac{4 R_{f}^{4}}{A l_{p l}^{4}} \frac{A h c}{16 R_{4 D} R_{f}}-\frac{4 R_{i}^{4}}{A l_{p l}^{4}} \frac{A h c}{16 R_{4 D} R_{i}}\right]\left[\frac{16 R_{4 D} R_{f} R_{i}}{A h c\left(R_{i}-R_{f}\right)}\right]=$
$=-C_{V}\left[\frac{h c R_{f}^{3}}{4 l_{p l}^{4}}-\frac{h c R_{i}^{3}}{4 l_{p l}^{4}}\right]\left[\frac{16 R_{f} R_{i}}{\operatorname{Ahc}\left(R_{i}-R_{f}\right)}\right]=$
$=-\frac{C_{V} h c}{4 l_{p l}^{4}}\left(R_{f}^{3}-R_{i}^{3}\right)\left[\frac{16 R_{f} R_{i}}{\operatorname{Ahc}\left(R_{i}-R_{f}\right)}\right]$
And, knowing that $C_{V(4 D)}=4 R_{4 D}$, and further simplifying, we obtain the sought-after equation:

$$
\begin{equation*}
\Delta S_{\text {rev. isochoric }}=\frac{16 R_{4 D} R_{f} R_{i}\left(R_{f}^{3}-R_{i}^{3}\right)}{l_{p l}^{4} A\left(R_{f}-R_{i}\right)} \tag{E.6}
\end{equation*}
$$

## Entropy Variation in a Reversible Isothermal Transformation in the 4D Universe

In this case, we have:

$$
\begin{equation*}
\Delta S_{\text {rev.isothermal }}=\frac{\Delta Q}{T}=\frac{Q_{f}-Q_{i}}{T} \tag{E.7}
\end{equation*}
$$

Furthermore, because $\Delta U=0$, the Eq. E. 3 becomes:

$$
\Delta Q=\Delta W
$$

We substitute Eq. 50 for W:

$$
\Delta Q=\Delta W=\frac{\Delta\left(\Pi V_{4}\right)}{1-\gamma_{4 D}}
$$

And, substituting it into Eq.E.7, we obtain:

$$
\begin{equation*}
\Delta S_{\text {rev.isothermal }}=\frac{Q_{f}-Q_{i}}{T}=\frac{\Pi_{f} V_{4_{f}}-\Pi_{i} V_{4_{i}}}{\left(1-\gamma_{4 D}\right) T} \tag{E.8}
\end{equation*}
$$

Considering the general gas law in the 4 D universe (Eq.42):

$$
\Pi V_{4}=-n R_{4 D} T
$$

Note that the negative sign appears because the pressure during expansion is negative.
Substituting $\Pi V_{4}$ into Eq.E.8, we obtain:

$$
\Delta S_{\text {rev.isothermal }}=\frac{-n_{f} R_{4 D} T-\left(-n_{i} R_{4 D} T\right)}{\left(1-\gamma_{4 D}\right) T}
$$

From which:

$$
\Delta S_{\text {rev.isothermal }}=\frac{R_{4 D}\left(n_{f}-n_{i}\right)}{\left(\gamma_{4 D}-1\right)}
$$

The previous equation can be simplified considering that $n$ is related to the radius of the 4 D universe by Eq. 41 .

$$
\Delta S_{\text {rev.isothermal }}=\frac{R_{4 D}}{\left(\gamma_{4 D}-1\right)}\left(\frac{4 R_{f}^{4}}{A l_{p l}^{4}}-\frac{4 R_{i}^{4}}{A l_{p l}^{4}}\right)
$$

And by factoring out we obtain the following equation:

$$
\Delta S_{\text {rev.isothermal }}=\frac{4 R_{4 D}}{\left(\gamma_{4 D}-1\right) A l_{p l}^{4}}\left(R_{f}^{4}-R_{i}^{4}\right)
$$

And knowing that $\left(\gamma_{4 D}-1\right)=\frac{1}{4}$, see Table 3 , and by factoring out and simplifying, we obtain the sought-after equation:

$$
\Delta S_{\text {rev.isothermal }}=\frac{16 R_{4 D}}{A l_{p l}^{4}}\left(R_{f}^{4}-R_{i}^{4}\right)
$$

Finally, substituting equations E. 6 and E. 9 into Eq.E.2, we obtain the equation that allows us to calculate the variation of entropy in the 4D universe (see Eq.53)

$$
\Delta S_{\text {irrev.adiabatic }}=\Delta S_{4 D}=\frac{16 R_{4 D}}{A l_{p l}^{4}}\left[\frac{R_{f} R_{i}\left(R_{f}^{3}-R_{i}^{3}\right)}{\left(R_{f}-R_{i}\right)}+\left(R_{f}^{4}-R_{i}^{4}\right)\right]
$$

If we consider $R_{i}=l_{p l} \cong 0$, then Eq. 53 simplifies to the following:

$$
\begin{equation*}
\Delta S_{4 D} \cong \frac{16 R_{4 D}}{A l_{p l}^{4}} R_{f}^{4} \tag{E.10}
\end{equation*}
$$

Equation E. 10 highlights that the entropy of the 4D universe increases proportionally to the fourth power of its radius.


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    Received:- 29 March, 2024, Manuscript No. puljmap-24-7010; Editor assigned:- 1 April, 2024, Pre-QC No. puljmap-24-7010 (PQ); Reviewed:- 4 April, 2024, QC No. puljmap-24-7010 (Q); Revised: - 8 April, 2024, Manuscript No puljmap-24-7010 (R); Published: - 20 April 2024, DOI: 10.37532.2023.7.1.1-17

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