RESEARCH

UNE application de la théorie des groupes

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ABSTRACT
This article present a new conjecture that define a method to calculate a sort of integral numeric in using the group theory.

Key words: *Groups; Surjection; Polynomial sum; Logic*

INTRODUCTION

Let G_b a group of order b and $G_b = \{g_0, g_1, \cdot, g_{b-1}\}$ where g_0 is the neutral element of G.

The purpose is to made a surjection, using the group G, between $\mathbb{N}$ and the set of integers {0, 1, 2, . . . , b − 1} and to do a conjecture with it [1].

In this introduction I will use an example to illustrate this surjection with $G = \mathbb{Z}/4\mathbb{Z}$.

Let $n \in \mathbb{N}$. We convert n in base b = 4. So the integer is equal to: $n = \sum_{i=0}^{\alpha} c_i \times b^i$ where any c_i is an element of the set {0, 1, 2, 3} and where:

$$\exists! \alpha, \forall i > \alpha \; c_i = 0$$

We decide now $\forall i <= \alpha$ that all c_i equals to 0 are replaced by g_0 and the 1 replaced by g_1 and so on respectively the 2 replaced by g_2 and the 3 replaced by g_3.

We can name this new value C_i'

$$\forall i C_i' \in G = \mathbb{Z}/4\mathbb{Z}$$

And after we made the surjection f with:

$$\forall_n \in \mathbb{N} \; f(n) = \sum C_i'$$

This sum is made with the + that is the internal law of G.

Remark 1.1.
If the internal law of G is *x*, we can do a product.

And so all *f(n)* are element of G, here in your example $\mathbb{Z}/4\mathbb{Z}$

Finally we associate all the *f(n)* equals g_0 to the integer 0 and so on in the same order for all members of G with an bijection of identification that we name g

In your example of $G = \mathbb{Z}/4\mathbb{Z}$ and all $G = \mathbb{Z}/b\mathbb{Z}$ we can evidently view that:

Lemma 1.2.
If $G = \mathbb{Z}/b\mathbb{Z}$ also

$$\forall n \in \mathbb{N} \; (g \; o \; f)(n) \equiv \sum C_i \mod b$$

Remark 1.3.
We have used $G = \mathbb{Z}/b\mathbb{Z}$ but it's clearly evident that the surjection $g \; o \; f$ can be defined with the same process with any group G (Figure 1,2).

MAIN PROPERTY OF THE SURJECTION

In this section we will proove that there is *b* periode of b^{b-1} elements in the results of the surjection $g \; o \; f$

De N =	à N =	From the law of the Groupe G				
0	b-1	0+0	0+1	0+2	.....	0+b-1
b	2b-1	1+0	1+1	1+2	.....	1+b-1
......						
(b-1)b	b²-1	b-1+0	b-1+1	b-1+2	.....	b-1+b-1
On a donc b² nombre						
De N =	à N =	d'après le table de la loi du groupe				
b²	b^2+b-1	1+0	1+1	1+2	.....	1+b-1
b^2+b	b^2+b+b-1	1+1+0	1+1+1	1+1+2	.....	1+1+b-1
.....						
b^2+(b-1)b	b^2+(b-1)b+(	1+b-1+0	1+b-1+1	1+b-1+2	.....	1+b-1+b-1
.....						
(b-1)b^2+(b-1)b	(b-1)b^2+(b-1	b-1+b-1	b-1+b-1+1	b-1+b-1+2	.....	b-1+b-1+b-1

So we have b³-b² numbers and they are not the same as above

And so on until

De N =	à N =
b^(b-2)	...
.....	.....
.....	.....
	b^(b-1)-1

In this last array b^(b-1)-b^(b-2) numbers

So I thus described the first periodicity that I call 0-period and we have b^(b-1) number in all

Figure 1) *0-periode.*

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The 1-period starts with the second line of the first table above (at the very top) and I describe it below

De N =	à N =	From the law of the Groupe G				
b^(b-1)	b^(b-1)+b-1	1+0	1+1	1+2	.....	1+b-1
b^(b-1)+b	b^(b-1)+b+b-	1+1+0	1+1+1	1+1+2	.....	1+1+b-1
....						
b^(b-1)+(b-1)b	b^(b-1)+(b-1)b	1+b-1+0	1+b-1+1	1+b-1+2	.....	1+b-1+b-1

We have b^2 numbers

Then we have the same type of tables which always start with the second line of the first table of the 1-period and We obtain in all a 1-period of b^(b-1) number

For the i-period the tables start with the ith line of the 1st table of the 0-period.

And we thus obtain b different periodicities

Il reste maintenant à prouver que (b)-période est une (1+0)-période

The (b th)-period starts with b^b and will have the same values as the 1-period

the (j th)-period starts with j*b^b and j can be decomposed in base b and the decomposition of j in base b is the start Of one of the b first periods.

This was to be demonstrated

Figure 2) *Other-periode.*

Theorem 3.1.

If we define the b periods with a group G of order b and if we assign as in the section above to all element of the b periods, one element of the natural element {0, 1, 2, ·, b−1} with the surjection $g\ o\ f$ then ∀ polynom P of degree $< b$ we have this equality

$$\sum_{i=0}^{i=b^b-1} P(i) = b * \sum_j as\ g\ o\ f\ (c_j)\ 0^{P(j)}$$

Proof.

Not yet perfomed but this conjecture do a definition of an integral numeric $\sum_j as\ g\ o\ f\ (c_j)\ 0^{P(j)}$

ANNEXES

This script calculate a sum with all their terms in 43 s or calculate the same with 1 term over 6 in 6 s

```
clear all
format long
% b si votre pc a beaucoup de mémoire vive peut-être augmenté sur le mien
% non
b=6
X=0;
tic
for i=(1:(b^b-1))
X=X+vpi(num2str(i)^5);
end
t=toc
tic
B=(0:(b-1));
indice1=zeros(1,b^2);
k=0;
for i=(0:(b-1))
indice1(i*b+1:i*b+b)=mod(B+k,b);
k=k+1;
end
indicetous=zeros(b,b^2);
indicetous(1,:)=indice1;

for j=(1:(b-1))
k=0;
B2=indice1(j*b+1:j*b+b);
for i=B2
indicetous(j+1,k*b+1:k*b+b)=indice1(i*b+1:i*b+b);
k=k+1;
end
end
for h=(3:b-1)
indicetous=indicetous';
indice=zeros(b,b^(h));
indice(1,:)=reshape(indicetous,1,b^(h));
for j=(1:(b-1))
k=0;
B2=indice1(j*b+1:j*b+b);
for i=B2
indice(j+1,k*b^(h-1)+1:k*b^(h-1)+b^(h-1))=indice(1,i*b^(h
-1)+1:i*b^(h-1)+b^(h-1));
k=k+1;
end
end
clear indicetous;
indicetous=indice;
clear indice
end
ind=zeros(1,b^b);
ind=reshape(indicetous',[],b^b);
indfinal=find(ind==0)-1;
% t=toc
s=0

for i=indfinal
s=s+vpi(num2str(i))^5;
end
s=s*b
X-s
t2=toc
```

REFERENCES

1. Armstrong MA. Groups and Symmetry. Springer-Verlag New York Inc. 1988.